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Prediction of the failure mechanism of arches under base motion using DEM based on the NSCD method

Przewidywanie mechanizmów awarii łuków spowodowanych ruchami podłoża za pomocą metody elementów dyskretnych z wykorzystaniem metody NSCD

Keywords: Dynamic analysis, Masonry Analysis, Discrete Element Method, Numerical Modelling, Non-Smooth Contact Dynamics

Słowa kluczowe: analiza dynamiki, analiza konstrukcji murowych, metoda elementów dyskretnych, modelowanie numeryczne, dynamika zagadnień kontaktowych niegładkich

1. INTRODUCTION

1.1. Predicting the seismic behaviour of vaulted structures

The understanding of how vaulted structures behave under seismic loading is a world-wide issue that requires addressing in the pursue of two main objectives: the preservation of traditional built heritage and the saving of human lives. Vaults, arches and domes feature in a vast proportion of the world's historical architecture. Many of these buildings have survived for hundreds of years subject to everyday use and wear. Nonetheless, in earthquake prone areas, this otherwise convincing evidence is insufficient to assure the stability of the building in the future, for an earthquake could have devastating effects.

The engineering community currently lacks effective, safe methods for anticipating the seismic response of vaulted masonry buildings. The ongoing research presented in this paper aims to further the development of such methods. This work focuses on computational modelling of the dynamic response of vaulted structures subject to ground motion, using the Discrete Element Method (DEM).

1.2. Exploring DEM as an analysis tool

Discrete element methods are a family of numerical algorithms for computing the motion of particles, where the motion is principally governed by interaction forces between the elements at their interfaces. The methods have become popular for the engineering analysis of granular materials such as soils, but are also more broadly applicable to any physical system where the behaviour is strongly dependent on discontinuities. Masonry, comprised of a collection of rigid blocks laid dry or bound with mortar, complies with this pattern of behaviour, making DEM and appropriate modelling tool [1].

The present paper explores the applicability of DEM to modelling the behaviour of a semi-circular arch made of voussoirs laid dry that is subject to cyclic ground motion. The DEM code used is LMGC90, developed by the University of Montpellier [2] and based on the theory of Non-Smooth Contact Dynamics (NSCD). [3] A validation of the software for analysing masonry structures has been carried out, based on analytically solved problems such as the minimum thickness arch.

1.3. Ongoing experimental work

Alongside computational modelling, a series of experiments are being performed to explore the response of arches and vaults under ground motion. Model arches and cross vaults are being tested on a shaking table, subject to cyclic (sinusoidal) base motion [4].

Two types of models have been studied according to the material: 1) monolithic structures made of a plaster and sand mix; 2) block structures made of timber. The second group comprises a semicircular arch made of discrete wooden voussoirs, with a thickness-to-radius ratio $t/r = 0.15$. The DEM models presented in this paper are based on this arch; the same geometry and base motion are used in both experimental and computational models. The results are thus compared.

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2. THE DISCRETE ELEMENT METHOD AND NSCD

2.1. Discrete Element Method (DEM)

DEM are inherently appropriate for modelling ensembles of bodies whose deformability is governed by discontinuities. The main characteristics that the many existing numerical approaches share are [1]:

a) DEM can model systems in which the main mode of deformation is by relative movement between the elements, i.e. movement concentrates at the joints.
b) Through contact detection algorithms, contact between elements can be assessed throughout the analysis. It can be lost and later regained.
c) Large displacements can be effectively analyzed.
d) DEM is a non-linear dynamic method; while it can be applied to quasi-static problems, the solution is always performed via a time-stepping algorithm.

2.2. Non-Smooth Contact Dynamics (NSCD)

The Non-Smooth Contact Dynamics numerical method is a DEM approach devised by Jean and Moreau [5].

2.2.1. Unilateral constraints

Many discrete element formulations are compliant, allowing interpenetrability of the particles; if particles move during a time step such that their boundaries overlap, the contact force is calculated as a restoring force that is a function of the amount of penetration. Essentially, the particles behave as stiff springs.

