Paper

Minimum Array Elements for Resolution of Several Direction of Arrival Estimation Methods in Various Noise-Level Environments

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Abstract—The resolution of a Direction of Arrival (DOA) estimation algorithm is determined based on its capability to resolve two closely spaced signals. In this paper, authors present and discuss the minimum number of array elements needed for the resolution of nearby sources in several DOA estimation methods. In the real world, the informative signals are corrupted by Additive White Gaussian Noise (AWGN). Thus, a higher signal-to-noise ratio (SNR) offers a better resolution. Therefore, we show the performance of each method by applying the algorithms in different noise level environments.

Keywords—covariance matrix, direction of arrival, geolocation, resolution, noise, smart antenna.

1. Introduction

Direction-of-arrival (DOA) estimation [1], [2] aims essentially to find the direction of arrival of multiple signals, which can be in the form of electromagnetic or acoustic waves, impinging on a sensor or antenna array. The requirement for DOA estimation arises from the needs of locating and tracking [3] signal sources in both civilian and military applications, such as search and rescue, law enforcement, sonar, seismology, and emergency call locating.

A large amount of work has been performed on DOA algorithms, e.g. [4]–[6]. In [2] Krim *et al.* presented an interesting comparative study between a set of DOA estimation algorithms, such as beamforming techniques and subspace-based methods. The basic idea of beamforming techniques [7]–[9] is to steer, electronically, the array in one direction at a time and measure the output power, so when the steered direction coincides with a DOA of a signal, the maximum output power will be observed. The scheme leads essentially to the formation of an appropriate form of output power that will be strongly related to the DOA.

Although beamforming techniques are simple to implement and require low computational time and power, they suffer from their poor resolution. For this reason, we introduce the concept of subspaces and propose the subspace-based methods [10], [11] that use the decomposition of the output data covariance matrix to benefit from the orthogonality of the two subspaces: the signal subspace and the noise subspace. Other methods have been proposed recently to overcome the computational load provided by the decomposition of the data covariance matrix, such as the propagator [12], [13] and the partial propagator [14].

Obviously, it has been proven [1], [15] that the accuracy and resolution of DOA estimation can be affected by several factors such as the number of the impinging sources, the number of array elements, the SNR, number of snapshots and angle differences [16]. In this paper, we focus on a study of the resolution capability of several DOA estimation algorithms by selecting the minimum array elements needed to separate closely spaced signals in different noise level environments. Our aim is to analyze the resolution performance of those methods, and at the same time, show their sensitivity against the noise. The study is restricted to one-dimensional signals that are assumed to be narrowband [17] and corrupted by a uniform Additive White Gaussian Noise (AWGN), impinging on a Uniform Linear Array (ULA).

2. Problem Modeling

Before presenting the data model, authors consider the same assumptions taken in [1]:

- isotropic and linear transmission medium,
- far-field,
- narrowband,
- the noise is AWGN.

Consider a ULA consisting of M identical elements that are aligned and equally spaced on a line by a distance Δ, receiving a wavefield generated by d narrowband sources in the presence of an AWGN, as presented in Fig. 1 [1].

The data received by the antenna array elements can be expressed as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

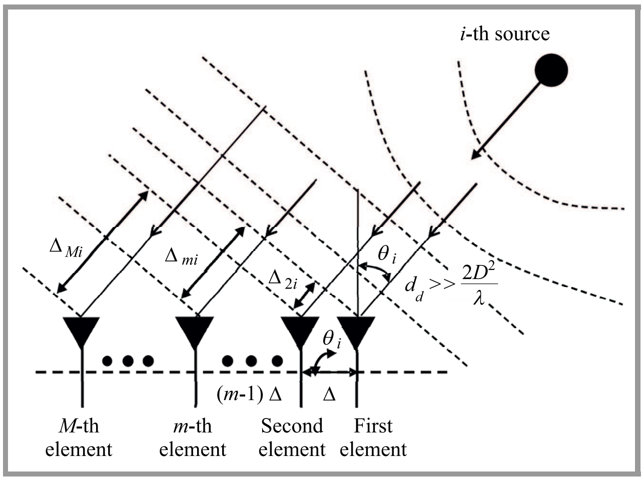

Fig. 1. Data model for DOA estimation of d sources with a linear array of the M element.

where $\mathbf{x}(t) = [x_1(t)\ x_2(t) \dots x_M(t)]^T$ denotes the received array data vector, $\mathbf{s}(t) = [s_1(t) \dots s_d(t)]^T$ denotes the source waveform vector, $\mathbf{n}(t) = [n_1(t)\ n_2(t) \dots n_M(t)]^T$ is the vector of the uncorrelated additive noise in the array.

