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A comparison of fuzzy logic and cluster renewal approaches for heat transfer modeling in a 1296 t/h CFB boiler with low level of flue gas recirculation

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Abstract The interrelation between fuzzy logic and cluster renewal approaches for heat transfer modeling in a circulating fluidized bed (CFB) has been established based on a local furnace data. The furnace data have been measured in a 1296 t/h CFB boiler with low level of flue gas recirculation. In the present study, the bed temperature and suspension density were treated as experimental variables along the furnace height. The measured bed temperature and suspension density were varied in the range of 1131–1156 K and 1.93–6.32 kg/m^3, respectively. Using the heat transfer coefficient for commercial CFB combustor, two empirical heat transfer correlation were developed in terms of important operating parameters including bed temperature and also suspension density. The fuzzy logic results were found to be in good agreement with the corresponding experimental heat transfer data obtained based on cluster renewal approach. The predicted bed-to-wall heat transfer coefficient covered a range of 109–241 W/(m^{2}K) and 111–240 W/(m^{2}K), for fuzzy logic and cluster renewal approach respectively. The divergence in calculated heat flux recovery along the furnace height between fuzzy logic and cluster renewal approach did not exceeded ±2%.

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Keywords: Fuzzy logic; Heat transfer coefficient; Cluster renewal approach; Flue gas recirculation; Circulating fluidized bed

Nomenclature

C_D – drag coefficient
CFB – circulating fluidized bed
c – specific heat, kJ/(kg K)
c_g – specific heat of gas, kJ/(kg K)
c_p – specific heat of bed particle, kJ/(kg K)
D_h – hydraulic diameter, m
d_p – mean bed particle size, mm
E_r – relative error, %
e – emissivity
FGR – flue gas recirculation
f – fractional of wall covered by clusters
G_s – solid circulation flux, kg/(m^{2}s)
g – gravitational acceleration, m/s^2
H – furnace height, m
h – bed-to-wall heat transfer coefficient, W/(m^{2}K)
h_c – cluster heat transfer coefficient, W/(m^{2}K)
h_g – gas convection heat transfer coefficient, W/(m^{2}K)
h_p – particle convection heat transfer coefficient, W/(m^{2}K)
h_w – wall contact heat transfer coefficient, W/(m^{2}K)
k – thermal conductivity, W/(m K)
MCR – maximum continuous rating
n – number of heat transfer data
p – pressure, kPa
Pr – Prandtl number
q – heat flux recovery, W/m^2
Re_p – particle Reynolds number
SH – superheater
T – temperature, K
t_c – cluster residence time, s
U_g – superficial gas velocity, m/s
U_{mf} – minimum fluidization velocity, m/s
U_t – terminal velocity of bed particles, m/s
Y – fraction of particles in the dispersed phase
Z – height above air distributor, m

Greek symbols

Δp – pressure drop, kPa
ΔT – furnace temperature difference, K

Δz – distance between pressure taps, m
δ – wall layer thickness, m
δ_g – gas gap thickness, m
ε – cross-sectional bed average voidage
η – goodness of fit
μ – membership functions
ρ – density, kg/m^3
ρ_b – suspensiondensity, kg/m^3
σ – Stefan Bolzmann's constant, W/(m^{2}K^4)

Superscripts and subscripts

ad – air dried basis
ar – as received
b – bed
c – cluster phase
cra – cluster renewal approach value
d – dispersed phase
fl – fuzzy logic value
g – gas
max – maximum value
mf – minimum fluidization
p – particle
rc – radiation from cluster phase
rd – radiation from dispersed phase
w – wall

1 Introduction

The circulating fluidized bed (CFB) fuels combustion technology has been widely used to heat and power generation in an environmentally acceptable manner. CFB combustors have many advantages in comparison with the conventional pulverized coal combustion boilers such as fuel flexibility (i.e., ability to burn a combination of fuels with varying heating values, ash content and moisture content), low nitrogen oxides emissions due to the lower furnace temperature, direct sulphur dioxide capture *in situ* with a sorbing calcium-based agent ($CaCO_3$), high efficiency (net efficiency near 45%), low operating cost and also high availability. In industrial CFB boilers operated under oxy-fuel combustion conditions to achieve reasonable furnace temperature, the flue gas recirculation is used to control the reaction temperature and heat transfer process between the fluidized bed and active heat transfer surfaces inside the furnace chamber. In addition, recycled flue gases flowing back to furnace chamber allow us to maintain a uniform temperature profiles, both horizontal and vertical profiles along height of the

CFB combustor. Knowledge of heat transfer characteristics is an essential factor for better design, modeling and scale-up of heat exchangers in large-scale circulating fluidized bed reactors. In the recent years, several research works concerning the fundamental analysis of the heat transfer process between the fluidizing medium and water tubes have been carried out both in laboratory and pilot-scale CFB facilities, for example: Chinsuwan *et al.* [1], Nag *et al.* [2], Lockhart *et al.* [3], and Luan [4]. A few information is available on heat transfer data in large-scale circulating fluidized beds in the published literature were focused on: (i) application of artificial neural network approach to predict the bed-to-wall heat transfer coefficient, as discussed by Krzywanski *et al.* [5]; (ii) utilization of cluster renewal approach in detailed heat transfer analysis inside CFB furnace, Basu *et al.* [6], Blaszczuk *et al.* [7–9], and Dutta *et al.* [10,11]; (iii) heat transfer experimental methods, as described by Andersson *et al.* [12]; and also (iv) heat transfer performance in CFB boilers with different configurations and loads, for instance in Wedermann *et al.* [13] and Cheng *et al.* [14]. To authors knowledge, there are no literature data about the heat absorption from bed particles to the vertical membrane walls during flue gas recirculation into CFB furnaces in large-scale. Besides, information about the evaluation of heat transfer inside CFB facilities using a fuzzy logic has not been reported. To overcome this gap in research knowledge related to heat transfer study in CFB boilers, the authors examine this issue, which is very important for high performance of the circulating fluidized bed combustor and also in many engineering applications such as drying, coating and granulating. Until now, application of fuzzy logic systems in CFB boilers were focused on: (i) advanced control methods for reducing nitrogen oxides, as discussed by Lentunen [15]; (ii) main steam pressure control, for instance in: Karppanen [16] and Yin *et al.* [17]; (iii) compensation of fuel quality fluctuation, as described by Karppanen [16]; (iv) fuel feed optimization, Karppanen [16] and Jaronen *et al.* [18]; (v) O_2/CO optimization, as discussed by Jaronen *et al.* [18]; (vi) the bed temperature control, for example in Jaronen *et al.* [18] and Yang *et al.* [19]; and also (vi) the bed pressure control, as described by Li *et al.* [20]. The first paper, which introduced a way of predicting the local heat transfer coefficient in the combustion chamber of a circulating fluidized bed combustor by the fuzzy logic approach was proposed by Krzywanski and Nowak [21].

Several mechanistic models have been proposed in literature to describe the bed-to-wall heat transfer coefficient in circulating fluidized beds. These models can be classified broadly as single particle models, cluster renewal models, and also continuous film models. Basu *et al.* [22] and Glicksman [23] have presented comprehensive reviews on heat transfer studies in CFB units. In a single particle models, heat transfer process is considered at the particle level. The primary concern of the single-particle models is to treat the first layer of particles adjacent to the active heat transfer surface. The single-particle model proposed by Di Natale *et al.* [24] considers the average surface void fraction as the only regression parameter for description of the effect of pressure, temperature, particle diameter, and also solid/gas physical properties in a wide range of experimental conditions. An alternative model in order to explain the heat transfer behaviour of circulating fluidized bed is the cluster renewal approach. Many authors (Gupta *et al.* [25], Dutta *et al.* [11], Chen *et al.* [26]) proposed the mechanistic model based on cluster renewal approach to predict the bed-to-wall heat transfer coefficient in CFB furnace. Major feature of the cluster renewal model is the assumption that clusters formed from bed particles downward travel a certain distance and then dissolve or detach themselves from the heat transfer surface. A description of cluster renewal model is discussed in Sec. 2 in detail. In further heat transfer investigations in CFB facilities, Karimipour *et al.* [27] developed the concept of temperature penetration depth to evaluate heat transfer behaviour near the wall of gas-solid fluidized beds, according to the cluster renewal approach. Unlike the cluster renewal approach, the continuous film model assumes that the walls of CFB furnace are always covered by a homogeneous film of gas and solids. Downward movement of solids take place through an annulus region. Any part of the wall does not come in contact with the up-flowing gas jointly with the dispersed phase. Chen *et al.* [28], Leckner [29] and Mahalingam *et al.* [30, 31] have used the continuous film model to calculate the heat transfer coefficient.