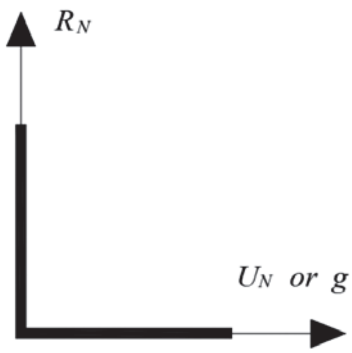

Fig. 1 Signorini condition

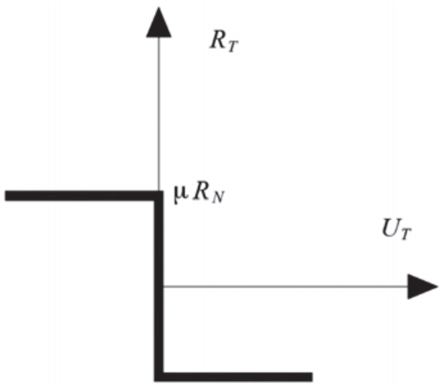

Fig. 2 Coulomb friction law

This results in a smooth variation of particle velocities but can lead to very stiff equations which require explicit solvers. These are conceptually simple, but generally require very short time steps to ensure stability, resulting in long run times.

The key principle underlying the NCSD method is that of *unilaterality*, i.e. no interpenetrability of the particles is permitted, so that the gap between particles must always be greater than or equal to zero. Rather than the interaction being represented as a finite spring stiffness, contact occurs in forms such as hard impact and frictional sliding at boundaries. As a result, the element velocities can no longer be treated as varying smoothly. While this approach may be analytically more complex, it facilitates the use of efficient, implicit numerical time integration schemes.

Mathematically, the unilateral constraint can be expressed using the Signorini Condition. This states that the normal component of the force between two bodies in contact (i.e. with zero gap) must be positive, meaning a repulsive rather than an attractive force, and becomes zero when contact is lost (i.e. the gap between the bodies becomes greater than zero). There is no mapping between force and gap size; the reaction force is not a function of the gap and vice versa.

2.2.2. Dry friction

Coulomb friction is adopted: a certain tangential force may be exerted on the interface between two bodies without any sliding, sliding occurring only when the force overcomes a certain threshold. This threshold is proportional to the normal component of the reaction between bodies and the threshold value is maintained during sliding.

3. LMGC90. APPLICATION TO MASONRY ANALYSIS

3.1. Past applications of LMGC90 on masonry

NSCD, through software LMGC90, has been applied to the analysis of vaulted masonry structures in the work by

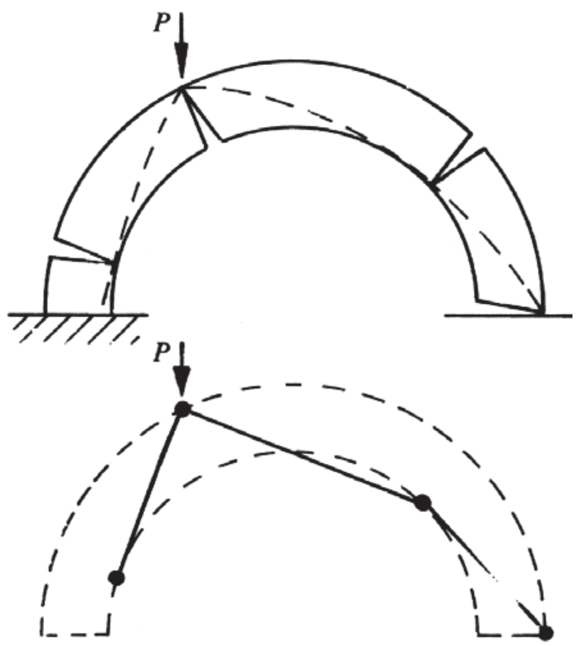

Fig. 3 Collapse mechanism of a ring arch under a point load

Rafiee [6, 7]. In these studies the dynamic response of various masonry structures is explored, ranging from a self-standing ring arch to complex ensembles such as the Nîmes Arena. [6] explores the behaviour of a ring arch made of 13 voussoirs subject to ground motion. Various different contact laws (dry contact vs. cohesion, where cohesion refers to tensile strength) are implemented and the collapse mechanisms are given. This interesting work lacks, however, a validation of the LMGC90 computations against either experimental or analytical solutions.

