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Comparison of Two Algorithms of Evaluating Fractal Dimension for Sea Bottom Typing

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The fractal dimension of received echo envelope is a useful parameter in a sea bottom classification procedure. As fractal structure of bottom is believed to transfer onto the shape of echo envelope, fractal dimension may describe properly some physical features of seabed, as surface roughness or complexity of layered structure. The paper presents and compares the results of two methods of fractal dimension calculation from echo envelope given as finite set of discrete values. The first method evaluates box dimension and the second is based on the relation between fractal dimension and Lipschitz exponent, which describes some properties of autocorrelation function of investigated signal. The obtained results show that at least one type of fractal dimension method may be useful in a sea bottom recognition task.

1. Introduction

The problem of sea bottom identification and classification is important in many fields. Acoustic methods of bottom characterisation are non-invasive and more cost effective than other methods, e. g. geological cores. Among various acoustic techniques for characterising and classifying the bottom type, the methods of normal incidence – utilising backscatter data from a single-beam echosounder – have achieved special attention, due to their simplicity, accessibility and versatility. They can involve several approaches such as:

- measurement of energy ratio of the first and second bottom echo [2], [3],
- comparison of theoretically modelled and measured echo patterns [1], [10], [13],
- analysis of a set of values of acoustic and statistical parameters of the echo envelope using cluster analysis [5], [11], artificial neural networks [8], [12] or fuzzy logic [8],
- evaluating the fractal dimension of echo envelope [6] or deconvolved bottom impulse response [7].

The last method is based on the assumption, that in many cases seafloor may exhibit fractal structure, which is transferred onto the shape of scattered echo envelope or bottom impulse response. In such a case, fractal dimension of echo envelope may describe properly some physical features of investigated seabed, as surface roughness when acoustic wave is scattered dominantly on bottom surface, or complexity of layered structure of sediments when more deep penetration of bottom by transmitted signal is assumed.

This paper concerns the problem of choosing the method for fractal dimension estimating for echo envelope, which is usually given in digitised form as a finite vector of samples. Two methods: box dimension method and Lipschitz exponent method were investigated and compared with use of the same acoustic data, assuming that echo envelope contains mainly information about bottom surface.

2. Fractal Dimension and Methods of Its Calculation

It was found that the shapes and structures of many objects in nature usually show no regularities characteristic for simple Euclidean figures. On many occasions, however, nature has proved to accommodate various types of elements with the fractal structure, e. g. the structure of plants' leaves, corrugated sea surface or bottom surface [4], which suggests that fractal analysis methods are proper for studying and describing such elements.

Fractal sets, like Koch snowflake or Cantor set [4], are defined as scale-invariant (self-similar) geometric objects: they can be written as a union of rescaled copies of themselves.

To investigate and describe real objects in nature with use of the fractal analysis, one should have a method of measuring the magnitude of their dimensions and comparing them. Standard methods, that consist of measuring a length or area of 1^D and 2^D figures, are not appropriate here. One of defined fractal dimensions, so-called Hausdorff dimension [4], [9] may be the solution to this problem, as it can be used as a measure of many very general sets, including fractals. The Hausdorff dimension of a subset X of Euclidean space is defined as a limit

$$D = \lim_{r \to 0} \frac{-\log N(r)}{\log r}, \tag{1}$$

where $N(r)$ denotes the smallest number of open balls of radius r needed to cover subset X; an open ball $B(p, r) = \{x: \mathrm{dist}(x, p) < r\}$, where $\mathrm{dist}(x, p)$ is the distance between points x and p.

It is easy to see that the dimension defined by formula (1) measures of the complexity of a given figure. In the case of a sea bottom echo envelope, it may be an indicator of the complexity or variability of this waveform, which may imply its use as a signature of the type of investigated seabed. As it is known, that bottom roughness is related to its hardness, the authors predict, that fractal dimension of echo envelope from harder seabed, like rock, should have greater value than that of echoes from softer bottom, i. e. mud.

