

Tomasz Kowalczyk\* Paweł Ziółkowski, Daniel Sławiński,  
Małgorzata Cisak and Janusz Badur

## A role of the heat and work uncompensated transformations in the balance of entropy and the turbomachinery efficiency

*Energy Conversion Department, Institute of Fluid-Flow Machinery, Polish Academy of Sciences, Fiszerza 14, 80-231 Gdańsk, Poland*

### Abstract

The notion of heat uncompensated transformation has early been introduced by Clausius in 1854 and next, after fifty years of forgetting, in 1904 Duhem has revalorized it and combined it with a new notion of work uncompensated transformation [3]. In this way the so-called Clausius-Duhem inequality has been established. In our paper we wish to present a novelized procedure of estimating the role of the uncompensated transformations of heat and work within the flow of viscous and heat conducting working fluid like water stream. The original procedure was introduced by Puzyrewski and it is essential in expressing of a local, in time and space, balance of entropy. Furthermore, this unique approach leads to redefinition of the efficiency notion, as is usually applied to fluid-flow machineries, to a new one important in computational fluid dynamics (CFD) three-dimensional modeling. As a result, it is shown that usage of the polytropic efficiency, instead of the isentropic efficiency, is more convenient and seems to be natural in CFD approach. Helpfully, we have also found a correlation between those two efficiency definitions with usage of proposed new dimensionless (criterion) number.

**Keywords:** Entropy balance; Entropy sources; Uncompensated transformations; Isentropic efficiency; Polytropic efficiency; Eulerian fluid, Turbine stage, Puzyrewski number

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\*Corresponding author. E-mail address: tomasz.kowalczyk@imp.gda.pl

## Nomenclature

$A$	–	area, m <sup>2</sup>
$\vec{b}$	–	mass force ( $\vec{b} = -g \vec{e}_z$ , $g = 9,81$ ), m/s <sup>2</sup>
$\mathcal{B}$	–	working body with capacity $V$
CFD	–	computational fluid dynamics
$c_p$	–	specific heat capacity at constant pressure, J/kg K
$d\theta$	–	increase of temperature from standard conditions, K
$\overset{\leftrightarrow}{d}$	–	rate-of-strain tensor
$dQ$	–	total heat transfer ( $dQ = \iint_{A_{heat}} \vec{F}_{heat} \cdot \vec{n} dA dt$ ), J
$e = u + \kappa + \psi$	–	specific total energy, J/kg
$\vec{e}$	–	versor
$\vec{F}_{work}$	–	flux of mechanical energy
$\vec{F}_{heat}$	–	flux of thermal energy (commonly designated as: $\vec{q}$ ), W/m <sup>2</sup>
$\vec{g}_\theta$	–	temperature gradient
$\vec{h}$	–	entropy flux, W/K m <sup>2</sup>
$i$	–	medium enthalpy, [kJ/kg]
$\overset{\leftrightarrow}{I}$	–	Gibbs unit tensor ( $\overset{\leftrightarrow}{I} = \delta_{ij} \vec{e}_i \otimes \vec{e}_j$ , where $i, j = x, y, z$ )
$I_d$	–	trace of $\overset{\leftrightarrow}{d}$ ( $I_d = \text{tr } \overset{\leftrightarrow}{d} = d_{ii}$ )
$\dot{m}$	–	real mass flow rate
$N$	–	Clausius total uncompensated heat transformation, [J/K]
$N_{u,CFD}$	–	circumferential power of a stage obtained by CFD approach
$\vec{n}$	–	unit vector normal to the surface $\partial V$
$n_\eta$	–	entropy sources such as losses, dissipation
$n_s$	–	uncompensated heat and work transformation (total entropy production), W/kg K
$n_\mu$	–	uncompensated viscous work transformation (entropy production by mechanical effects), W/kg K
$n_\lambda$	–	uncompensated heat transformation (entropy production by heat transport effects), W/kg K
$p$	–	Pascal-Boyle thermodynamic pressure, N/m <sup>2</sup>
Pu	–	Puzyrewski number
$\overset{\leftrightarrow}{R}$	–	turbulent stress tensor ( $\overset{\leftrightarrow}{R} = R_{ij} \vec{e}_i \otimes \vec{e}_j$ )
$s$	–	specific entropy (state parameter explained in terms of kinetic theory of gases)
$S$	–	Clausius total entropy, J/K
$t$	–	local time referring to the concept of transformation
tr	–	trace
$T$	–	thermodynamic temperature (Kelvin absolute temperature scale)
$\overset{\leftrightarrow}{t}$	–	total momentum flux ( $\overset{\leftrightarrow}{t} = -p \overset{\leftrightarrow}{I} + \overset{\leftrightarrow}{\tau} + \overset{\leftrightarrow}{R}$ )
$u$	–	specific internal energy, J/kg
$u_0$	–	specific internal energy at standard conditions for temperature and pressure, J/kg
$\vec{u}$	–	blade drift velocity vector, m/s,
$\vec{v}$	–	velocity vector, m/s
$v$	–	specific volume ( $v = \rho^{-1}$ ), m <sup>3</sup> /kg