In the present work, the main aim is to evaluate the heat transfer in a large-scale CFB boiler with thermal capacity 966 MW_{th}. Experimental research is needed to elucidate the heat transfer characteristic during the recirculation of flue gases back to the furnace through star-up burners. The fuzzy logic and cluster renewal approach were used to predict bed-to-wall heat transfer coefficient. This work provides a correlation between heat transfer coefficient and bed temperature whether or solid suspension density. The heat transfer data obtained in identical operative conditions for fuzzy logic and cluster renewal approach afterwards were compared with each other. These data are useful in the design and scale-up of heat exchangers operated in a fast fluidization flow regime within the combustion chamber. This work would stimulate discussion and development of these approaches for heat transfer modeling in large-scale CFB combustors. So far, all heat transfer data were obtained in a CFB furnace with small cross-sectional area and low thermal capacity (> 900 MW$_{th}$). Thus, this work addresses an existing gap in the CFB literature data. In addition, the reported heat transfer data for an industrial scale unit will be useful in the assessment of the impact of changes in the operating variables on heat transfer conditions in a CFB furnace.

2 Heat transfer in CFB furnace chamber

Heat transfer in the circulating fluidized bed is a complex process due to the intensity of physical and chemical processes [32], which occur inside furnace chamber as indicated in Fig. 1 [33]. Heat transfer from fluidized bed to active heat transfer surface depends upon hydrodynamics regions in CFB combustor. In CFB furnace, core-annulus flow structure comes under the fast fluidization regime with group B particles of the Geldart classification. According to the definition proposed by Basu and Fraser [34], the fast fluidization is a regime of up-flowing gas-solid suspension with a high degree of refluxing of bed particles. From a macroscopic view point the fast fluidization consists of two different regions along the furnace height (Fig. 1). The core is a central region along the height of the furnace chamber with a low bed particles concentration or clusters presence in which solid particles are entrained by a velocity higher than the superficial gas velocity. The annulus region or thermal boundary layer is much denser and smaller in the vicinity of the vertical heat transfer surface in which bed particles congregate and fall down as mixture of the clusters and the relatively dilute suspension. The stream of particles in the form of clusters (or strands) is responsible for majority of bed particle traverse a distance from the core region to annulus with heat transfer to the vertical membrane wall. Due to the mixing of fluidized solids at near the wall, the bed particles/clusters contribute in the enhancement of heat transfer to the furnace wall. The annulus is replenished by the high temperature bed particles and by the descending burning fuel particles, which they come from the core region. The published information on cluster formation and properties in the CFB literature is reported by Blaszczuk *et al.* [8] and will not be repeated in this work. The extent of each region depends on the bed operating conditions

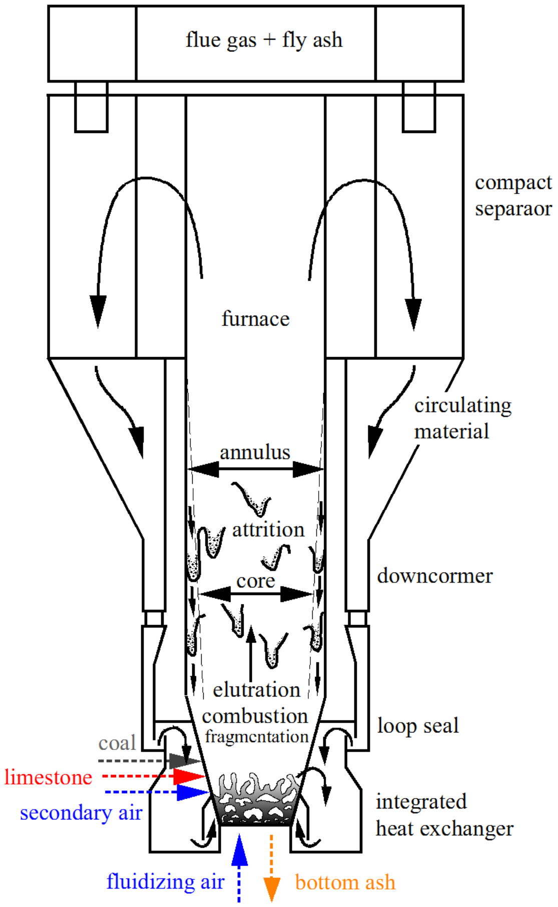


Figure 1: Schematic of the flow pattern inside a CFB furnace with the indication of certain basic processes [34].

such as the suspension density, superficial gas velocity, and also the solid circulation rate. Details and information on core-annulus flow structure of CFB reactors is discussed by Johansson *et al.* [35], Zhang *et al.* [36], Harris *et al.* [37], Horio [38], and Blaszczuk *et al.* [39].

Thermal energy generated by the burning fossil fuel is absorbed by the bed particles and gas. In CFB combustors, heat is mainly transferred from the core to the membrane wall by several dissimilar mechanisms, which can be expressed as

$$h = f h_p + (1-f)\, h_g + f h_{rc} + (1-f)\, h_{rd}\ , \tag{1}$$

where h denotes the bed-to-wall heat transfer coefficient, the subscripts p, g, rc, and rd represent particle convection, gas convection, radiation from clusters and radiation from dispersed phase, respectively. In this heat transfer study, dispersed phase is defined as the mixture of gas and a dilute gas-particle suspension. The parameter f is a fraction of time during which any point on the wall is covered by clusters and is calculated by hydraulic diameter and furnace height of the CFB boiler as follows [13]:

$$f = 1 - \exp\left[4300\,(1-\varepsilon)^{1.39}\left(\frac{D_h}{H}\right)^{0.22}\right] , \tag{2}$$

where ε is the cross-sectional bed average voidage. The values of coefficients in Eq. (2) were obtained on the basis of heat transfer data from four large-scale CFB boilers with a different equivalent diameter D_h: 1.6 m (12 MW_{th}), 5.2 m (20 MW_e), 6.2 m (145 MW_{th}), and 10.6 m (170 MW_e) having heights, h, above the secondary air injection of 11.5 m, 25 m, 26 m and 30 m.

In this case the heat transfer study, mechanistic model based on cluster renewal approach has been proposed to determinate individual components of the overall heat transfer coefficient. A brief description of all components of the bed-to-wall heat transfer coefficient, including both convective and radiative heat transfer modes, is given below.