It is not easy to calculate the fractal dimension of a figure following the definition of the Hausdorff dimension (1). However, some methods are known, which in a quite simple way, under some assumptions lead to evaluation of some quantity, that is equal or related to Hausdorff dimension, even when the data are in discretised form. Two of such methods are presented in this paper.

2.1. Box Dimension Method

The box dimension [4] can replace the Hausdorff dimension for many sets, including shapes of echo envelopes. The box dimension of a plane figure of investigated echo waveforms is defined as follows. Let $N(\Delta s)$ denote the number of boxes in a grid of the linear scale Δs which meet the set X on a plane. Then X has a box dimension

$$D = \lim_{\Delta s \to 0} \frac{-\log N(\Delta s)}{\log \Delta s}. \tag{2}$$

The method of evaluating the box dimension of a bottom echo envelope is explained in Fig. 1. The grid of square boxes of side Δs is superimposed on the graph of echo envelope and the number of boxes which consist of the fragments of the envelope graph is counted and denoted as $N(\Delta s)$.

It must be pointed out, that the accurate definition of box dimension with limit given by formula (2) cannot be used here, because a digitised echo pulse consists of a finite set of straight sections and it is not a real fractal. That is why one would always obtain the box dimension value equal to 1 in such a case. However, it is possible to approximately evaluate this dimension calculating it not as a limit, but assuming Δs to have a finite fixed value taken from a range of scale for which the investigated digitised shape is assumed to have fractal properties:

$$\widetilde{D}_{box\,\Delta s} = \frac{-\log N(\Delta s)}{\log \Delta s}. \tag{3}$$

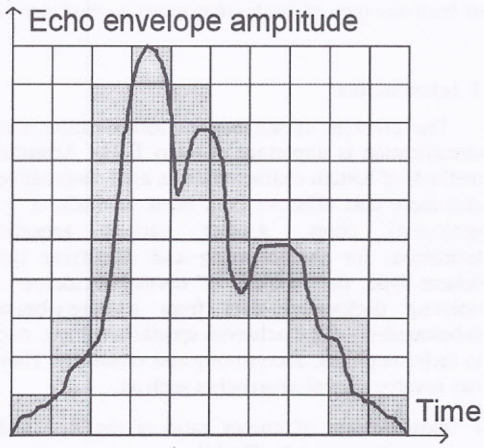


Fig. 1. Illustration of the box dimension evaluation. In presented case, $\Delta s = 0.1$, $N_s = 10$, $N(\Delta s) = 30$.

When using the above method, it is a problem of different lengths of particular echoes. The possible solution is to normalise all echoes from dataset to standard length prior to superimposing the grid. Alternatively, the concept of moving window and averaging may be used to maintain the constant number of samples per unit box in the grid [6].

Previous investigations [6] show no significant differences between the results of applying that two ideas in estimation the box dimension as a distincive seabed parameter. In this work, the echoes normalising to standard length was used and Δs was 1/36.

2.2. Lipschitz Exponent Method

The Lipschitz exponent method of calculating Hausdorff dimension is based on some properties of the autocorrelation function of echo envelope $y(t)$. It is known, that fractal process $y(t)$ obeys Lipschitz-Holder condition [9]:

$$|y(t+\tau)-y(t)| \approx c\tau^{\alpha}, \tag{4}$$

where c is constant and τ is time lag. For small increments τ the exponent α is called the Lipschitz exponent. Mandelbrot [9] showed that α is related to the Hausdorff dimension of a graph of $y(t)$ via simple formula:

$$D = 2 - \alpha. \tag{5}$$

α (or D) can be treated as a measure of complexity "roughness" of $y(t)$. On the other hand, it can be shown that for small lags τ