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$\vec{x}$	–	position vector
$x_z$	–	distance in line of gravitation, m
$z$	–	high (position in line of gravitation), $z = x_z$
$\partial V$	–	the contact area of the solid structure with the working medium, $\partial V = A = A_{work} \cup A_{heat}$
$0D$	–	zero-dimensional algebraic model of flow based on integral balances of mass, momentum and energy
$3D$	–	three-dimensional model based on differential equations, which requires complete geometry of a flow channel
$\otimes$	–	dyadic multiplier

### Greek symbols

$\eta$	–	efficiency, Eqs. (26)–(30); specific entropy (the notation of state parameter by Carnot, Rankine, Reech, Gibbs, Duhem), kJ/kg K
$H$	–	Carnot total entropy, Eq. (1)
$\theta$	–	thermodynamic temperature (Rankine temperature scale)
$\kappa$	–	specific kinetic energy, J/kg; Rankine Universe temperature, $\kappa = 2.4K$
$\overset{\leftrightarrow}{\lambda}$	–	tensor of conduction coefficients, $\overset{\leftrightarrow}{\lambda} = \lambda_{ij} \vec{e}_i \otimes \vec{e}_j$
$\lambda$	–	volume bulk viscosity
$\mu$	–	shear (dynamic) viscosity
$\rho$	–	fluid density
$\overset{\leftrightarrow}{\tau}$	–	viscous stress tensor
$\tau$	–	time, the parameter of integration in the cycle
$\psi$	–	specific potential energy, J/kg
$\zeta$	–	losses

### Subscripts and superscripts

$in$	–	inlet
$out$	–	outlet
$p$	–	polytropic
$s$	–	isentropic
$t$	–	theoretical
$u$	–	circumferential
$z$	–	direction in line of gravitation in the Cartesian coordinate system
$0p, 1p, 2p, \dots$	–	polytropic points of process
$0s, 1s, 2s, \dots$	–	isentropic points of process
$0, 1, 2, \dots$	–	real points of process

## 1 Introduction

Proposition of the entropy by Sadi Carnot as a new thermodynamic state parameter, which was originally intended to descriptions of the movement of heat, quickly leads to serious implications in science and technology, and even everyday life. The entropy conservation law in heat engines, proposed by him in 1824, says that [1]:

*“after summarizing all entropy changes that occur during the work cycle, caused by heating and working process, the entropy of the working medium returns to the initial state”*

that is mathematically presented as

$$\oint \left( \frac{d}{dt} H \right) d\tau = \oint \left( \frac{d}{dt} \iiint_V \rho \eta dV \right) d\tau = 0. \quad (1)$$

Further development of this line of thinking led to derivation of a formula of the law of conservation local in time and space. It can be obtained by using Carnot’s suggestion that in an ideal cycle the entropy,  $\eta$ , entropy flux,  $\vec{h}$ , and entropy source,  $n_\eta$ , should be conserved independently. It means that we can summarise  $\eta$ ,  $\vec{h}$  and  $n_\eta$  into one scalar equation

$$\iiint_V \left( \rho \frac{d}{dt} \eta = \operatorname{div} \vec{h} + \rho n_\eta \right) dV \quad (2)$$

should be noted that, in the original Carnot paper the uncompensated heat transformation,  $n_\eta$ , was described only by words such as ‘dissipation’, ‘losses’ during his dissipation on work of real thermal engines.

Soon later, the works of Sadi Carnot led to the improvement of other, competing and even antagonistic lines of development in thermodynamics. A thesis competitive to the law of conservation of entropy in thermodynamic cycles was proposed by Rudolf Clausius (1865), that sounded as follows [2]:

*“Universe entropy is a half-conserved, it could not decrease or be stable, it can only increase”.*

It is the law of half-conservation of entropy. In time interval  $\tau_0$ – $\tau$  Clausius has written it as inequality

$$S - S_0 \leq \int_{\tau_0}^{\tau} \frac{dQ}{T} d\tau + N_{0-\tau}, \quad (3)$$

where  $N_{0-\tau} = \int_{\tau_0}^{\tau} [\iiint_V \rho n_\eta dV] d\tau$ .