3.2. Theoretical framework: limit analysis of masonry structures

Heyman [8] validated the traditional analysis of masonry structures by setting it within the frame of plastic theory structural analysis. Three hypotheses are applied in this analysis: zero tensile strength, infinite compressive strength and impossibility of sliding failure. These lead to the conclusion that the only possible failure is through loss of stability by the formation of enough hinges in the structure.

According to the fundamental Safe Theorem of plastic theory, if it is possible to find a distribution of internal stresses that is in equilibrium with the external loads and nowhere violates the strength conditions of the material, the structure is safe. Applied to masonry structures considering the above hypotheses, this theorem translates as "if it is possible to find a thrust line for the given loads that is contained within the section of the structure throughout, the structure is safe".

3.3. Validation case: minimum thickness arch

The work on LMGC90 began with the study of a number of classic masonry mechanics problems of known analytical solution. We present here one of the chosen problems, that of the minimum thickness semicircular arch [9].

The computational model for the semicircular arch comprises 20 equal voussoirs, rigid, each embracing a 9° angle. The first and last voussoirs rest on two further elements devised as rigid foundations. The structure is subject to self weight only. The contact law is a simple, inelastic quasi-shock law with Coulomb Friction. This is effectively a dry contact law (no cohesion, i.e. no tensile strength at the interface). The selected friction coefficient is 0.5 throughout. The density of the material is constant throughout the structure and is set to be 18 kN/m^3 (although the stability of the arch is independent of this value).

The theoretical minimum thickness-to-radius ratio for a semi-circular arch subject to its own weight is t/R_{ext} = 0.1021. This arch is on the verge of collapse by the formation of a symmetric 5-hinge mechanism, with hinges forming at the worst possible locations, (hinges opening anywhere in the structure): hinges at 0°, 54.5°, 90°, 125.5° and 180°.

LMGC90 has yielded a result of t/R_{ext} = 0.1025 minimum thickness-to-radius ratio for a stable arch as described above. The error is below 0.5%, and can be explained by the hinge positions, which are restricted to the existing joint locations in the DE model. With voussoirs embracing 9° angles, the hinges of the 5-hinge mechanism opened at 0°, 54°, 90°, 126° and 180°. The 2nd and 4th hinges are therefore slightly shifted from the theoretical position. This is considered to be a good result overall.

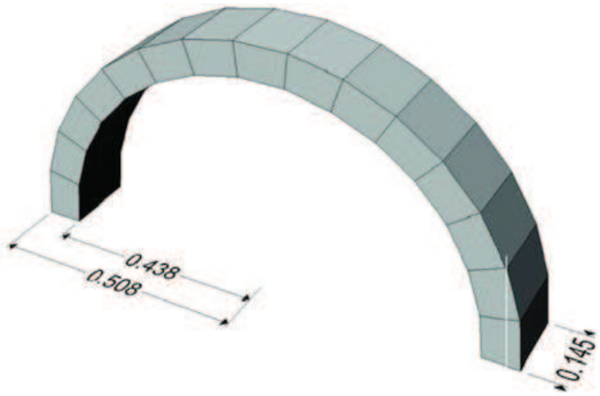

Fig. 4 Geometry of arch with 15 voussoirs and 180° angle of embrace

4. DEM BASE MOTION MODELLING

4.1. Geometrical model

The geometry of the arch model is based on a ring arch with t/R_{med} = 0.15, subdivided into voussoirs embracing 12° angles each. The overall dimensions of the model respond to the size of the shaking table used for the experimental tests. The external radius is R_{ext} = 0.508 m and the thickness t = 0.071 m. The width of the arch (perpendicular to the plane of the arch) is b = 0.145 m. The full 180° angle of embrace model is shown in Fig. 4.

4.1.1. Varying angle of embrace

The DEM analysis has been applied to 4 arch models based on the above geometry, introducing variations in the overall angle of embrace. This was achieved maintaining the initial configuration, but removing the bottom voussoirs in a symmetrical fashion. The 4 models are described in Table 1.