$$R_{yy}(\tau) \approx R(0) - c_1\tau^{2\alpha}, \tag{6}$$

where $R_{yy}(\tau)$ is the autocorrelation function given by

$$R_{yy}(\tau) = \mathrm{E}[y(t)y(t+\tau)]. \tag{7}$$

E is a symbol of averaging operator and c_1 is a constant. By normalising (6) and taking the logarithm of both its sides we obtain:

$$\ln\left(1-\overline{R}_{yy}(\tau)\right) = 2\alpha \ln\tau + c_2, \tag{8}$$

where c_2 is a constant and $\overline{R}_{yy}(\tau)$ is the normalised autocorrelation function

$$\overline{R}_{yy}(\tau) = R_{yy}(\tau)/R_{yy}(0). \tag{9}$$

The Lipschitz exponent α and subsequently the Hausdorff dimension D can be calculated from the slope of a log-log plot of $1-\overline{R}_{yy}(\tau)$ versus τ (for small τ) using linear regression algorithm.

It is important to choose the proper range of lag τ. Taking into account the small τ condition, the authors selected $\tau = 0.24$ ms as the upper limit of the τ range. This is because the autocorrelation properties of echo envelopes were different for τ below and above this value. The lower limit was selected to be twice the sampling period, that is $\tau = 0.048$ ms.

3. Results and discussion

The bottom echoes data used to test the method were recorded during acoustic surveys on Lake Washington by the digital DT4000 BioSonics echosounder with operating frequency 120 kHz, pulse length 0.4 ms and sampling frequency 41.66 kHz. Data acquisition was performed both while the vessel was anchored and along transects.

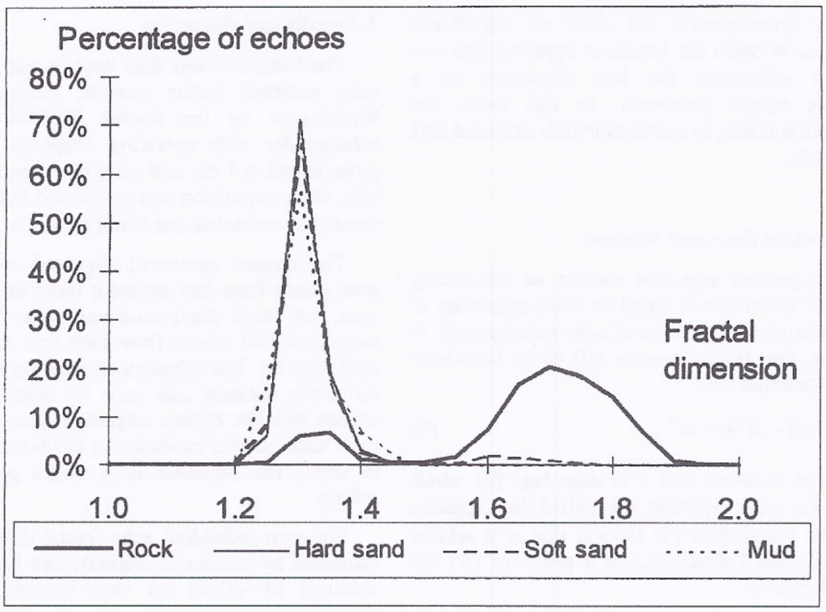
The dataset contained digitised envelopes of echo pulses from four different types of sea bottom: mud, soft sand, hard sand and rock. There were more than 600 echoes from each type of bottom in each data set. The calculations were performed once for entire datasets and once for selected 10% of echoes with the highest amplitude level, e. g. of the most likely normal incidence to the bottom, in order to check the influence of a ship's pitching and rolling.

For each individual echo fractal dimension was estimated by two above methods and histograms of obtained its values for each bottom type were constructed and analysed.