2.1 Particle convection component

Particle convection coefficient is much greater than the gas convection coefficient for small bed particles operating at near atmospheric pressure and at relatively low furnace temperatures due to the large heat capacity of solids. The particle convection heat transfer coefficient contains heat conduction into the semi-infinite cluster and the conduction through the thin gas layer residing between the cluster and the furnace wall. Thus, heat transfer by particle convection refers to the energy transfer due to continuous bed particle movement in the lateral direction between the core region and the annulus region (thermal boundary layer). Therefore, the particle convection component can be calculated in this heat transfer model based on cluster renewal approach as follows [41]:

$$h_p = \frac{1}{\left(\frac{1}{h_c}+\frac{1}{h_w}\right)} = \frac{1}{\left(\frac{t_c\pi}{4k_c\rho_c c_c}\right)^{0.5} + \left(\frac{d_p\delta}{k_g}\right)} , \tag{3}$$

where h_c is the cluster heat transfer coefficient, h_w means the wall contact heat transfer, d_p denotes the mean bed particle size, and k_c represents the thermal conductivity of the cluster. In Eq. (3), the thermal conductivity of the cluster can be taken from a graph given by Baskakov [41] or can be estimated from the expression given by Gelperin and Einstein [42]. In order to completely evaluate the particle convection heat transfer using the above equation it is necessary to find suitable values of physical parameters for the cluster and gas phase such as: cluster residence time, t_c, density of cluster, ρ_c, specific heat of cluster, c_c, thermal conductivity of gas, k_g and thickness of gas layer, δ_g. Correlations for these physical and thermal properties for each medium phase (i.e., the cluster and the gas) in the vicinity of the furnace wall are given by Blaszczuk *et al.* [8].

2.2 Gas convection component

As explained earlier, the bed-to-wall heat transfer coefficient within a combustion chamber of a CFB boiler contains the gas convection component as one of the three heat transfer mechanisms towards the membrane walls. Under operating conditions of the CFB furnace at large Archimedes numbers or low solids concentration, heat transfer by gas convection is a significant component of the overall heat transfer coefficient. Heat transfer by gas convection refers to the energy transport across the annulus to the active heat transfer surface whereas the vertical furnace wall is uncovered by clusters, but the water membrane wall is contacted by gas or a very dilute particle-gas mixture. The gas convection heat transfer coefficient to furnace walls is estimated by the modified equation of Wen and Miller [43] for the dust-laden gas, which was given in [44]:

$$h_g = \frac{k_g c_p}{d_p c_g} \left(\frac{\rho_d}{\rho_p}\right)^{0.3} \left(\frac{U_t^2}{g d_p}\right)^{0.21} \mathrm{Pr} \,, \tag{4}$$

where ρ_p denotes particle density, g represents the gravitational acceleration due to gravity, Pr means Prandtl number, c_g and c_p are specific heat of gas and specific heat of particle, respectively. Other denotations in Eq. (4) are explained in the Eq. (3). It is important to note that the thermal conductivity of gas, k_g, should be evaluated at the mean gas-film temperature, as suggested by Basu and Nag [22]. The term U_t indicates terminal velocity of bed particles, and it is given in this work as follows:

$$U_t = \left(\frac{4 d_p g \left(\rho_p - \rho_g \right)}{3 \rho_g C_D} \right)^{0.5} , \tag{5}$$

where C_D is the drag coefficient and it depends on the flow regime [44]. When calculating the drag coefficient, the intermediate law is used at the particle Reynolds number covering the range of 0.4–500.

In the present heat transfer investigation, to calculate the dispersed phase density, ρ_d, the correlation

$$\rho_d = \rho_p Y + \rho_g \left(1 - Y \right) . \tag{6}$$

is used at assumption that solid fraction, Y, is not equal to zero. The value of volumetric concentration of particles in the dispersed phase is recommended as 0.001% [46].

2.3 Radiation from clusters

In order to completely calculate of cluster radiation coefficient, the cluster phase is assumed to constitute a continuous absorbing, emitting and scattering medium. The thermal radiation between the cluster and the heat absorbing surface can be calculated similar to two parallel planes, where gray body expression may be used:

$$h_{rc} = \frac{\sigma \left(T_c^4 - T_w^4 \right)}{\left(T_c - T_w \right) \left(\frac{1}{e_w} + \frac{1}{e_c} - 1 \right)} , \tag{7}$$

where, σ means the Stefan-Boltzmann constant, T_c and T_w denote the cluster and wall temperatures, respectively, e_c and e_w are emissivities of the cluster and the wall, respectively. The cluster temperature can be estimated from the local temperatures expression proposed by Golriz [46] or the relationship given by Borodulya and Teplitsky [47]. In the current heat transfer study, the furnace wall emissivity is the property of the surface and is assumed to have a constant value 0.8, as for oxidized steel. The emissivity of the cluster is calculated by the following relation proposed by Grace [48], which takes into account multiple reflection of the bed particles surface:

$$e_c = 0.5 \left(1 + e_p \right) , \tag{8}$$

where e_p is the emissivity of bed particle surface and is equal to 0.7 in the present heat transfer study. As can be seen from the above relation, the cluster emissivity ($e_c = 0.85$) is larger than the particle emissivity ($e_p = 0.7$).

2.4 Radiation from dispersed phase

At the high furnace temperature ($T > 973$ K) and low bed density ($\rho_b < 30$ kg/m^3) [49], radiation mode from the dispersed phase is dominant in the heat transfer mechanisms between the fluidized bed and the furnace wall. It is evident from earlier heat transfer studies in a large-scale CFB unit, as confirmed by Blaszczuk *et al.* [8,39]. The thermal radiation between the dispersed phase and the wall is considered as that of two parallel planes similar to the cluster and the wall. For the portion of the vertical membrane wall uncovered by clusters the dispersed phase radiation heat transfer coefficient can be written as given by Gorliz and Sunden [50], in the form

$$h_{rd} = \frac{\sigma\left(T_b^4 - T_w^4\right)}{\left(T_b - T_w\right)\left(\frac{1}{e_w} + \frac{1}{e_d} - 1\right)} \,, \tag{9}$$

where T_b denotes the bed temperature, e_d represents the emissivity of the dispersed phase, other denotations are explained in Eq. (7).

In the case of a large-scale CFB boiler, the particle scattering has significant effect on the radiation from the dispersed phase. Thus, for calculation of the effective dispersed phase emissivity in a large circulating fluidized bed boiler, the Brewster correlation with consideration of scattering can be used, as follows [51]:

$$e_d = \sqrt{\frac{e_p}{\left(1 - e_p\right)B}\left[\frac{e_p}{\left(1 - e_p\right)B} + 2\right]} - \frac{e_p}{\left(1 - e_p\right)B} \,, \tag{10}$$

where parameter B may be taken as equal to 0.5 for isotropic scattering.

3 Description of fuzzy logic

The designing and scale-up of heat exchangers is quite complicated, as it needs correct analysis of heat transfer rate apart from issues such as long-term performance and economic aspect. The major challenge in designing and scaling-up a heat exchanger is to make the heat transfer surface allowing for control and maintaining the optimum furnace temperature for reduction of air pollutions from fossil fuel combustion processes. Thus, there is a need for the increase in the efficiency of the heat exchanger, through an augmentation of heat transfer with simultaneously a considerably saving in the energy generation costs. Fuzzy logic is a method which can be helpful to reduce its costs and also allows us to predict experimental results for tests, which have not been carried out or are not possible to conduct due to some restrictions such as commercial reason, labour consumption of tests or financial costs.

In the current heat transfer study, fuzzy logic was proposed to predict the local heat transfer coefficient between the vertical membrane wall and the fluidized bed inside a 1296 t/h CFB boiler in a large-scale. In order to perform fuzzy logic modeling, the commercial and closed-source application for the FuzyLite Libraries QtFuzzyLite version 5 [52] software was used (grafical user interface). The aim of this fuzzy logic is to consider the effect of three main factors of heat transfer. The fuzzy logic modeling is divided into four subfunctions, as described by Krzywanski *et al.* [21] and Gopal [53], namely (i) fuzzyfication, (ii) fuzzy rule-base, (iii) inference engine, and also (iv) defuzzyfication. The parts of the fuzzy system are presented in Fig. 2.

