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Impact of Pre-Stress on Stability and Vibration of Geometric Nonlinear Column at a Load Force Directed to the Positive Pole

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Abstract

The subject of the results of theoretical and numerical studies measures to designate the effect of pre-stress on the vibration of geometrically non-linear column exposed to the load force directed to the positive pole. Considering the total mechanical energy of the system the equation of motion and the boundary conditions necessary to solve the boundary problem were determined. Based on the kinetic stability criterion the range of values of internal compression forces was determined in which the growth of critical load of the column above the local loss of stability was achieved. In the research the geometrically non-linear system was analysed with variable asymmetry of bending stiffness and at selected values of geometrical parameters of the head realizing the load.

Keywords: free vibrations, geometrically non-linear system, pre-stressing

1. Introduction

The geometrically non-linear slender systems are the subjects of many scientific papers in which considered the issue of their stability and free vibrations at different ways of load and setup. In terms of stability testing of slender systems different load cases were investigated including conservative load (Euler's – [1]) and specific ([2]) and non- conservative load (generalized of Beck - 3). For columns which are geometrically non-linear critical load was determined using linear ([4]) and curvilinear ([5]) form of static balance. The course of changes in frequency of free vibrations as a function of the external load ([4,5]).

Another issue are the study of local and global instability geometrically non-linear systems ([4]). In this case, comparative analyses on the value of bifurcation load of geometrically non-linear columns and critical load of the respective linear columns. Under consideration of this system load cases the value of the external load at which loss of linear static balance was obtained, a function of the asymmetry factor bending stiffness between the rods geometrically non-linear column was determined. The local loss of stability occurs with much lower coefficients of bending stiffness asymmetry of models geometrically non-linear systems. In this case, the value bifurcation load of these models is less than the critical force of suitable linear model.

2. The physical model of the system

Figure 1 shows the physical model of geometrically non-linear column (**KN**) realizing the load force directed to the positive pole ([6]). The column consists of two pairs of rods (1,2) with symmetrical distribution of bending stiffness$(EJ)_1$, $(EJ)_2$, compressive stiffness $(EA)_1$, $(EA)_2$ and mass per unit length $(\rho A)_1$, $(\rho A)_2$. Linear system (**KL**) ([7]) was built with two rods with a total bending stiffness $(EJ)_1$ (without internal rods). The column is loaded by a Q-force by redundancy beam (5) and tension member (3) with a length l_B, whose angle with respect to the undeformed column axis x has a value of β. The rods of column are rigidly constrained ($x = 0$). At the free end ($x = l$) are hingedly connected to tie rod (3) by means of redundancy cubes (4) having a mass m. The direction of the external load P passes through a fixed point P, lying on the undeformed axis of the column. Variable position of the pole O was achieved by mechanical system (6).

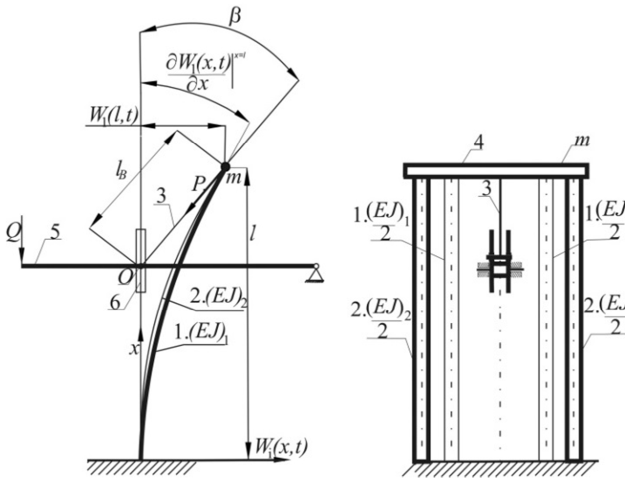

Figure 1. Physical model of geometrically non-linear column (**KN**) loaded with force tracking directed to the positive pole ([6])