$\mathbf{A} = [\mathbf{a}(\theta_1)\,\mathbf{a}(\theta_2) \dots \mathbf{a}(\theta_d)]$ denotes the steering matrix containing the steering column vectors $\mathbf{a}(\theta_i)$ defined as:

$$\mathbf{a}(\theta_i) = \begin{bmatrix} 1 & e^{\frac{j2\pi\Delta}{\lambda}\sin(\theta_i)} & \dots & e^{(M-1)\frac{j2\pi\Delta}{\lambda}\sin(\theta_i)} \end{bmatrix}^T ,$$

where Δ is the element spacing which satisfies $\Delta \leq \frac{\lambda}{2}$, λ is the wavelength of the propagating signals, and θ_i is the unknown direction of arrival of the i-th source.

The noise is assumed to be uncorrelated between array elements, and to have identical variance σ^2 in each element. Under this assumption, the $M \times M$ spatial covariance matrix of the data received by an array can be defined as:

$$\mathbf{R_{xx}} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma^2 I_M , \qquad (2)$$

where $(.)^H$ is the conjugate transposition, E is the expectation operator and $\mathbf{R}_{ss} = E[\mathbf{s}(t)\mathbf{s}^H(t)]$ is the $d \times d$ signal covariance matrix.

In practice, the exact $\mathbf{R_{xx}}$ is hard to find, due to the limited number of data sets received by the array, but it can be estimated by:

$$\mathbf{R_{xx}} \simeq \widehat{\mathbf{R}}_{\mathbf{xx}} = \frac{1}{N}\sum_{t=1}^{N} \mathbf{x}(t)\mathbf{x}^H(t) = \frac{1}{N}\mathbf{X}\mathbf{X}^H , \qquad (3)$$

where $\mathbf{X}$ denotes the noise corrupted signal (or data) matrix composed of N snapshots of $\mathbf{x}(t)$, $1 \leq t \leq N$. Many DOA estimation algorithms basically try to extract the information from this array data covariance matrix.

Knowing the data model, and before dealing with our principal aim, which is to study the resolution capability of several popular DOA estimation techniques by showing the minimum array elements they require to split two nearby sources, here is a brief overview about these techniques.

3. Algorithms

3.1. Conventional Beamforming

Conventional beamforming [7], also known as the Bartlett spectrum, is one of the beamforming techniques which are based on an electronic steering of the array in one direction at a time, and measure the output power, so when the steered direction coincides with a DOA of a signal, the maximum output power is observed.

An array can be steered electronically just as an antenna can be steered mechanically by designing a weight vector $\mathbf{w}$ and combining it with the data received by the array elements to form a single output signal $\mathbf{y}(t)$:

$$\mathbf{y}(t) = \mathbf{w}^H\mathbf{x}(t) . \qquad (4)$$

By taking N snapshots, the total averaged output power of an array is given by:

$$\mathbf{P}(\mathbf{w}) = \frac{1}{N}\sum_{n=1}^{N} |\mathbf{y}(t_n)|^2 = \frac{1}{N}\sum_{n=1}^{N} \mathbf{w}^H\mathbf{x}(t_n)\mathbf{x}^H(t_n)w = $$
$$= \mathbf{w}^H\widehat{\mathbf{R}}_{\mathbf{xx}}w . \qquad (5)$$

The conventional beamforming method consists of $\mathbf{w} = \mathbf{a}(\theta)$ with θ being the scanning angle, and the steering vector $\mathbf{a}(\theta)$ is defined as:

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 & e^{\frac{j2\pi\Delta}{\lambda}\sin(\theta)} & \dots & e^{(M-1)\frac{j2\pi\Delta}{\lambda}\sin(\theta)} \end{bmatrix}^T ,$$

where Δ is the element spacing which satisfies $\Delta \leq \frac{\lambda}{2}$, λ is the wavelength of the propagating signals.