Table 1. Arch models: angle of embrase and no. of voussoirs

Model	Angle of embrace	Number of voussoirs
1	180°	15
2	156°	13
3	132°	11
4	118°	9

4.2. Input motion

The base motion input to the models is a sinusoidal displacement with a period of attack (increasing amplitude), a period at full size and a period of decay (decreasing amplitude). It is comprised of a regular sinusoidal oscillation multiplied

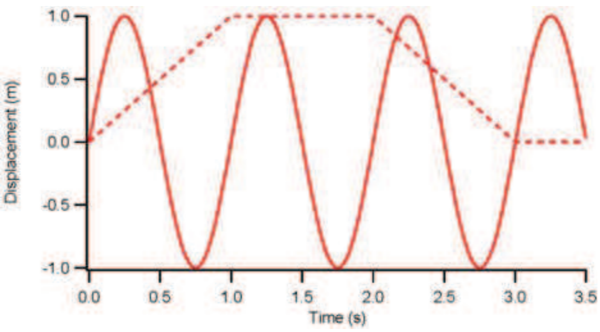

Fig. 6 Displacement wave components

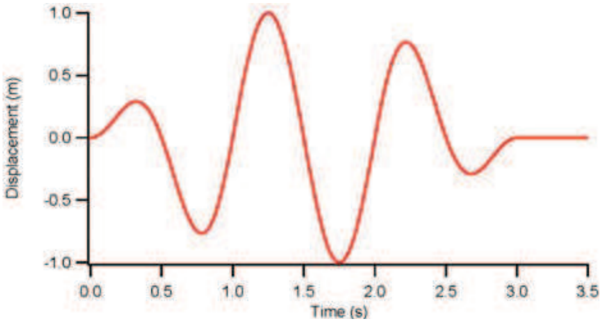

Fig. 7 Resulting displacement wave sinusoid & envelope

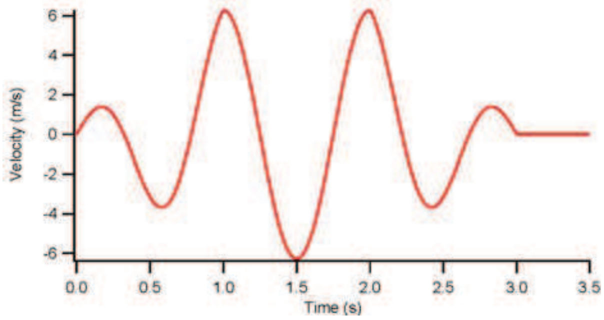

Fig. 8 Velocity wave input motion to DE models

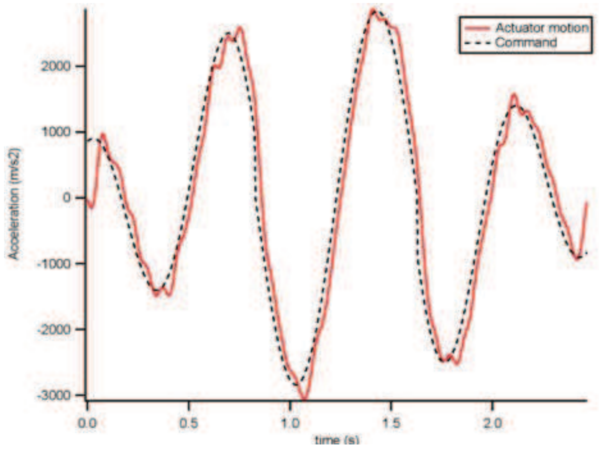

Fig. 9 Acceleration wave: ideal command wave (obtained from the mathematical expression for displacement) vs. actual actuator motion (obtained by numerical differentiation of the actuator position data)

by an envelope (Figs. 5 & 6) The attack and decay pulses are introduced to prevent possible large acceleration spikes appearing at the start and end of the motion.

Input motions in LMGC90 are described as velocity functions. Thus, the displacement wave is differentiated (see Fig. 7) to obtain the input for the DEM model.