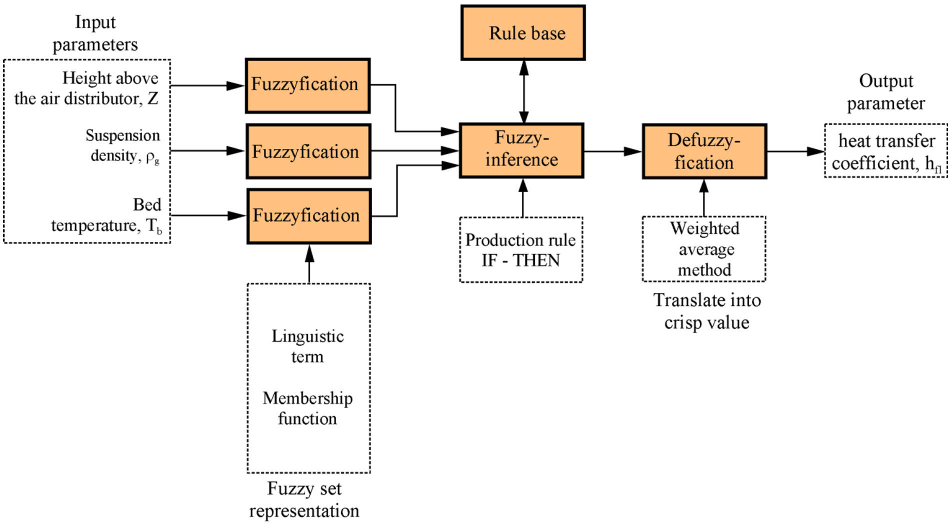
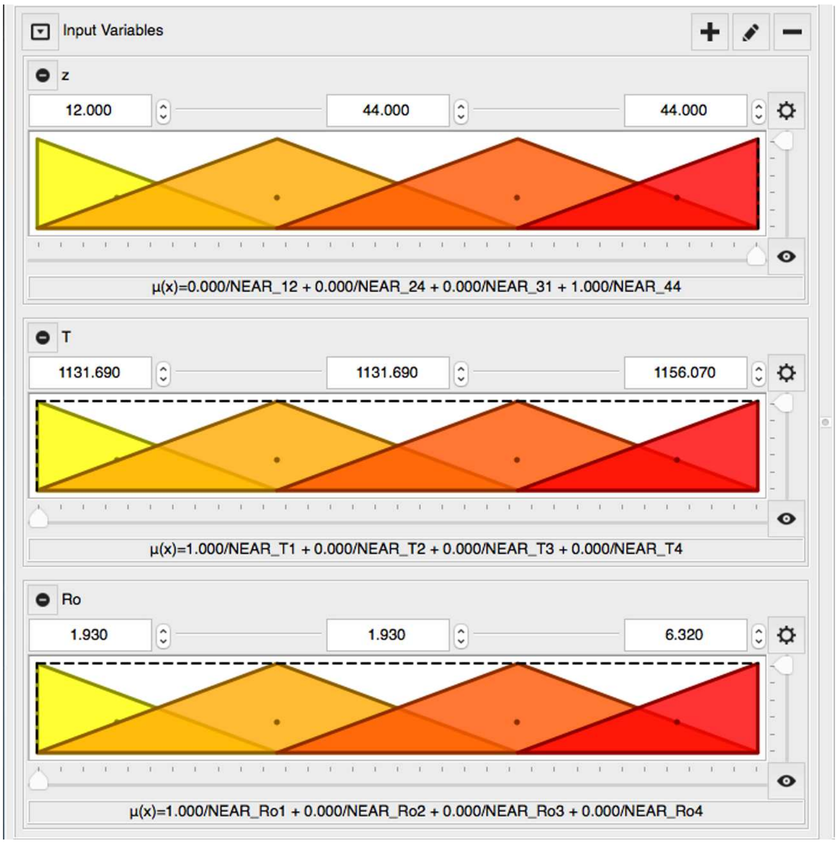


Figure 2: A fuzzy system structure used in this heat transfer study.

The first step in a fuzzy system is selection and definition of appropriate input and output parameters and also their levels. Nondimensional distance Z/H from the air distributor in 4 levels ranging from 0.25 to 0.87 ($12 < Z < 48$ m), suspension density from 1.93 to 6.32 kg/m^3, bed temperature from 1131 to 1156 K as input variables and overall heat transfer coefficient, h, as an output variable were chosen. The value of the output parameter was found by using the QtFuzzyLite software. The fuzzyfication is the next step whereas the three real-valued variables (Z, ρ_b, T_b) are classified to fuzzy sets. Each fuzzy set is characterized by suitable membership functions. In the fuzzy set theory, the most commonly the triangular-shaped membership function is used, because it is the most economic one, as suggested by Driankov *et al.* [54]. Figure 3 shows symmetric membership functions for input variables used in this work. Similarly, Fig. 4 depicts membership functions for the output parameter. The membership function of each of the considered parameter varies in the range from 0 to 1. The set membership function is a continuous real value between 0 or 1, describing the degree to which the element fulfils the measures of a full membership. The value 0 represents the worst status while the value 1 represents the absolute optimum. In the fuzzyfication process, the fuzzy variable values (i.e., three input and one output fuzzy sets) on the horizontal axis are mapped onto the membership value on the vertical axis. For fuzzyfication of these variables the linguistic term such as: very low (VL), low (L), high (H), and very high (VH) are used for input parameters and to describe the output parameter h. Similarly, five linguistic terms, i.e.: very low (VL), low (L), high (H), and very high (VH) are used for the overall heat transfer coefficient.

In the second step the fuzzy rule inference with some sets of fuzzy logic operators and production rules are defined. For the three inputs and one output, a fuzzy associated memory or decision is formed as regulation rules by using *If – Then* rules. A fuzzy rule base comprises a set of fuzzy rules, where each rule may have multiple inputs and one output variable. A total of 16 fuzzy rules are developed in the same way. The fuzzy rule base is shown in Tab. 1. Here, all the values of linguistic variables are fuzzy number i.e. [0, 1], not the exact process value.

The exact numerical crisp value of the output heat transfer coefficient for the given inputs is calculated using the defuzzyfication techniques, which is the last stage of fuzzy modeling. The value of output heat transfer coefficient is calculated from the weighted average defuzzyfication technique,



12.000 44.000 44.000

μ(x)=0.000/NEAR_12 + 0.000/NEAR_24 + 0.000/NEAR_31 + 1.000/NEAR_44

1131.690 1131.690 1156.070

μ(x)=1.000/NEAR_T1 + 0.000/NEAR_T2 + 0.000/NEAR_T3 + 0.000/NEAR_T4

1.930 1.930 6.320

μ(x)=1.000/NEAR_Ro1 + 0.000/NEAR_Ro2 + 0.000/NEAR_Ro3 + 0.000/NEAR_Ro4

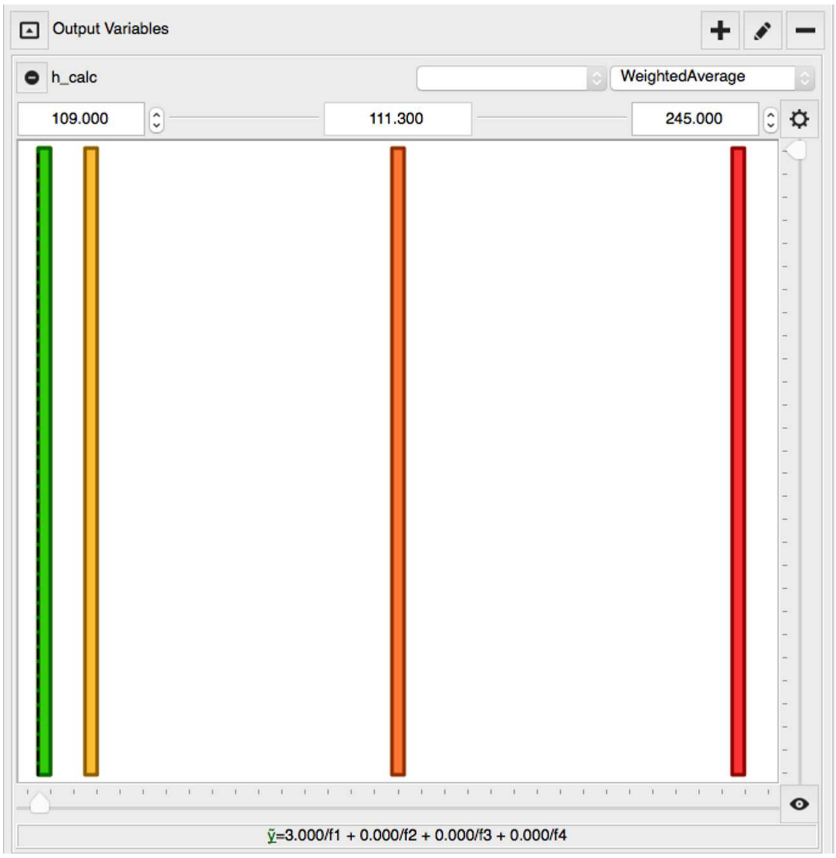
Figure 3: Symetric triangular-shaped membership functions for input parameters.