In the description of the column (**KN**) is defined asymmetry value of bending stiffness μ:

$$\mu = \frac{(EJ)_2}{(EJ)_1}, \tag{1}$$

assuming that the sum of the bending stiffness geometrically non-linear system (**KN**) is constant:

$$T = \sum_{i=1}^{2} \frac{(\rho A)_i}{2} \int_0^{l_i} \left[\frac{\partial W_i(x_i,t)}{\partial t} \right]^2 dx_i \tag{2}$$

$$\sum_{i=1}^{2}(EJ)_i = idem \tag{3}$$

The rigidity in bending of the column rods (**KL**) is the same as the stiffness of rods the column index 1 (**KN**) with accepted asymmetry of bending stiffness of the model of geometrically nonlinear column described by coefficient μ.

Taking into account the physical model of the column shall be determined according with Bernoulli - Euler's theory of bending components of kinetic and potential energy. Kinetic energy T is the sum of kinetic energy of the individual column rods and body with a concentrated mass m :

$$T = T_1 + T_2 = \frac{1}{2}\sum_{i=1}^{2}(\rho A)_i \int_0^l \left[\frac{\partial W_i(x,t)}{\partial t}\right]^2 dx + \frac{1}{2} m\left[\frac{\partial W_1(l,t)}{\partial t}\right]^2. \tag{4}$$

The total potential energy of the system is composed of energy: internal forces, elasticity in bending and components of external load.

$$\begin{aligned} V = {} & \frac{1}{2}\sum_{i=1}^{2}(EJ)_i \int_0^l \left[\frac{\partial^2 W_i(x,t)}{\partial x^2}\right]^2 dx + PU_1(l,t) + \frac{P}{2l_B}\left(W_1(l,t)\right)^2 + \\ & + \frac{1}{2}\sum_{i=1}^{2}(EA)_i \int_0^l \left[\frac{\partial U_i(x,t)}{\partial x} + \frac{1}{2}\left(\frac{\partial W_i(x,t)}{\partial x}\right)^2\right]^2 dx. \end{aligned} \tag{5}$$

where in: $W_i(x,t)$, $U_i(x,t)$ are appropriately transverse and longitudinal movement, and - the pair of rods of the geometrically non-linear system.

3 The wording of problems, the equations of motion, the boundary conditions

The issue of stability and vibration of geometrically nonlinear column solved using Hamilton's principle ([8]):

$$\delta\int_{t_1}^{t_2}(T - V)\,dt = 0 \tag{6}$$

where: δ means the operator of variations

Taking into account dependences (3) and (4) in equation (5) the prior use property of the commutative integration operation (with respect to x and t) and calculating the variation of mechanical energy, the equation of motion were obtained ([9]):

$$(EJ)_i \frac{\partial^4 W_i(x,t)}{\partial x^4} + S_i(t)\frac{\partial^2 W_i(x,t)}{\partial x^2} + (\rho A)_i \frac{\partial^2 W_i(x,t)}{\partial t^2} = 0 \quad i=1, 2, \tag{7a,b}$$

and the equation of longitudinal displacements of the individual rods of system:

$$U_i(x,t) = -\frac{S_i(t)}{(EA)_i}x - \frac{1}{2}\int_0^x \left[\frac{\partial W_i(x,t)}{\partial x}\right]^2 dx \quad i{=}1,\,2, \tag{8a,b}$$

where in dependencies (6a,b) and (7a, b) included the definition of the longitudinal force

$$S_i(t) = -(EA)_i\left(\frac{\partial U_i(x,t)}{\partial x} + \frac{1}{2}\left(\frac{\partial W_i(x,t)}{\partial x}\right)^2\right). \tag{9}$$