In its form this law of half-conservation is integral both in time and space. Nevertheless, from a mathematical point of view, it is identical with the Carnot local law of entropy conservation (2). This equation was adopted in the literature; it constitutes the basis while learning the second law of thermodynamics. In this

inequality, the postulate spoken by Clausius that “*uncompensated transformation of heat is always positive*”, stands out [2]

$$N_{0-\tau} > 0. \quad (4)$$

Despite these fundamental differences in the definition of the law of conservation of entropy Eqs. (1) and (3), thermodynamics has developed itself only at an integral level for many years, because mainstream thinking through the Carnot function (later called thermodynamic efficiency) was used for the design of heat cycle. From the other hand, local (field) approach, now known as 3D, represented by the thermodynamics of viscous fluid, which transfers heat and is subject (field of study) to flows, turbulence, phase, and chemical transformations, basically did not exist. Up to now we have not any expression  $N_{0-\tau}$  in thermodynamics of 3D fields. Hence, the question what is the amount of uncompensated transformation of work  $n_\mu$  and heat  $n_\lambda$  has never arose [3].

The question about the uncompensated transformation of heat was caused by irreversible flow of heat energy, was undertaken by Stokes (1851) and Lamé (1852), who have already dealt with anisotropic heat flow and thermal conductivity coefficients and they discovered that the tensor of thermal conductivity can be asymmetrical [3]. This allows to formulate the Fourier constitutive equation as follows:  $\vec{F}_{heat} = \overset{\leftrightarrow}{\lambda} \vec{g}_\theta$ , where  $\vec{g}_\theta$  denotes temperature gradient. The conduction coefficients tensor  $\lambda$  according to Stokes and Lamé, could be asymmetrical, which means that his mathematical structure consists of nine independent constants. It has happened so until 1931, when Onsager in [4] protested against this wasting of creative forces of nature and enforced a symmetry for the coefficients. That solution was called as a ‘fourth principle of thermodynamics’ and opened the gates for the fifth, sixth, etc., ontologically worthless principles [3]. Józef Wierusz-Kowalski<sup>1</sup> in his paper *The conditions for a constant thermal conductivity should be satisfied* (1890) focused on inequality saying that the heat uncompensated during the conduction process should be consistently positive and, therefore, constants for heat conduction (the tensor of lambda) must satisfy the requirements [5]:

$$\lambda_{11} > 0, \quad \text{and} \quad \lambda_{11}\lambda_{22} - \frac{1}{4}(\lambda_{12} + \lambda_{21})^2 > 0. \quad (5)$$

Further conditions satisfy the quadratic form of the Jacob formula.

<sup>1</sup>He is well known Polish thermodynamics; the initiator of social events where Pierre Curie and Marie Skłodowska met each other.

In 1901 Duhem suggested an inequality for uncompensated transformation of work, which was very similar to those of Wierusz-Kowalski's and was referred to as [6]

$$n_\mu = \overset{\leftrightarrow}{\tau} \cdot \overset{\leftrightarrow}{d} = \left( \lambda I_d \overset{\leftrightarrow}{I} + 2\mu \overset{\leftrightarrow}{d} \right) \cdot \overset{\leftrightarrow}{d} = \lambda I_d^2 + 2\mu d_{ij}d_{ji}, \quad (6)$$

wherein the tensor of viscous stress is determined by two viscosities: volume bulk viscosity,  $\lambda$ , and shear viscosity,  $\mu$ . Jacobi quadratic form, corresponding to the constitutive relations, will be positive if the following two conditions are fulfilled:

$$\mu > 0, \quad \text{and} \quad 3\lambda + 2\mu > 0. \quad (7)$$

The Wierusz-Kowalski inequality and the above-mentioned Duhem inequality are the first practical use of the Clausius laws of half-conservation of entropy in the field-theoretic models.

## 2 The uncompensated transformation of heat and work in the local balance of entropy

Nowadays, constructing high-performance and reliable fluid-flow machinery without using the computational fluid dynamics (CFD) modeling is hard to imagine. CFD and thermal fluid-structure interaction (FSI) have become a major tool to design and estimating stage efficiency [7]. The study of thermodynamic phenomena in terms of local level (three-dimensional model based on differential equations) is especially needed to create ever more accurate models of physical phenomena. As Adrian Bejan wrote in his treatise [8], the identification of local entropy production allows to all over again build further principles of so-called constructional laws leading to the final design.