The purpose of the models is to find the minimum acceleration that will cause the failure of the arch. A frequency is selected (in general, the same frequency that is observed to cause failure in the experimental tests – see §5) and the amplitude of the displacement wave is ramped up until failure takes place.

5. SHAKING TABLE EXPERIMENTAL WORK

As part of the ongoing research project, a series of shaking table experimental tests are being carried out. The DE models described in this paper are based on a set of tests carried out on a model of a ring arch constructed of wood and divided into voussoirs, with a geometry as described in §4.1.

The shaking table constructed for these tests comprises a steel platform approximately 1 m square, running on linear bearings. It is driven by a single 10 kN Instron servo-hydraulic actuator that can apply uni-directional horizontal shaking at displacement amplitudes up to 75 mm. The mass of the test specimens and the performance envelope of the actuators meant that it could operate at full capacity at frequencies up to around 5 Hz.

Test control was achieved using an Instron 8800 closed loop control unit. The simple waveforms described in §5.2 were applied using Instron's software RS-Plus.

The position data of the Instron actuator is recorded. The actual acceleration has been computed from this data, after filtering the displacement for frequencies higher than 20Hz, finding a discrepancy with the theoretical acceleration derived from the analytical formula for the displacement input shown in Fig. 8. There is a small delay and, most relevant for our analysis, the peaks of the actual motion exceed in magnitude the acceleration formula.

6. RESULTS AND DISCUSSION

6.1. Collapse accelerations

The results obtained from the computational modelling and the experimental tests run in parallel are summarised in Table 2. This table presents three sets of results: the experimental results and two sets of DE model results. The first DEM set corresponds to the model of the idealised arch – un-calibrated- as described in §5. The DE model was then calibrated using the arch with angle of embrace 156º, obtaining the results given in the final set.

The data given for each set comprises the oscillation frequency at which collapse takes place, the nominal displacement amplitude of the full-size cycle and the resulting maximum acceleration amplitude. The peak acceleration given for the experimental test results is the actual value, greater than the theoretical value, as shown in Fig. 8.

Table 2 Collapse frequency, displacement amplitude and acceleration amplitude for experimental tests and DE models

Angle of embrace	Experimental tests			LMGC90			LMGC90 Calibrated		
	Freq.	Peak Displac. Amplitude	Accel. Max	Freq.	Peak Displac. Amplitude	Accel. Max	Freq.	Peak Displac. Amplitude	Accel. Max
180º	1.00 Hz	40 mm	1.67 m/s^2	1.00 Hz	41 mm	1.62m/s^2	0.80 Hz	17 mm	0.67m/s^2
156º	1.25 Hz	46 mm	3.05 m/s^2	1.25 Hz	66 mm	4.07m/s^2	1.25 Hz	49 mm	3.02m/s^2
132º	1.60 Hz	56 mm	6.20 m/s^2	2.00 Hz	51 mm	8.50m/s^2	1.60 Hz	59 mm	5.90 m/s^2
118º	2.50 Hz	46 mm	11.36 m/s^2	2.50 Hz	61 mm	15.05m/s^2	2.50 Hz	50 mm	12.34 m/s^2

The discrepancies between the un-calibrated DE models and the shaking table tests, excluding the case of the 180º angle of embrace -which shall be considered independently-, are approximately 25%. Although this may seem like a large error, it is common in this type of experiments on block structures due to the numerous imperfections that can arise, mainly in the setting up of the model structure.

There is no soft interface between the blocks that can assure contact is distributed throughout the whole faces of the blocks. Furthermore, the theoretical model assumes that contact between two blocks that are rotating with respect to each other occurs along their edges, which rarely happens in reality. These and other issues reduce the performance of the physical model with respect to the theoretical idealised solution.

The calibration of the DE models consisted on finding an equivalent reduced arch thickness that would yield results comparable to the experimental tests. This calibration was performed on the 156º angle of embrace arch. It was found that a reduction to 82% of the original thickness gave a very close approximation to the experimental results. The reduction to 82% of the thickness was then applied to all arches and the minimum collapse accelerations computed.