Geometrical boundary conditions considered system:

$$W_1(0,t) = W_2(0,t) = U_1(0,t) = U_2(0,t) = 0, \tag{10a÷d}$$

$$U_1(l,t) = U_2(l,t),\; W_1(l,t) = W_2(l,t)\,, \tag{10e,h}$$

$$\left.\frac{\partial W_1(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial W_2(x,t)}{\partial x}\right|_{x=0} = 0 \quad \left.\frac{\partial W_1(x,t)}{\partial x}\right|^{x=l} = \left.\frac{\partial W_2(x,t)}{\partial x}\right|^{x=l}, \tag{10i,j}$$

substituted into the equation (5) to give the other boundary conditions necessary to solve the boundary problem:

$$\sum_{i=1}^{2}(EJ)_i\left.\frac{\partial^3 W_i(x,t)}{\partial x^3}\right|^{x=l} + P\left[\left.\frac{\partial W_1(x,t)}{\partial x}\right|^{x=l} - \frac{W_1(l,t)}{l_B}\right] - m\,\frac{\partial^2 W_1(l,t)}{\partial t^2} = 0, \tag{10k}$$

$$\sum_{i=1}^{2}(EJ)_i\left.\frac{\partial^2 W_i(x,t)}{\partial x^2}\right|^{x=l} = 0 \tag{10l}$$

4. The results of calculations

Considering the solution of boundary value problem obtained on the basis of equations (6a, b), (7a, b) and the boundary conditions (9a-j) numerical studies on stability and free vibration on considered system was obtained.

In the figure 2a presented change of the force values of geometrically nonlinear bifurcation column (**KN**) and the critical load of the linear column (**KL**) as a function of the asymmetry bending stiffness factor μ. Critical load parameter λ^{*}_{cr} system (**KN**) and (**KL**) refers to the total bending stiffness of the system (**KN**) (Formula 10). It has been shown that this geometrically nonlinear system is characterized by a local and a global loss of stability. In terms of changes in values $\mu \in (0, \mu_{gr})$ bifurcation load (loss of linear static balance) is less than the critical load of the column (**KL**). For the local loss of stability corresponds instability of pair of rods with a smaller bending stiffness. Removal of geometrically nonlinear column of said bars causes a rapid increase in the critical force (transition from point A_1 to point A_2). Consequently in terms of the variation of asymmetric distribution coefficient to bending stiffness of the rods $\mu \in (0, \mu_{gr})$ there is local instability of the system. For $\mu > \mu_{gr}$ there is a global loss of stability.

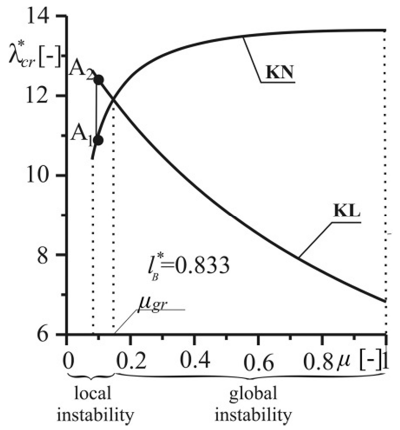

Figure 2a. Change of the critical parameter of load λ^*_{cr} in function of asymmetry factor bending stiffness distribution μ

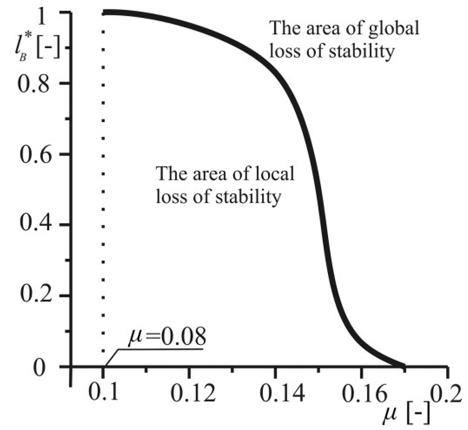

Figure 2b. Change the factor of length of the tendon l_B^* as a function of the limit distribution of the asymmetry factor u to the bending stiffness of the column