Table 1: Fuzzy rule base for input data and output parameter.

Input data	Z, m	VL	L	H	VH
	ρ_b, kg/m^3	VH	H	L	VL
	T_b, K	VH	H	L	VL
Output parameter	h_{fl}, W/(m^{2}K)	VH	H	L	VL

VL – very low, L – low, H – high, VH – very high

which is mostly used as an efficient method in the fuzzy logic system, as suggested by Ross [55]. Thus, the one crisp output is calculated by using



ỹ=3.000/f1 + 0.000/f2 + 0.000/f3 + 0.000/f4

Figure 4: Membership functions for the output parameter.

the following relation:

$$h_{fl} = \frac{\sum b_i \mu_i}{\sum \mu_i} \,, \tag{11}$$

where b represents the position of a single tone and μ denotes membership functions. The subscripts i and fl mean an input variable number and fuzzy logic value, respectively. In this analysis, the predicted bed-to-wall heat transfer coefficient has been obtained for the parameter i equal to 3 since three input parameters are considered during the study.

4 Input data to heat transfer modeling

In order to perform heat transfer modeling by means of fuzzy logic or cluster renewal approach, a performance test was carried out in a 1296 t/h CFB boiler under a low level of flue gas recirculation. During the test, the volumetric flow of flue gas recirculation into the furnace chamber was kept at the level of 20.5 $\mathrm{m}_n^3/\mathrm{s}$. The analyzed CFB boiler is described in detail in the works by Blaszczuk *et al.* [7,8,31,38,56], particularly presenting the construction data, cross-sectional area, geometry of membrane wall, arrangement of measuring points and also data acquisition system. Besides, more detailed information on the flue gas recirculation system, flue gas analysis system and properties of such flue gases fed to the combustion chamber are described by Blaszczuk [57]. Before test, the experimental conditions referred to this heat transfer study were stabilized for four hours. The detailed experimental data for heat transfer modeling are given in Tab. 2.

Table 2: Experimental conditions referred to this heat transfer study.

Parameter	Unit	Value
Superficial gas velocity, U_g	m/s	4.27
Terminal velocity, U_t	m/s	1.99
Minimum fluidization velocity, U_{mf}	m/s	0.01643
Solid circulation flux, G_s	$\mathrm{kg/(m^2s)}$	23.7
Particle Reynolds number, Re_p	–	7.57
Particle density, ρ_p	$\mathrm{kg/m^3}$	2700
Sauter mean particle diameter, d_p	mm	0.246
Bed pressure, p_b	kPa	7.7
Bed temperature, T_b	K	1126

The test lasted eight hours in order to establish the repeatability of furnace data. For heat transfer characteristics inside the furnace chamber of the CFB combustor, the performance test was conducted jointly by Czestochowa University of Technology and Lagisza Power Plant, including some measurements of the bed temperature and static pressure difference between the two ports within the combustion chamber. The performance test was conducted with a maximum-continuous-rating (MCR) load of 80% and it was necessary to validate the fuzzy logic expert system.

Furnace data used in this heat transfer investigation are given in a dimensionless scale. The metal temperature varied between 680 K and 730 K. All local measurement results are presented as 30s averages. Both bed temperature and solid suspension density were normalized by a maximum value obtained during the experimental work. The local furnace data are drawn as individual data points, for bed temperature and suspension density, respectively. In Figs. 5 and 6, the solid lines represent approximation of the experimental data by means of a cubic B-spline method using the Origin Pro software in version 8 [58].

Figure 5 depicts the bed temperature profile in the core region along furnace height of the CFB boiler in a large-scale. While the furnace height is higher than 3 m, the bed temperature is kept almost unchanged, with the exception of experimental data at the bottom region above the air distributor level ($0.25 < Z \leq 3.0$ m) and also at the furnace height of 31–48 m. The bed temperature in the core region reaches the maximum at the furnace height of 12 m. This fact is due to the combustion process of fuel particles in this part of the furnace chamber. The experimental data on the bed temperature in Fig. 5 indicate that the temperature difference within the combustion chamber equals 40 K. At the furnace height of 31–48 m the bed temperature profile in the core region had a nonlinear character. The bed temperature in the upper part of the transport zone diminishes monotonously due to the localization of radiant superheaters (SH) II inside a CFB furnace. An additional active heat transfer surface in the form of SH II has a length of 20 m from the roof towards the air distributor. Whereas, the discrepancy of the bed temperature below flue gas recirculation level can be explained by local hydrodynamic conditions in the bottom region of the slope section. Thereby, temperature gradients have a speedy increase in the lower part of furnace, for instance 335 K/m on average.

Figure 6 shows a vertical profile of the solid suspension density in the furnace chamber at the superficial gas velocity of 4.27 m/s. In our heat transfer study, solid suspension density was calculated by the following empirical correlation:

$$\rho_b = -\frac{1}{g}\frac{\Delta p}{\Delta z} , \tag{12}$$

where Δp represents the pressure drop and Δz denotes the distance between pressure taps. Attrition and gas-particle acceleration effects were neglected in Eq. (12). The extremely high suspension densities recorded

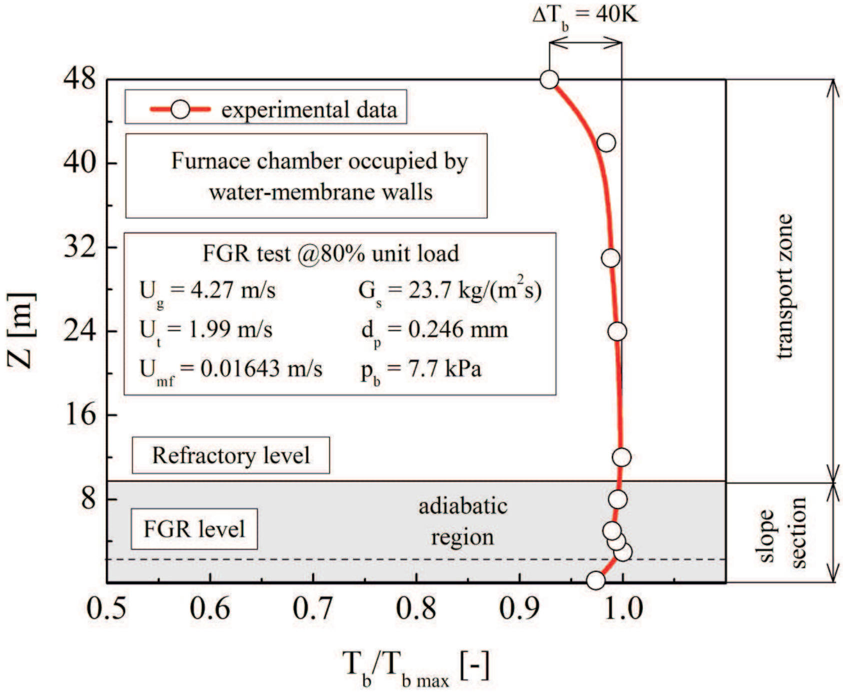


Figure 5: Normalized bed temperature as a function of the distance from the air distributor.

below the flue gas injection level near the fluidization grid are significantly affected by the bed particle acceleration as well as by the solids (fuel, sorbent, circulating material) entry configuration. Above this region of the combustion chamber, the suspension density decreases exponentially and approached the lowest value at furnace height of 42 m. Our experimental data on the suspension density clearly show a decrease in pressure gradients with the increasing furnace height.