To determinate equations describing local sources of entropy, in terms of the uncompensated transformation of heat and work within the flow, we shall use as a basis Romuald Puzyrewski's procedure [9, 10]. This procedure is based on the following set of equations describing viscous and heat conductive fluid in the working fluid volume  $V$ :

- balance of entropy

$$\frac{d}{dt} \iiint_V \rho s dV = \iint_{\partial V} \vec{h} \cdot \vec{n} dA + \iiint_V \rho n_s dV,$$

- balance of momentum

$$\frac{d}{dt} \iiint_V \rho \vec{v} dV = \iint_{\partial V} \overset{\leftrightarrow}{t} \vec{n} dA + \iiint_V \rho \vec{b} dV ,$$

- balance of mass

$$\frac{d}{dt} \iiint_V \rho dV = 0$$

- balance of angular momentum:

$$\iiint_V \rho \vec{v} \times \vec{x} dV = \iint_{\partial V} (\overset{\leftrightarrow}{t} \vec{n}) \times \vec{x} dA ,$$

- balance of energy:

$$\frac{d}{dt} \iiint_V \rho(u + \kappa + \psi) dV = \iint_{\partial V} \left( \vec{\mathcal{F}}_{work} + \vec{\mathcal{F}}_{heat} \right) \cdot \vec{n} dA .$$

Using the Reynolds procedure for time differentiation over moving volume and the Green procedure that turns a surface integral into volume integral of the divergence, we attain set of balance equations in a conservative and local form [10, 11]:

- balance of mass:  $\partial_t \rho + \text{div}(\rho \vec{v}) = 0$  ,
- balance of momentum:  $\partial_t(\rho \vec{v}) + \text{div}(\rho \vec{v} \otimes \vec{v}) = \text{div} \overset{\leftrightarrow}{t} + \rho \vec{b}$  ,
- balance of angular momentum:  $\overset{\leftrightarrow}{t} = \overset{\leftrightarrow}{t}^\top$  ,
- balance of entropy:  $\partial_t(\rho s) + \text{div}(\rho s \vec{v}) = \text{div} \vec{h} + \rho n_s$  ,
- balance of energy:  $\partial_t(\rho u + \rho \kappa + \rho \psi) + \text{div}[\rho \vec{v}(u + \kappa + \psi)] = \text{div}(\vec{\mathcal{F}}_{heat} + \vec{\mathcal{F}}_{work})$  .

The equation of the balance moment of momentum will be further used implicitly by taking into account symmetry of the Cauchy stress tensor.

The balance equations are supplemented with the following constitutive equations:

- stress tensor:  $\overset{\leftrightarrow}{t} = -p \overset{\leftrightarrow}{I} + 2\mu \overset{\leftrightarrow}{d} - \frac{2}{3}\mu I_d \overset{\leftrightarrow}{I}$ ,  $I_d = \text{tr} \overset{\leftrightarrow}{d}$ ,
- vector of mass forces:  $\vec{b} = -9.81\vec{e}_z$  (the Earth gravitational field),
- thermal energy flux:  $\vec{\mathcal{F}}_{heat} = \theta\vec{h}$
- mechanical energy flux:  $\vec{\mathcal{F}}_{work} = \overset{\leftrightarrow}{t} \vec{v}$ ,
- Fourier equation for thermal energy flux:  $\vec{\mathcal{F}}_{heat} = \lambda\vec{g}_\theta = \overset{\leftrightarrow}{\lambda} \vec{g}_\theta$ ,
- internal energy equation:  $u = u(v, s)$ , for example  $u = u_0 + c_p d\theta$ ,
- kinetic energy equation:  $\kappa = \frac{1}{2}\vec{v} \cdot \vec{v}$ ,
- gravitational potential energy:  $\psi = zg = \vec{x} \cdot \vec{b} = x_z \vec{e}_z \cdot (-9.81\vec{e}_z) = -9.81x_z$ ;

and following kinetic equations:

- the tensor of deformation rate:  $\overset{\leftrightarrow}{d} = \frac{1}{2}(\text{grad}\vec{v} + \text{grad}^\top\vec{v}) = d_{ij}\vec{e}_i \otimes \vec{e}_j$ ,
- the trace of deformation rate:  $I_d = \text{tr}(\overset{\leftrightarrow}{d}) = \text{div}\vec{v}$ ,
- the temperature gradient:  $\vec{g}_\theta = \text{grad}\theta$ .

A separate equation, with a special status, is the Gibbs equation, which puts together state parameters into one formula

$$\frac{d}{dt}u = \theta\frac{ds}{dt} - p\frac{dv}{dt}, \quad (8)$$

where Gibbs derivative is here interpreted to be the time derivative of the internal energy  $u(v, s)$ . The (material) derivative appearing above leads to serious implications and to use the set of the balance equations in nonconservative form, in which the material derivative has a leading role.