$$\lambda^*_{cr} = \frac{P_{cr} l^2}{\sum_{i=1}^{2} (EJ)_i} \tag{11}$$

Characteristic of curve describing the range of variation values of the distance to the pole l_B^* (implementing head load parameter) based on the limit value of the asymmetry factor μ_{gr} bending is shown in Figure 2b. Discussed curve describes the value of the parameters (l_B^*, μ_{gr}), and where there is equality of bifurcation load column (**KN**) and the critical load corresponding column (**KL**). On the basis of the presented curve range of local and global stability loss rectilinear form of static equilibrium system has been designated (KN).

In a paper considered the problem of the impact of pre-stress on the stability of the geometrically non-linear (**KN**). Figure 3 presents value range of critical load of geometrically non-linear column at initial pressurization (**KNW**) as a function of pre-stress (solid line). Calculations were performed at the selected asymmetry bending stiffness μ and given l_B^* parameter of implementing load head. Pre-stressing was achieved by introducing an additional force which initially stretched rods of smaller bending stiffness $(EJ)_2$ In this case the pair of rods - index (1) is subjected to a compression by force S_0. Taking into account the description of the phenomenon of pre-stress, equal longitudinal displacement at the free end of the system (cf. formula (9e)) and Hooke's law, distribution of internal forces in each pair of rods of the geometrically non-linear system was defined (**KNW**).

$$S_1 = S_0 + \frac{P(EA)_1}{\sum_{i=1}^{2}(EA)_i}, \quad S_2 = -S_0 + \frac{P(EA)_2}{\sum_{i=1}^{2}(EA)_i}, \qquad (12a,b)$$

where in $S_i > 0$ - strut, $S_i < 0$ – tension rebar

The lines number 3 and 4 mean the distribution parameter of internal forces S_1^* (curve 3), S_2^* (curve 4) corresponding to the critical load. S_0^*, S_1^*, S_2^* are defined as follows:

$$S_j^* = \frac{S_j l^2}{\sum_{i=1}^{2}(EJ)_i} \quad j = 0,1,2. \qquad (13)$$

If $S_0^* > 1.45$ hat the loss of stability occurs only as a result of the instability of rods bending stiffness $(EJ)_1$. In this case, the parameter S_2^* internal force in the rods of the bending stiffness $(EJ)_2$ is negative (tensile). It has been shown that in the range of pre-stress $S_0^* \in (S_0^{'}, S_0^{''},)$ to an increase of the critical load $(\lambda^*{}_{cr})_{KNW}$ of the column (**KNW**) above the critical force contribution $(\lambda^*{}_{cr})_{KL}$ line (line 2) - "out" from the scope of the local loss of stability. At point *C*, the column loses the stability as a result of the exclusive action of pre-stress.

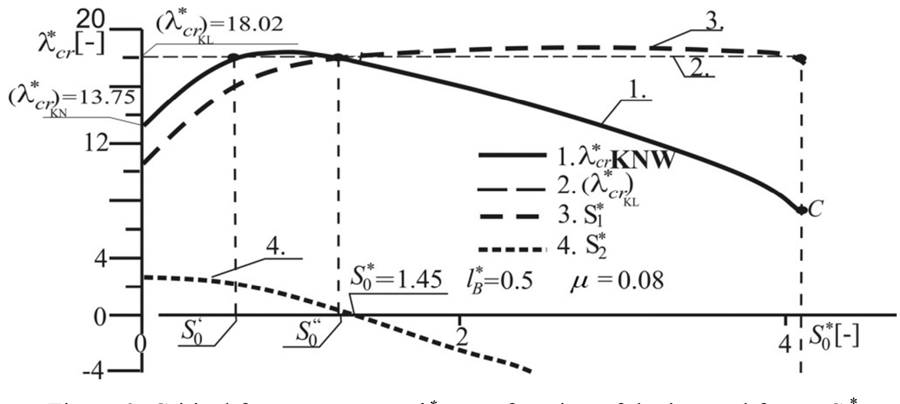

Figure 3. Critical force parameter λ^*_{cr} as a function of the internal forces S_0^*