One longitudinal profile of suspension density follows an L-shaped profile, with a dense phase at the bottom and a dilute phase at the entire region of the combustion chamber. The typical profile of the suspension density is S-shaped, which is generally the case of CFB systems. The different profiles of suspension density reported in the CFB literature likely resulted from an essential impact of operating conditions on the gas-solid flow structure inside the CFB combustor. However, the shape of the suspension density profile in Fig. 6 depends upon the ratio of primary to secondary air

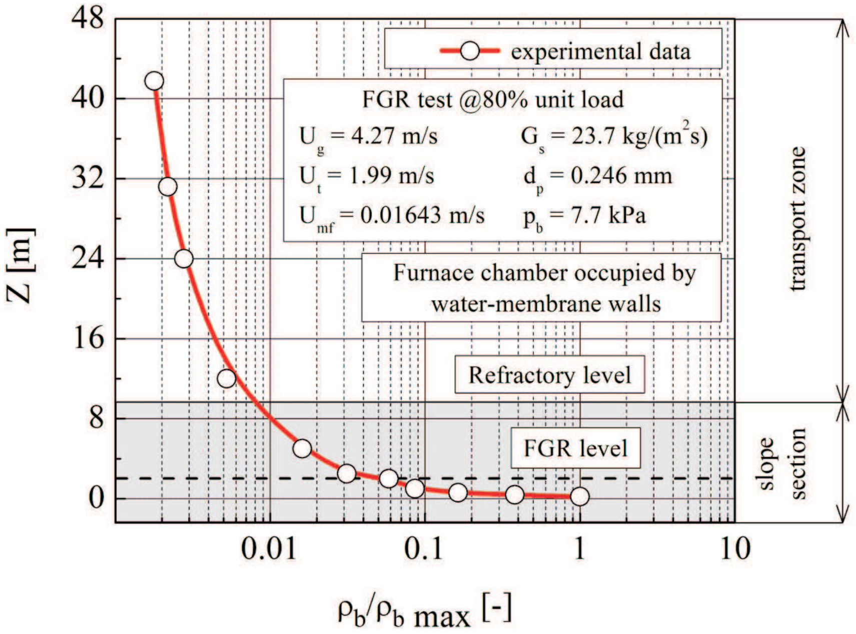


Figure 6: Normalized suspension density variation with furnace height of the CFB boiler.

PA/*SA*. During the FGR test, the parameter *PA*/*SA* was kept at high level (i.e. $PA/SA = 0.67$). Hence, the bed particles from the dense slope section were transported to the top section with the dilute lean phase, because the superficial gas velocity was high enough ($U_g = 4.27$ m/s). The solid suspension density profile confirmed that above the secondary air injection levels bed particles transport occurred without a splash zone. This trend was also confirmed by Blaszczuk *et al.* [7] and Błaszczuk [57] in several previous studies of solids flow structure.

5 Results and discussion

The evaluation of heat transfer in a 1296 t/h CFB boiler was performed in the transport zone of the furnace chamber, where main heat transfer surfaces are located in the form of water-membrane walls. In the current heat transfer study, the transport zone is defined as a part of furnace chamber above the slope section (i.e., adiabatic region of combustion chamber) between the refractory line and exit region. Heat transfer due to heat exchange by direct contact of fluidized bed particles with steam/water tubes was studied by means of the fuzzy logic and cluster renewal approach, respectively. At the given location above air distributor, the local furnace data (i.e., bed/wall temperature and suspension density) were used for the prediction of overall heat transfer coefficient in a circulating fluidized bed boiler. The co-variation between the bed-to-wall heat transfer coefficient and the local furnace data is investigated. The furnace data presented in this section were normalized by the maximum value obtained during the performance test. In the case of Figs. 7, 8, and 9, the solid curves represent approximation of heat transfer data by means of regression analysis using the Origin Pro 8 software. In this study, the comparison has been done between the fuzzy logic and cluster renewal approach results. The correlation between fuzzy logic and cluster renewal approach values of bed-to-wall heat transfer coefficient at different distances from the grid have been shown in Fig. 10. Statistical methods based on the relative error and fuzzy logic system goodness of fit were used for comparison. Comparison of heat flux recovery has been carried out under identical operating parameters of the CFB boiler for two analyzed approaches.

5.1 Bed-to-wall heat transfer coefficient distributions

The variations of bed-to-wall heat transfer coefficient at different furnace heights for fuzzy logic and cluster renewal approach are shown in Fig. 7. The bed-to-wall heat transfer coefficient along furnace chamber of the CFB boiler in a large-scale is generated based on experimental conditions given in Tab. 2. The heat transfer data points are drawn as individual marks for fuzzy logic and cluster renewal approach, respectively. As the furnace height increases, the bed-to-wall heat transfer coefficient reaches a lower value due to a decrease in suspension density and fractional of wall covered by clusters. Otherwise, the reason for this is that furnace temperature slightly decreases as the furnace height increases. These conclusions are consistent with the comments in the previous Sec. 4. The overall heat transfer coefficient approaches the highest value 241 W/(m^{2}K) at 12 m distance from the fluidization grid, whereas the lowest value of 109 W/(m^{2}K) is found at the exit region of the furnace (42 m).

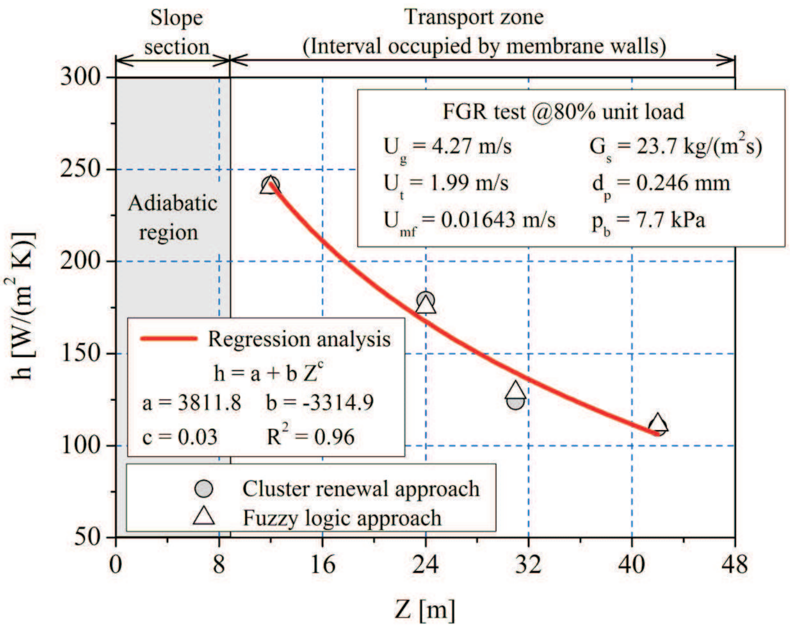


Figure 7: Bed-to-wall heat transfer as a function of distance from the airdistributor.

This situation is caused by the hydrodynamic behavior of the CFB combustor. Regardless of positioning of the active heat transfer surface within the combustion chamber, the effect of solid suspension density and bed temperature on bed-to-wall heat transfer coefficient is significant. It is discussed in more detail with further part of work, especially in the case of Figs. 8 and 9. The heat transfer data obtained using fuzzy logic and cluster renewal approach were correlated by a nonlinear function using regression analysis. The heat transfer red curve in Fig. 7 shows a steeper slope in the transport zone of the furnace chamber. In addition, the reported local heat transfer coefficients as a function of elevation above the fluidization grid have a satisfactory correlation with the correlation coefficient equal to 0.96.