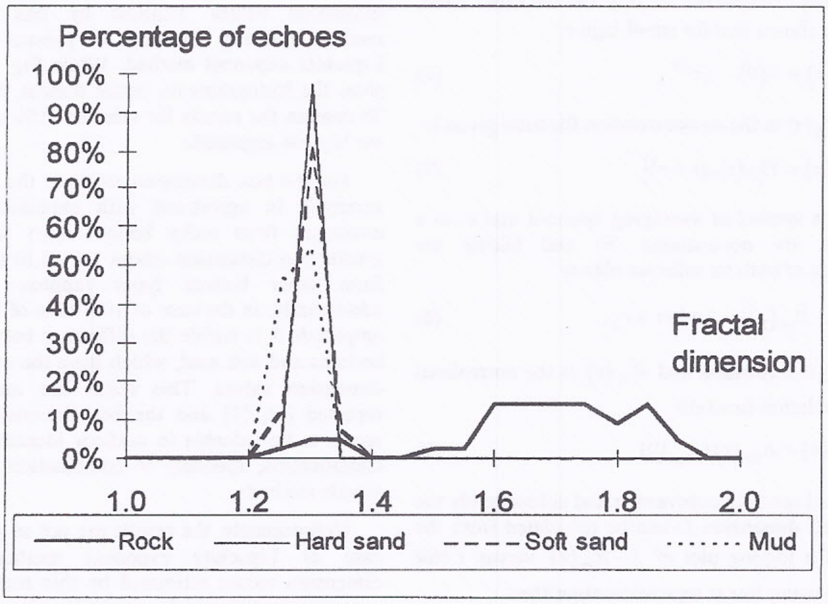
The results are shown in Fig 2 and Fig. 3. Fig. 2a and 2b contain the histograms of fractal dimension values obtained by box dimension method and Fig. 3a and 3b present results of Lipschitz exponent method. While Fig. 2a and 3a show the histograms for entire dataset, Fig. 2b and 3b contain the results for selected 10% echoes with the highest amplitude.

For the box dimension method, the results are generally in agreement with expectations. Echo envelopes from rocky bottom have significantly greater box dimension (about 1.6 - 1.8) than echoes from other bottom types (approx. 1.3) and additionally, in the case of 10% data of the highest amplitude it is visible the difference between sandy bottoms and soft mud, which have the smallest box dimension values. This result has already been reported [6], [7] and the box dimension method seems to be valuable in seafloor identification and classification, specially in combination with other simple methods.

Unfortunately, the results are not so good in the case of Lipschitz exponent method. Fractal dimension values estimated by this method are to some extent different for particular types of bottom, but the relations between them are not in line with expectations and with box dimension method results. Moreover, the visible tendency is rather opposite: the harder bottom, the less fractal dimension. It means, that in domain of small lag τ, where application of the Lipschitz exponent method is allowed, the investigated echo envelopes do not have fractal properties, what was additionally checked by analysing the echoes' shapes themselves

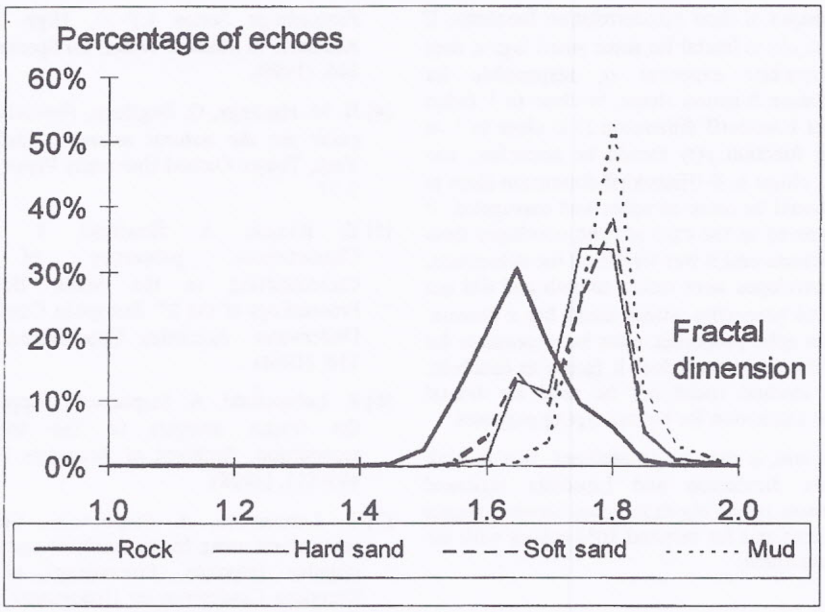


a)