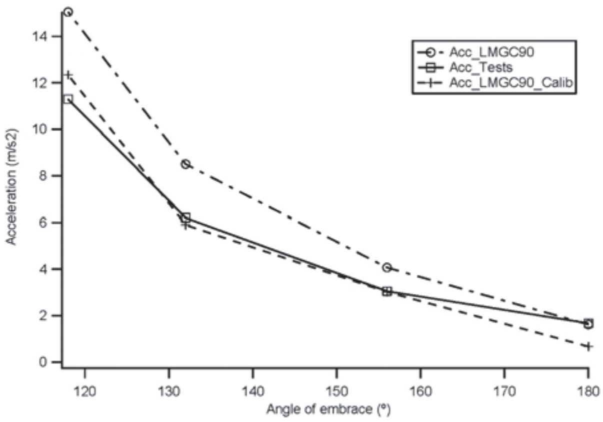

Fig. 10 Collapse accelerations for the 4 arch models, as obtained in experimental tests, and DE calibrated and un-calibrated models

The resulting errors between the calibrated DE models and the experimental tests range between 8% (118º angle of embrace) and -5% (132º angle of embrace). Thus, the discrepancy between collapse acceleration for calibrated and un-calibrated DE models is between 20 and 25%. The case of the arch with 180º angle of embrace is an exception, obtaining a reduction in of acceleration magnitude of 40% in the calibrated model with respect to the un-calibrated one. These results suggest that, although the calibration proves effective in obtaining comparatively good results with the experimental testing, the calibration of the different models should be carried out individually and analysed for consistency.

Returning to the case of the arch with 180º angle of embrace, in the experimental tests, this arch was at risk of sliding taking place at the supports. Such sliding was prevented installing end brackets on the outside of the bottom voussoirs. However, this solution introduced a problem, hindering the rotation of these voussoirs around their base. As a result, there was no free rotation at the base of the arch, and the collapse mechanisms that formed differed from the theoretical minimum energy mechanism. The structure took a higher level of acceeration than would have been expected [10, 11].

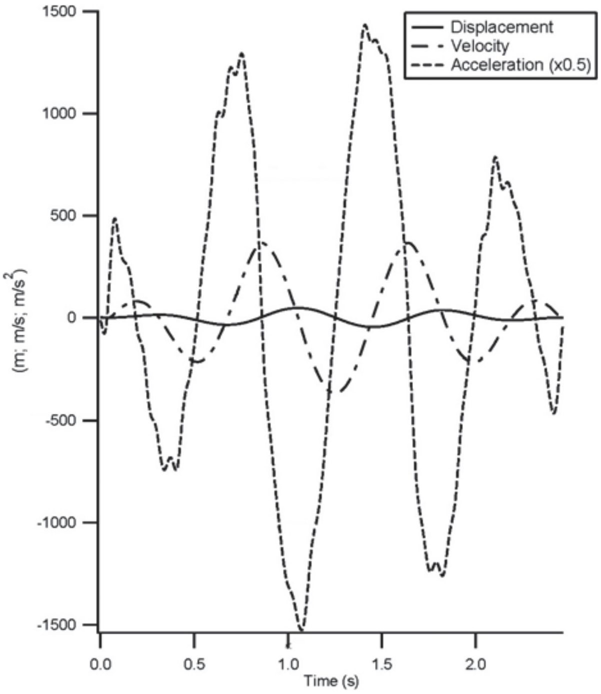

Fig. 11 Collapse base motion for 156º angle of embrace experimental test: Displacement, Velocity and Acceleration

6.2. Collapse mechanism

DEM takes into account the key properties of the structural behaviour of masonry: it effectively models discontinuities, enabling large displacements, and the no-tension property (i.e. zero cohesion) is easily implemented. The failure of the masonry assessed using DEM must therefore respond to a loss of stability. The DE models predict a collapse sequence for the arches when subject to horizontal base motion.

We here analyse the collapse sequence predicted by LMGC90 for one of the models to illustrate the general pattern of behaviour predicted by the computations under the given sinusoidal base motion.

The arch selected for this study is that with 156º angle of embrace. Figure 10 shows the displacement, velocity and acceleration input to the foundations of the model arch in the experimental test. Figure 12 shows the collapse sequence of both the experimental and the computational models.