As can be seen from Fig. 7, there is not much difference between experimental data points for the fuzzy logic and the cluster renewal results in the experimental data region (i.e., $12\ \mathrm{m} < Z < 42\ \mathrm{m}$). The highest difference in calculated heat transfer data has been recognized at the distance from the grid equal to 31 m. The calculated deviation between fuzzy logic data and cluster renewal data does not exceed $5\mathrm{W}/(\mathrm{m}^2\mathrm{K})$. Nonetheless, the fuzzy logic results agree well with the results from Eq. (1).

Figure 8 depicts the effect of normalized bed temperature on the bed-to-wall heat transfer coefficient at two different systematic approaches for estimating heat transfer on the water membrane wall. During heat transfer modeling, the bed was operated at fast fluidizing conditions and coal particle size was in the interval of 0.01–16.25 mm in terms of diameter. An increase in the overall heat transfer coefficient can be observed as the bed temperature increases. This fact is explained by the thermal radiation as a dominant mechanism of heat transfer between the core and the annulus region when the heat transfer coefficient is proportional to ${T_g}^4$.

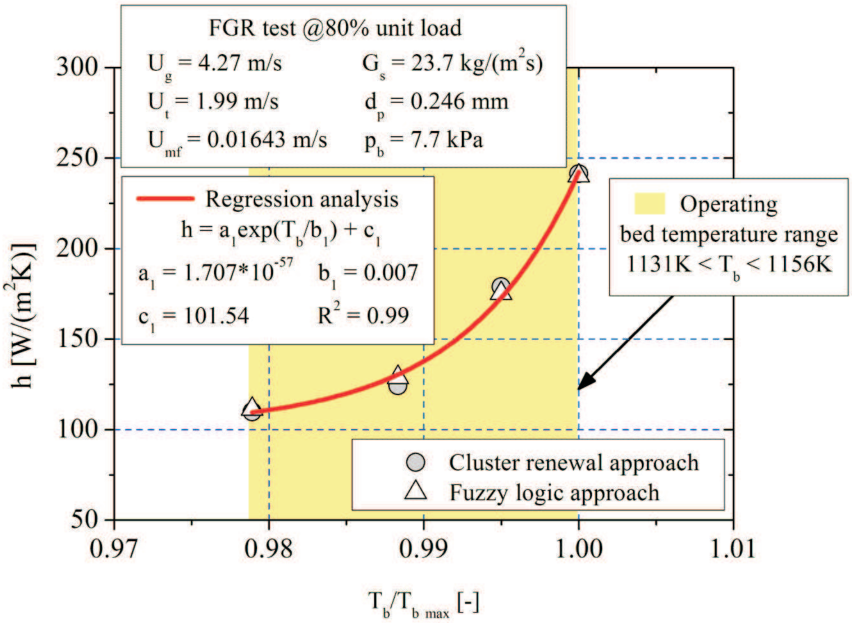


Figure 8: Effect of bed temperature on the bed-to-wall heat transfer coefficientfor fuzzy logic and cluster renewal approach.

From the Stefan-Boltzmann equation, it was expected that the bed-to-wall heat transfer coefficient is strongly dependent on heat source temperature.

Thus, the total heat transfer coefficient increases with bed temperature due to a higher thermal gas conductivity and higher radiation at increased temperatures. It happens due to the reduction in the convective component of the heat transfer inside the furnace chamber. This suggests that the furnace temperature has a greater influence on the heat transfer coefficient than the suspension density. A similar trend in heat transfer coefficient variation is observed both for laboratory/pilot units [20,58] and CFB boilers operating with low suspension densities (less than 20 kg/m^3) [7,8,60–63]. The heat transfer data for both fuzzy logic and cluster renewal approach are close to each other. As shown in Fig. 8, the coefficient for the fitted exponential curve for bed-to-wall heat transfer coefficients with non-dimensional bed temperature is very low. The fitted curve was found for a narrow range of bed temperature (1131–1156 K).

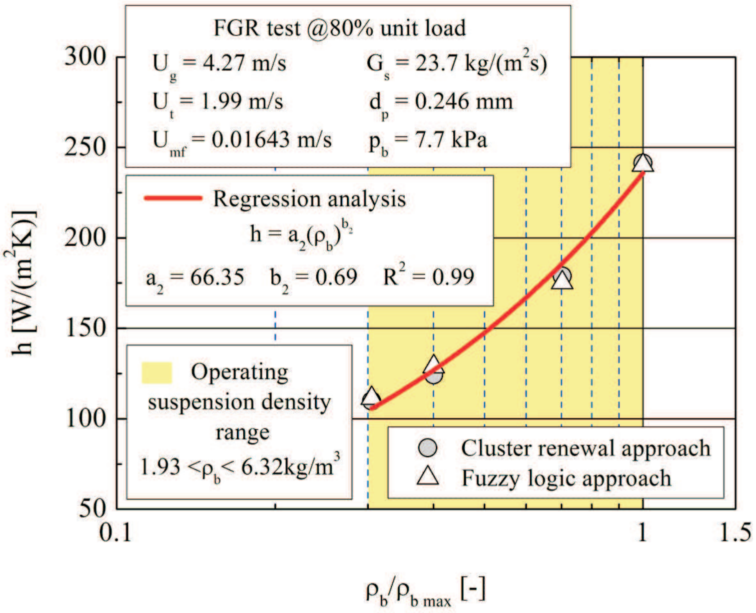


Figure 9: Effect of suspension density on the bed-to-wall heat transfer coefficient for fuzzy logic and cluster renewal approach.

The effect of normalized suspension density on the total heat transfer coefficient at fuzzy logic and cluster renewal approach is shown in Fig. 9. The bed-to-wall heat transfer coefficient increases from 109 to 241 W/(m^{2}K) as the suspension density increases from 1.93 to 6.32 kg/m^3. These heat transfer coefficients are lower than those for shorter heat transfer surfaces at the same suspension density due to the decreasing bed temperature difference between the core and the long cooling surface. It is well-known fact that the shorter tube length exhibits higher bed-to-wall heat transfer coefficient than the longer one, as indicated by Basu *et al.* [22] and Han *et al.* [64]. A number of researchers, for example: Blaszczuk *et al.* [7,9], Dutta *et al.* [8], Andersson *et al.* [9], Cheng [14], Han *et al.* [64], and Breitholtz *et al.* [65] found that the bed-to-wall heat transfer coefficient increased with the suspension density in large-scale CFB coal combustors. This is expected because the thermal capacity of solids is much higher than that of gas. Moreover, the bed hydrodynamics on membrane walls above the refractory line ($\rho_b = 6.32$ kg/m^3 at $Z = 12$ m) are different from that in the exit region, where active heat transfer surfaces are exposed to relatively dilute up-flowing solids ($\rho_b = 1.93$ kg/m^3 at $31 < Z < 42$ m). For the sake of variations in suspension density along furnace height, the local fluid dynamic conditions play an essential role in enhancement of heat transfer, especially for the radiation component of the bed-to-wall heat transfer coefficient. The lower suspension density arose from a insufficient superficial gas velocity for transport of the bed inventory from the slope section to the upper part of the furnace chamber. Eventually, the superficial gas velocity affected the total heat transfer coefficient. That dependence was also confirmed by Horio [38], Andersson [66], Patil *et al.* [67], and also Kolar *et al.* [68].