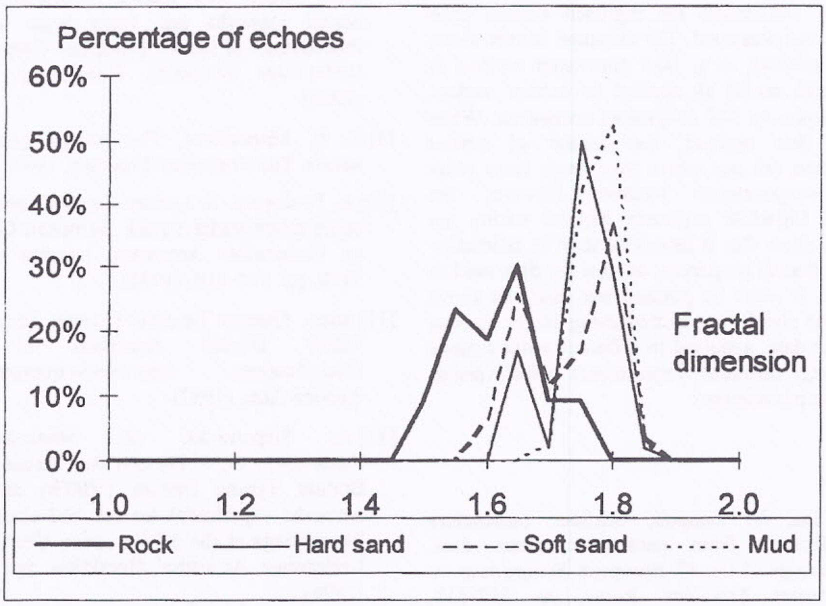


b)

Fig. 2. The histograms of fractal dimension calculated for echo envelopes for 4 types of bottom using box dimension method: a) for entire datasets, b) for selected echoes with the highest amplitude.



a)



b)

Fig. 3. The histograms of fractal dimension calculated for echo envelopes for 4 types of bottom using Lipschitz exponent method: a) for entire datasets, b) for selected echoes with the highest amplitude.

and the shapes of their autocorrelation functions. If the process $y(t)$ is fractal for some small lags τ, then if its Lipschitz exponent α, responsible for autocorrelation function shape, is close to 1 (what means that Hausdorff dimension D is close to 1 as well), the function $y(t)$ should be smoother, and when α is closer to 0 (Hausdorff dimension close to 2), $y(t)$ should be more complex and corrugated. It was not proved in the case of echo envelopes from this experiment and it was visible on the echograms, that the envelopes were rather smooth and did not show fractal properties within small lag τ domain. Here, other echo properties must be responsible for its Lipschitz exponent value. It forces to conclude, that this method could not be used for fractal dimension estimation for bottom typing purposes.

In the end, it may be pointed out, that in both cases, box dimension and Lipschitz exponent method, there are no big differences between results for all echoes and for selected 10% echoes with the highest amplitude.

4. Conclusions

The comparison of two methods for fractal dimension calculation for digitised bottom echo envelope was presented. The obtained results show, that one method, e. g. box dimension method is reliable and useful as applied to bottom surface shape complexity and roughness estimation. When applying that method, the results of seabed classification are not worse than those from other simple, one-parameter methods. However, the results of Lipschitz exponent method testing are worse and show that it cannot be used in estimation of bottom fractal properties, at least for data used in this work. It must be pointed out, that the above conclusions should be verified using larger amount of acoustic data, acquired in different water regions and during different experiments with various acquisition parameters.

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