In the computational analysis, the first mechanism forms after the attack cycle has been completed and the first peak of the acceleration full-size cycle is reached (at $t = 1.0$ s approx.). Such mechanism is depicted in Fig. 12. The position of the hinges corresponds to the mechanism that requires

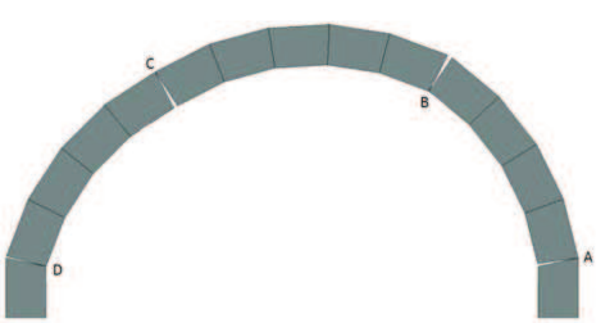

Fig. 12 156º angle of embrace arch: First mechanism mobilised at t = 1.0s approx (rotations amplified)

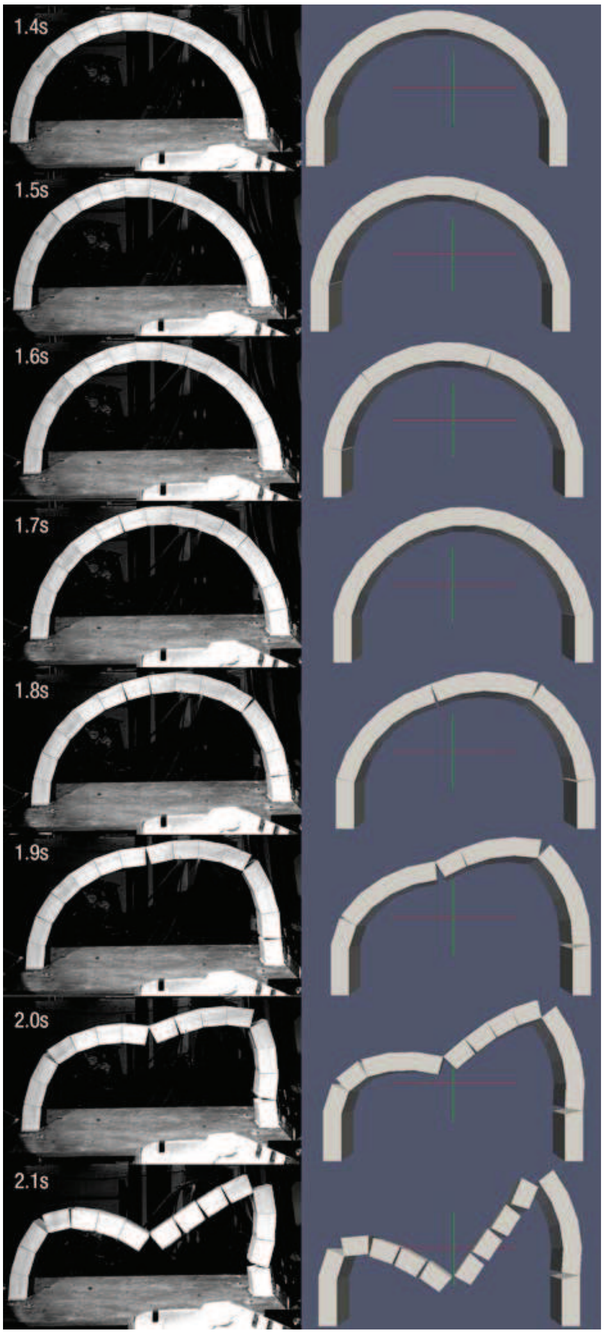

Fig. 13 Collapse sequence for experimental vs. DEM (un-calibrated) models. Angle of embrace of 156º

the least energy to form under horizontal acceleration [11]. The rotations generated are small and soon disappear as the acceleration decreases to 0. The acceleration then changes symbol and the symmetric mechanism starts forming, even before the maximum acceleration magnitude is reached (Fig. 12, 1.4s). Rotations for this second mechanism are larger, and the mechanism doesn't close when the value of acceleration reaches zero (Fig. 12, 1.6s). At this point, hinge C travels along to position itself between the 3rd and 4th voussoirs.