As can be seen in Fig. 9, there is not much difference between the data points for fuzzy logic and obtained results for cluster renewal approach at the suspension density varied in the range of 1.93–6.32 kg/m^3. The calculated difference at the same normalized suspension density does not exceed 4.6W/(m^{2}K). The heat transfer data obtained using fuzzy logic and cluster renewal approach were correlated by the non-linear function (i.e., solid curve in Fig. 9). The values of the parameters in the fitted curve, a_2 and b_2 were found by regression analysis using the *Origin Pro* 8 software. For the heat transfer red curve in Fig. 9, the coefficient a_2 is equal to 66.35, whereas the exponent b_2 for the fitted curve is found to be 0.69. This exponent is close to that observed by Basu and Nag [21] with 0.5, Divilio and Boyd [60] with 0.55, Wedermann and Werther [10] with 0.562, and also Breitholtz *et al.* [69] with 0.58. These values of exponent b_2 were found for the convective heat transfer coefficient for large-scale CFB boilers.

5.2 Comparison of heat flux recovery (heat transfer data)

A comparison of two approaches based on local furnace data from the commercial CFB combustor is plotted in Fig. 10. In this comparison, the same input data to heat transfer modeling are used in the fuzzy logic and the cluster renewal approach. In the present heat transfer study, the heat flux recovery along the furnace height was estimated from the following relationship:

$$q_j = h_j\,(T_b - T_w) \qquad j \equiv fl,\, cra\,, \tag{13}$$

where, q represents the heat flux recovery, h denotes the bed-to-wall heat transfer coefficient, T_b and T_w are bed temperature and wall temperature of active heat transfer surface, respectively. The subscripts *fl* and *cra* are the value of the fuzzy logic prediction and the value for the cluster renewal approach, respectively. Figure 10 compares heat transfer data predicted by fuzzy logic with those calculated by the mechanistic heat transfer model based on cluster renewal approach, for the membrane wall. All obtained data points shown in Fig. 10 are referred to the value of the heat flux recovery from the core region towards active heat transfer surface equal to 50 kW/m^2. In the present heat transfer study, the presentation form of the obtained values of heat transfer data is due to commercial reason. As can be seen the fuzzy logic results were quite close to the cluster renewal approach results. The middle solid line (straight line at 45 °) represents the line of perfect agreement. The agreement between fuzzy logic and cluster renewal approach results is within ±2% limit (dashed line), which is a good agreement for reliable prediction of bed-to-wall heat transfer coefficient in large-scale CFB units. The relative error, E_r, is obtained by using the following equation:

$$E_r = \sum_{i=1}^{n} \left| \frac{q_{cra} - q_{fl}}{q_{cra}} \right| \times \frac{100\%}{n}\,, \tag{14}$$

where n denotes the number of heat transfer data ($n = 4$), q_{cra} and q_{fl} are heat flux recovery for the cluster renewal approach and fuzzy logic, respectively. Moreover, the fuzzy logic system goodness of fit, η, is calculated by using the following equation:

$$\eta = \sqrt{1 - \frac{\sum\limits_{i=1}^{n} \left(q_{cra} - q_{fl}\right)^2}{\sum\limits_{i=1}^{n} \left(q_{cra} - \overline{q}_{cra}\right)^2}}\,. \tag{15}$$

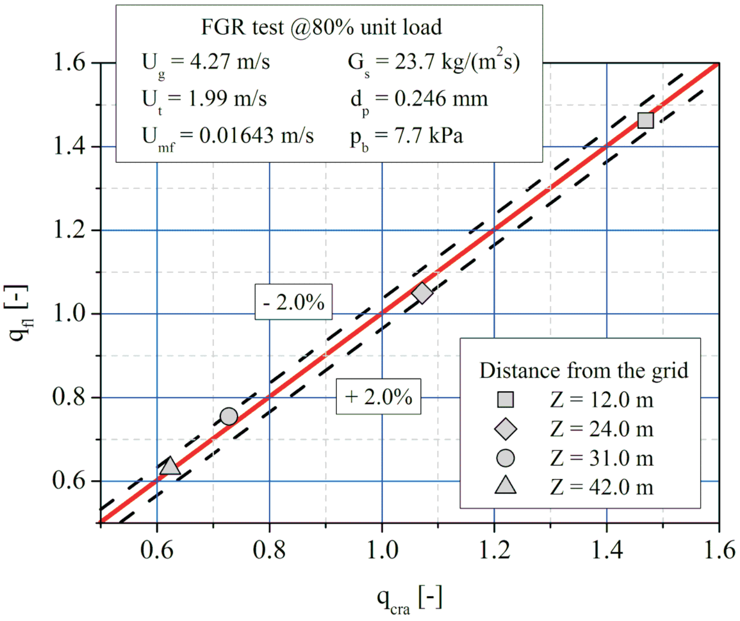


Figure 10: Comparison of relative heat transfer data between fuzzy logic and cluster renewal approach.

The highest value of goodness of fit is 1, and it indicates the capability of the induced system. At two elevations above the air distributor level, $Z = 1$ m and $Z = 42$ m, a perfect correspondence is observed between the fuzzy logic and cluster renewal approach. The correlation coefficient of heat flux recovery is obtained as 0.99. It indicates that the prognosis by means of fuzzy logic system is very good and it is possible to estimate the bed-to-wall heat transfer coefficient reliably. Moreover, the above results confirm also sufficient accuracy of the bed-to-wall heat transfer coefficient prediction. The high accuracy of fuzzy logic to estimate the heat transfer coefficient has been obtained on the basis of furnace data (about 100 data points) for the circulating fluidized bed boiler. Due to data encryption by the owner of the CFB unit in a large-scale, these data are not presented in this work.

6 Conclusions

In the present study, a fuzzy logic and cluster renewal approach have been formulated to predict bed-to-wall heat transfer coefficient characteristics in a large-scale 1296 t/h circulating fluidized bed combustor. The effects of parameters such as bed temperature and suspension density on the overall heat transfer coefficient are studied. The predicted and calculated heat transfer data were generated from the same local furnace data for the CFB boiler in a large-scale. It was observed a decline in bed-to-wall heat transfer coefficient along furnace height. These variations of local heat transfer coefficient arose from solid suspension density variations together with distance from the air distributor. It's worth noting, that the bed temperature has more a significant effect on the bed-to-wall heat transfer coefficient than the suspension density. This is due to the fact of higher thermal gas conductivity and higher radiation at increased temperatures. Nevertheless, the overall heat transfer coefficient depends upon suspension density and bed hydrodynamic conditions in the direct vicinity of the membrane wall. The relationship between the bed-to-wall heat transfer coefficient and bed temperature or suspension density was proposed for a large-scale CFB boiler. These variation trends of the heat transfer coefficient are consistent with the available literature data for large-scale CFB boilers. Both fuzzy logic and cluster renewal approach results are obtained to be valid within the allowable limits. The value of relative error is found to be within the acceptable limits of 2%. From the fuzzy logic system, the goodness of the fit of prediction values is found to be 0.99 for heat flux recovery, which is close to 1 as anticipated. The fuzzy logic model is a good tool which gives some possibility of estimating the bed-to-wall heat transfer coefficient based on operation conditions of the CFB combustion system in a large-scale. The main advantage of such fuzzy logic involves simple investigations of several parameters having a direct or indirect influence on values of local heat transfer data. The predicted and calculated results of heat transfer data are in accordance with typical process data for CFB units in the large-scale. The fuzzy logic and cluster renewal approach were introduced in the work to predict the bed-to-wall heat transfer coefficient during the recirculation of flue gases back to the furnace through star-up burners. Therefore this study will be helpful in the further development of heat transfer models for modern, more economical, large-scale CFB reactors, and will allow us to substantially reduce the environmental emissions. The proposed fuzzy logic and cluster renewal approaches are useful tools for the given type and size of CFB boiler. The reported results indicate the high potential of the fuzzy logic and the cluster renewal approach to reliable prediction of the bed-to-wall heat transfer coefficient for commercial CFB combustor. The obtained heat transfer data is addressed to an existing gap in literature data.

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