The transition from conservative into nonconservative form involves inserting the material derivative into the balance equations:

- balance of mass:  $\frac{d}{dt}\rho + \rho \text{div}\vec{v} = 0$ ,
- balance of momentum:  $\rho\frac{d}{dt}\vec{v} = \text{div} \overset{\leftrightarrow}{t} + \rho\vec{b}$ ,
- balance of entropy:  $\rho\frac{d}{dt}s = \text{div}\vec{h} + \rho n_s$ ,
- balance of kinetic energy:  $\rho\frac{d}{dt}\kappa = \text{div}(\overset{\leftrightarrow}{t} \vec{v}) - \overset{\leftrightarrow}{t} \cdot \overset{\leftrightarrow}{d} + \rho\vec{b} \cdot \vec{v}$ ,



- balance of potential energy:  $\rho \frac{d}{dt} \psi = -\rho \vec{b} \cdot \vec{v}$ ,
- balance of total energy:  $\rho \frac{d}{dt} (u + \kappa + \psi) = \text{div}(\vec{\mathcal{F}}_{heat} + \vec{\mathcal{F}}_{work})$ ,
- balance of internal energy:  $\rho \frac{d}{dt} u = -\rho \frac{d}{dt} (\kappa + \psi) + \text{div}(\vec{\mathcal{F}}_{heat} + \vec{\mathcal{F}}_{work})$ .

We will introduce additive splitting of the stress tensor  $\overset{\leftrightarrow}{t}$  into the part responsible for the reversible phenomena (e.g., gas compression) and irreversible phenomena (e.g., due to gas viscosity). As we assume that the entire flux of entropy is responsible for entropy transport from place to place, therefore separating the flux of entropy into the reversible and irreversible part does not exist in this approach. So the Cauchy stress tensor has a form of  $\overset{\leftrightarrow}{t} = -p \overset{\leftrightarrow}{I} + \overset{\leftrightarrow}{\tau}$ , which is decomposition to a spherical tensor of thermodynamic pressure and the traceless tensor of viscous stress  $\text{tr} \overset{\leftrightarrow}{\tau} = 0$ . Therefore the balances of momentum and kinetic energy take the form

$$\rho \frac{d}{dt} \vec{v} + \text{grad} p = \text{div} \overset{\leftrightarrow}{\tau} + \rho \vec{b}, \quad (9)$$

$$\rho \frac{d}{dt} \kappa = \text{div}(\overset{\leftrightarrow}{\tau} \cdot \vec{v}) - \overset{\leftrightarrow}{\tau} \cdot \overset{\leftrightarrow}{d} - \text{div}(p \vec{v}) + p \text{div} \vec{v} + \rho \vec{b} \cdot \vec{v} \quad (10)$$

and the balance of internal energy

$$\rho \frac{d}{dt} u = \overset{\leftrightarrow}{\tau} \cdot \overset{\leftrightarrow}{d} + p \frac{d}{dt} v + \text{div}(\vec{\mathcal{F}}_{heat}). \quad (11)$$

Let us note the mathematical identities in equations:

$$\text{div} \vec{h} = \text{div}(\vec{\mathcal{F}}_{heat} \theta^{-1}) = \vec{\mathcal{F}}_{heat} \cdot \text{grad}(\theta^{-1}) + \theta^{-1} \text{div} \vec{\mathcal{F}}_{heat}, \quad (12)$$

$$\text{div} \vec{h} = -\theta^{-2} \vec{\mathcal{F}}_{heat} \cdot \vec{g}_\theta + \theta^{-1} \text{div} \vec{\mathcal{F}}_{heat}. \quad (13)$$

The size of uncompensated viscous work and uncompensated heat is defined now, following Puzyrewski, as

$$n_\mu = \overset{\leftrightarrow}{\tau} \cdot \overset{\leftrightarrow}{d} = \left( -\frac{2}{3} \mu I_d \overset{\leftrightarrow}{I} + 2\mu \overset{\leftrightarrow}{d} \right) \cdot \overset{\leftrightarrow}{d} = -\frac{2}{3} \mu (I_d)^2 + 2\mu \overset{\leftrightarrow}{d} \cdot \overset{\leftrightarrow}{d}, \quad (14)$$

$$n_\lambda = \lambda \vec{g}_\theta \cdot \vec{g}_\theta. \quad (15)$$

The further steps of procedure are simple – it involves substituting into the equation of entropy the individual relationships and to determine irreversibles designated as  $n_s$ . In order to keep the units, we multiply Gibbs equation by density while the entropy equation by temperature:

$$\rho \frac{d}{dt} u = \rho \theta \frac{ds}{dt} - \rho p \frac{dv}{dt}, \quad (16)$$

$$\theta\rho\frac{d}{dt}s = \theta\operatorname{div}\vec{h} + \theta\rho n_s. \quad (17)$$

Now, it is easy to find  $n_s$ :

$$\theta\rho n_s = \theta\rho\frac{d}{dt}s - \theta\operatorname{div}\vec{h} = \rho\frac{d}{dt}u + \rho p\frac{d}{dt} - \theta\operatorname{div}\vec{h}, \quad (18)$$

$$\theta\rho n_s = \overset{\leftrightarrow}{\tau} \cdot \overset{\leftrightarrow}{d} + \operatorname{div}\vec{\mathcal{F}}_{heat} - \theta\operatorname{div}\vec{h}. \quad (19)$$