The acceleration increases again (negative value) and the mechanism begins to close. As the magnitude of the acceleration increases further, it closes completely and a new mechanism opens (Fig. 12, 1.7s). The position of the hinges is initially different from the previous two mechanisms. The hinges travel to finally resemble the earlier mechanisms (Fig. 12, 1.8s) as collapse takes place (Fig. 12, 1.9s to 2.1s). The position of the hinges remains unchanged from the moment the acceleration reaches a new peak at 1.8s. Collapse follows as the sinusoidal movement continues.

The comparison of the collapse sequence obtained for the DE model with that observed in the experimental tests is particularly interesting for the fair agreement between them.

At the 1.5s instant we see how the very same mechanism has been mobilised (this is the lowest energy mechanism for this configuration, as described above). The arch recovers from this mechanism and a final mechanism opens around 1.7s.

Although the pattern of behaviour shown in LMGC90 and in the test is very similar, the final mechanism doesn't perfectly agree between the two. In the computation, hinges B and C (refer to Fig. 11) open between voussoirs 4 and 5, 8 and 9 respectively. In the test, however, these hinges open between voussoirs 3 and 4, 7 and 8 respectively.

The phenomenon of hinge travelling is observed in both collapse sequences, most clearly in the case of hinge D (Fig. 11). In the simulation, it initially opens between voussoir 13 and the support, then moving up one voussoir. Similarly, in the test, hinge D appears between voussoirs 12 and 13 and finally moves one voussoir up.

Furthermore, a small delay in the motion of the test with respect to the computation is observed. This responds to the delay described in Fig. 8, originated in the actuator system.

7. CONCLUSIONS

DEM is a valid tool for predicting the dynamic behaviour of masonry structures. The package LMGC90 has been validated for its use on arched structures by assessment of the well-known problem of the minimum thickness arch. The error obtained for this analysis is less than 0.5%.

LMGC90 has consequently been used to model the behaviour of arches under sinusoidal base motion. These models have been compared to a series of experimental tests performed on arches made of wooden voussoirs and subject to horizontal motion on a seismic table. Various arches with different angles of embrace have been explored. The sinusoidal base motion comprised three cycles: attack, full-size and decay.

The DE models overestimated the collapse acceleration by approximately 25%. A numerical calibration of the models was carried out based on the arch with an angle of embrace of 156º. It consisted in calculating what reduced thickness would cause the arch to fail under the same level of acceleration as the experimental test. The resulting thickness was 82% of the original value. This reduction was applied to all computational models. The calibration reduced the discrepancy with the experimental tests to a maximum of 8%. The calibration process ought to be revised.

The collapse sequence predicted by the DE models agrees very well with that observed in the experimental tests.

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Abstract

Many historic structures are located in regions of significant seismic activity and are vulnerable to earthquake damage. To assess the safety of these structures, an understanding of the seismic performance of key elements such as masonry arches, vaults and domes is vital. While static stability of masonry arches can be studied analytically using the principles of limit analysis, numerical methods are needed to deal with more complex geometries and/or dynamic load cases. A promising numerical approach is the Discrete Element Method (DEM). Currently, DEM codes are rather specialised research tools, and further development and validation are needed to confirm their suitability. In this paper, the DEM code LMGC90 is used to analyse the responses of arches subjected to dynamic base motions, with the results compared to shaking table experiments on arches built from wooden voussoirs. It is shown that the code is able to model the sequence of hinge openings and the final collapse mechanism extremely well. However, the numerical simulations overestimate the base acceleration amplitudes required to cause collapse. This is thought to be due to the impossibility of modelling the exact experimental set-up, which is substantially affected by small imperfections in the contacts between adjacent voussoirs. The resultant error is found to be quite consistent between tests, and has been compensated by a simple calibration of the experimental models. Future work will seek improved methods of dealing with this issue, and extension of the DEM approach to arches and vaults made of concrete-like materials, with non-zero tensile strength.