Comparing the values of the bifurcation load of system (**KN**) and the column load (**KNW**) at selected values of the parameter S_0^* (Fig. 4a) showed that pre-stressing should be used in a limited range of changes in the asymmetry factor of flexural rigidity μ such $\mu \in (0, \mu_2)$ for $S_0^* = 3.434$ $\mu \in (0, \mu_3)$ for $S_0^* = 1.717$. The positive effects of pre-stress are obtained when the critical load parameter λ^*_{cr} column (**KNW**) is larger than the column parameter (**KN**) (curve 2, 3). In the case of $S_0^* = 4.85$ (curve 1), the -pre-stressing should not be used.

Figure 4b shows an example of the course of the frequency of free vibrations the system under consideration. In terms of parameter changes the internal force of the rods S_1^*

the bending stiffness $(EJ)_1$ in the range of $S_1^* \in (0, S_0^*)$ is pre-compressed system. Then the rod is exposed to external load λ^* where in:

$$\lambda^* = \frac{Pl^2}{\sum\limits_{i=1}^{2} (EJ)_i}, \quad \Omega^* = \frac{\sum\limits_{i=1}^{2} (\rho A)_i l^4 \omega^2}{\sum\limits_{i=1}^{2} (EJ)_i}. \tag{14a,b}$$

The value of the critical load were obtained for Ω=0 parameters.

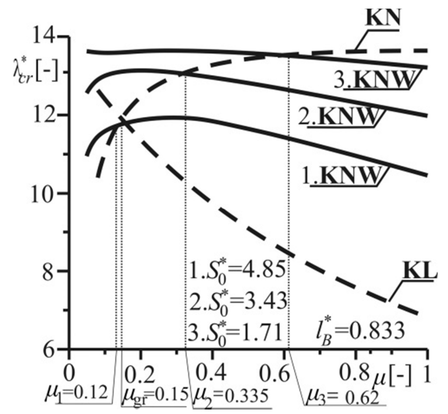

Figure 4a. Critical force parameter λ^*_{cr} as a function of variables internal forces S_0^*

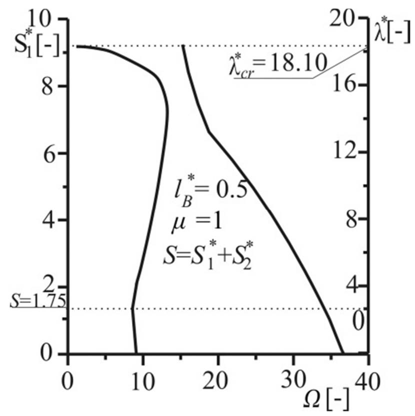

Figure 4b Mileage frequency of free vibrations of the system studied pre-compressed and loaded a dimensionless value of the external force λ^*

5. Conclusion

The subject of the paper was an analysis of vibration and stability of geometrical nonlinear column loaded with a force directed to the positive pole. The analysis of numerical results shows that the system under consideration depending on the value of the asymmetry factor decomposition bending stiffness is characterized by local or global loss of stability. Asymmetry parameter bending stiffness affects the value of the critical force of the geometrically nonlinear column. In terms of the influence of pre-stress on the vibration and stability of geometrically nonlinear column loaded with a force directed to the positive pole defines the scope of the pre-stressing for which it receives an increase in the critical load of the column above the limit of local loss of stability. It was found that the initial compression of the column in whole possible extent from the viewpoint of the value of bifurcation force is undesirable.

This applies especially to high pre-stress force values for which the results are opposite to expected (a significant reduction of the critical load). Pre-compression should be used for the columns characterized by local loss of stability.

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