Then, using expression for the divergence of entropy flux we obtain:

$$\theta\rho n_s = \overset{\leftrightarrow}{\tau} \cdot \overset{\leftrightarrow}{d} + \operatorname{div}\vec{\mathcal{F}}_{heat} - \theta(-\theta^{-2}\vec{\mathcal{F}}_{heat} \cdot \vec{g}_\theta + \theta^{-1}\operatorname{div}\vec{\mathcal{F}}_{heat}). \quad (20)$$

Finally, we read

$$\theta\rho n_s = n_\mu + \theta^{-1}n_\lambda \quad (21)$$

or

$$n_s = n_\mu(\rho\theta)^{-1} + n_\lambda\rho^{-1}\theta^{-2}, \quad (22)$$

which we read:

*Contribution to the entropy production comes from both the uncompensated transformation of work,  $n_\mu$ , as well as uncompensated transformation of heat,  $n_\lambda$ .*

The example of practical calculation proposed by Professor Puzyrewski indicates that in the case of the conductive viscous fluid flows in the adiabatic channel (for example in a turbine stage),  $n_\mu$  plays a dominating role [9, 10]. However, in the heated channels in which convective movement mainly appears (e.g. in the steam boilers), the uncompensated transformation of heat will dominate [12].

### 3 Entropy sources and the efficiency of fluid-flow machinery

Linking the increase of entropy in actual energy transformations, during which heating conversion becomes working, with the efficiency of the fluid-flow machinery is a typical issue. However, the reference point to which obtained actual decrement of enthalpy is compared to, becomes problematic. In design approach, which uses the zero-dimensional (0D) model, it is natural to reference the obtained value of the enthalpy losses to the enthalpy decrease during isentropic process [13–15]. In this case, the enthalpy of working medium at the outlet after

isentropic process,  $i_{out,s}$ , is theoretical enthalpy at the outlet  $i_{out,t}$  (e.g., outlet of one turbine stage or outlet of high pressure group of stages) and depends on the outlet pressure,  $p_{out}$ , and the outlet entropy ( $s_{out} = s_{in}$ ), which equals the entropy of working medium on the inlet,  $s_{in}$  (Fig. 1)

$$i_{out,s} = i_{out,t}(p_{out}, s_{in}). \quad (23)$$

However, the application of isentropic process is excessively obsolete and practically impossible for three-dimensional (3D) modeling using CFD tools. This happens because the commonly used computational codes simultaneously prefer to solve the system of equations for the flow based on discretization of flow area by finite element method (FEM) or finite volume method (FVM) and make it difficult to numerical-idealization of the thermodynamic process [16, 17]. In this case, the entropy at the outlet from the stage is a function of outlet pressure and outlet entropy

$$i_{out,p} = i_{out}(p_{out}, s_{out}). \quad (24)$$

It seems unnatural (eg., in 3D the the fow losses are taken into account, additionally outlet loss connected with the unused kinetic energy of fluid is included) to use isentropic process determined by 0D method in this place. So in order to determine the share of viscous (laminar and turbulent) and thermal conductivity on the production of entropy in the flow, we have to assume as a reference point for nonviscous and non-heat conducting fluid – the Euler model of fluid [17, 18]. Then the production of entropy associated with uncompensated transformation of heat and work (given in Eq. (22) by  $n_s$ ) equals to zero. However, due to the use of real gas model we attain the polytropic process (with the efficiency denoted as  $\eta_{p,CFD}$  and power  $N_{u,CFD}$ ) in opposite to isentropic process (power  $N_{u,tCFD}$ ), which is typical of the ideal gas model. In this process, the increase of entropy will be linked to the nature of transformation while the dissipation of energy will not occur. As a result we have got a reference point, which will be the entropy at the outlet of the stage after polytropic expansion,  $i_{out,p}$ , understood as the entropy after theoretical transformation,  $i_{out,t}$ , depending on the outlet pressure,  $p_{out}$ , and entropy  $s_{out,p}$  after the polytropic transformation for Euler's fluid

$$i_{out,p} = i_{out,t}(p_{out}, s_{out,p}). \quad (25)$$

The graphical interpretation of thermodynamic transformations in the turbine stage stator (pressure drop  $p_0 - p_1$  on stage nozzle) and rotor (pressure drop  $p_1 - p_2$  on stage: rotor wheel), respectively, of the turbine for isentropic, 0-2s, politropic, 0-2p and actual, 0-2, transformation is shown in Fig. 1. In this figure enthalpies:

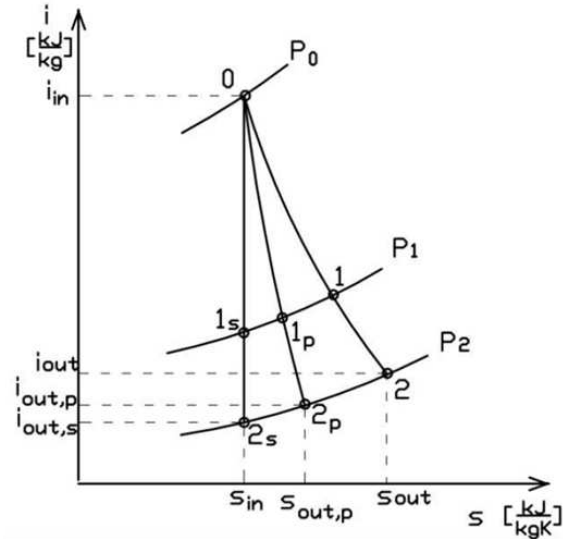


Figure 1: Enthalpy drop and entropy gain in the Molier diagram, where: 0, 1, 2 – real points of process; 1s, 2s– points of isentropic process; 1p, 2p– points of polytropic process.

isentropic  $i_{2,s} = i_{out,s}$ , and polytropic  $i_{2,p} = i_{out,p}$  calculated for a single turbine stage can be found.

For the thermal turbine stages the difference between the enthalpy after isentropic process,  $i_{out,s}$ , and the enthalpy after polytropic process for Euler's fluid,  $i_{out,p}$ , will be insignificant, as evidenced in [16,17]. However the next section will give some information about this issue.

## 4 The relationship between isentropic and polytropic efficiency

To explain further differences between isentropic and polytropic efficiencies, it is profitable to consider this aspect apart from the turbine stage, and look at it in a wider perspective, that is in the case of efficiency in a flow, where no heated and cooled or moving surfaces occur. It should be pointed out that the flow efficiency is connected with the general concept of the ratio of utilization of the energy stored, carried by the unit of mass of a working medium; it describes the relation of real enthalpy,  $i$ , change of the medium to some referential (theoretical) change

of energy,  $e$ , of the medium, which mathematically is proposed by Puzyrewski [10]

$$\eta_e = \frac{\int_{\tau, in}^{\tau, out} i d\tau}{\int_{\tau, in}^{\tau, out} e(s, v) d\tau} = \frac{i_{in} - i_{out}}{\int_{\tau, in}^{\tau, out} e(s, v) d\tau}, \quad (26)$$

where  $e(s, v)$  may be an internal energy, free energy, free enthalpy or any other energy expressing elastic properties of the fluid, and  $\tau$  is the equivalent time of thermodynamic process for example flow in the stage. For this case  $e(s, v)$  is the specific total energy depends from entropy destruction  $s$  and specific volume change  $v$  in the time from start of the process  $\tau_{in}$  to the end of process  $\tau_{out}$ .

Classical isentropic efficiency of a flow, computed in the 0D modeling, employs isentropic drop [13, 14], (subscript:  $e = s$ ):

$$\eta_s = \eta_{s,0D} = \frac{i_{in} - i_{out}}{i_{in} - i_{out,s}}. \quad (27)$$

This definition is practical for the 0D model, since it is related to the ideal state described by enthalpy  $i_{out,s}$  – it is easily interpreted in the Molier chart (Fig. 1.) and does not require the temperature  $T_{out}$  to be known [13, 14]. However, the definition of the isentropic efficiency does not have solid physical fundamentals for 3D approach, since the process  $i_{in} - i_{out,s}$  may not be adopted in the real fluid flow. On the other hand, the polytropic definition also does not require the temperature  $T_{out}$  to be known initially; energy dissipation is calculated in the 3D model solely for the real process  $i_{in} - i_{out}$ , or in the case of Eulerian fluid – for available energy drops to zero – the process becomes polytropic  $i_{in} - i_{out,p}$  [17, 18]. Hence, in the 3D modeling based in CFD methodology, the polytropic efficiency definition is [17]

$$\eta_{p,CFD} = \frac{N_{u,CFD}}{N_{u,tCFD}} = \frac{\iint_{A_u} (-p \vec{I} + \vec{\tau} + R) \vec{n} \cdot \vec{u} dA}{\iint_{A_u} (-p \vec{I}) \vec{n} \cdot \vec{u} dA} \cong \frac{(i_{in} - i_{out}) \cdot \dot{m}}{(i_{in} - i_{out,p}) \cdot \dot{m}_t}, \quad (28)$$

where  $\vec{u}$  is the structure velocity (e.g., blade drift velocity),  $\dot{m}$  and  $\dot{m}_t$  mass flow real rate and in the case of Eulerian fluid, respectively,  $A_u$  – surface of blade. However 0D approach polytropic efficiency  $\eta_{p,0D}$  [10] seems more natural than  $\eta_s = \eta_{s,0D}$  and is defined as proposed by Puzyrewski (subscript:  $e = s$ ) [10]

$$\eta_{p,0D} = \frac{i_{in} - i_{out}}{\int_{\tau, in}^{\tau, out} \frac{1}{\rho} \dot{p} d\tau}, \quad (29)$$

where  $\dot{p}$  is the rate of pressure change that is valid for any channel geometry, both convergent and divergent.

From Eq. (29), the difference between isentropic efficiency,  $\eta_s$ , and polytropic efficiency,  $\eta_{p,0D}$ , lies in a different referential (theoretical) state for the energy drop [10, 19]. In Puzyrewski monograph [10] evaluation of the difference is found:

$$\eta_s = \eta_{p,0D} \left( 1 + \zeta \frac{\bar{T}_{in-out} - \bar{T}_{out,s-out}}{\bar{T}_{out,s} - \bar{T}_{out}} \right). \quad (30)$$

where the over bar denotes averaged temperature of the thermodynamic process. From the above equation it reads that both efficiencies are close to one another by their values, if  $\eta_s = (1 + \text{Pu})\eta_{p,0D}$  [9, 10]; on the other hand, for CFD we write  $\eta_s = (1 - \text{Pu})\eta_{p,CFD}$  [17]. For both cases the Puzyrewski number, Pu, can be written as

$$\text{Pu} = \left( \frac{\bar{T}_{in-out}}{\bar{T}_{out,s-out}} - 1 \right) \zeta. \quad (31)$$

However for ideal situation, it can be assumed that  $\text{Pu} = 0$ . Hence Puzyrewski number says difference between design level ( $\eta_{s,0D}$ ) and real situation with 3D geometry ( $\eta_{p,CFD}$ ). For turbine stages it varies from 0.002 to 0.0001. This means in practice, that the computed polytropic efficiency is by 0.2 to 0.01% higher from the isentropic efficiency. For many cases, it is closely enough to justify assumption  $\eta_{s,0D} \cong \eta_{p,CFD}$  [17, 18].

The relationship between isentropic,  $\eta_s$ , and polytropic efficiency,  $\eta_{p,0D}$ , can be shown graphically, as presented in Fig. 2.

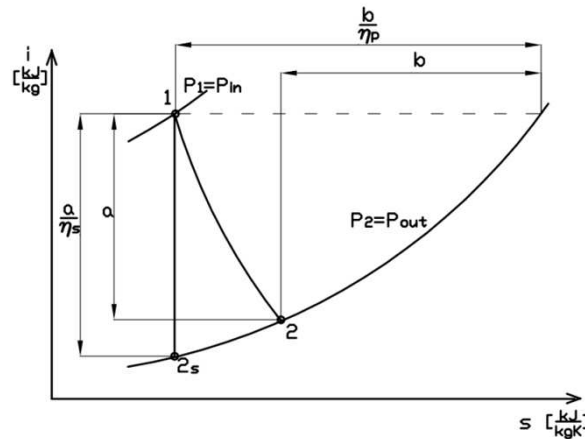


Figure 2: The relationship between isentropic and polytropic efficiency in case 0D for the case of expansion of working medium in turbine [16, 19].

## 5 Conclusions

Basing on all our historical knowledge about the balance of entropy, it should be said that in the phenomenological approach there are two competitive lines of reasoning leading, respectively, to the law of conservation of entropy and the law of half-conservation of entropy. The Clausius law of half-conservation of entropy became the starting point for the statistics and information theory [20]. Hence, in opposition to the entropy concept existing in everyday life understood as parameter measuring the degree of chaos, comes the entropy of Carnot concept describing the highest order of the thermal motion (order parameters: coertia and inertia) used by professionals like Duhem and Natanson [3, 21].

The detailed analysis of the enthalpy sources in the flow rationally justifies the usage of the Euler fluid to analyze the efficiency of energy machines designed and tested using three-dimensional methods. The application of polytropic transformation, instead of isentropic process, using CFD tools is more natural and more comfortable treatment at the same time, because we use the same computational tools to determine the reference point (reference value). In addition, thanks to polytropic transformation the difference in value between these two transformations is insignificant and can also be readily determined by a dimensionless Pu number. New areas to analysis Pu number are flow with a pressure increasing process [22] as new type of turbine inlet like in patents [23-25] with inlet scroll; steam turbine exhaust hoods [26,27]; modernized turbines [28,29]; ORC turbines [30,31]; Tesla microturbines [32]; blowers [33]; compressors [34,35] and also hydraulic turbines [36].

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