In practice, $\mathbf{w} = \mathbf{a}(\theta)$ is normalized as:

$$\mathbf{w}_{Bartlett} = \frac{\mathbf{a}(\theta)}{\sqrt{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}} . \qquad (6)$$

Thus, the output power is obtained as:

$$\mathbf{P}_{Bartlett}(\theta) = \frac{\mathbf{a}^H(\theta)\widehat{\mathbf{R}}_{\mathbf{xx}}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{a}(\theta)} . \qquad (7)$$

3.2. Capon's Beamformer

The conventional beamforming method has a poor resolution. We can increase the resolution by adding array elements, as will be shown further. However, to overcome this problem, Capon [8] proposed a method that uses the degrees of freedom to form a beam in the look direction and at the same time the nulls in other directions. For a particular look direction, Capon's method uses all but one of the degrees of the freedom to minimize the array output power while using the remaining degrees of freedom to constrain the gain in the look direction to be unity:

$$\min \mathbf{P}(\mathbf{w}) = 0 \ \text{ subject to } \ \mathbf{w}^H\mathbf{a}(\theta) = 1 . \qquad (8)$$

Thus, the weight vector is expressed as:

$$\mathbf{w}_{Capon} = \frac{\widehat{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\widehat{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{a}(\theta)} . \quad (9)$$

By combining this weight vector with the Eq. (5), the output power is:

$$\mathbf{P}_{Capon}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\widehat{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{a}(\theta)} . \quad (10)$$

3.3. Linear Prediction

Linear prediction [9] aims to minimize the mean output power of the array, subject to the constraint that the weight on a selected element is unity. The weight vector is given by:

$$\mathbf{w}_{LP} = \frac{\widehat{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{u}}{\mathbf{u}^H\widehat{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{u}} \quad (11)$$

and the power spectrum is:

$$\mathbf{P}_{LP} = \frac{\mathbf{u}^H\widehat{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{u}}{\left|\mathbf{u}^H\widehat{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{a}(\theta)\right|^2} , \quad (12)$$

where $\mathbf{u}$ is a column vector of all zeros except for the selected element, which is equal to 1. This selected element corresponds to the position of the selected element in the array. There is no criterion for the proper choice of this element.

3.4. Maximum Entropy

Maximum entropy [18] is similar to the linear prediction method, it is based on an extrapolation of the covariance matrix. The extrapolation is selected with maximized signal entropy, where its maximum is achieved by searching for the coefficients of an auto-regressive model that minimize the expected prediction error:

$$\mathbf{w} = \min \ \mathbf{w}^H\widehat{\mathbf{R}}_{\mathbf{xx}}\mathbf{w} \ \text{ subject to } \ \mathbf{w}^H\mathbf{e}_i = 1 , \quad (13)$$

where $\mathbf{e}_i$ is a column vector of all zeros except for the i-th element, which is equal to 1.

Developing the computations leads to achieving the following power spectrum:

$$\mathbf{P}_{MEM} = \frac{1}{\mathbf{a}(\theta)\mathbf{C}_i\mathbf{C}_i^H\mathbf{a}(\theta)} , \quad (14)$$

where $\mathbf{C}_i$ is the i-th column of the inverse of $\widehat{\mathbf{R}}_{\mathbf{xx}}$.

3.5. MUSIC

Multiple Signal Classification (MUSIC) [10] is considered as one of the most popular subspace-based techniques. It uses the property of orthogonality between the two subspaces, the signal subspace and the noise subspace. The eigen-decomposition of the covariance matrix can be expressed as:

$$\mathbf{R}_{\mathbf{xx}} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma^2\mathbf{I}_M = \mathbf{U}_s\Lambda_s\mathbf{U}_s^H + \sigma^2\mathbf{U}_n\mathbf{U}_n^H , \quad (15)$$

where $\mathbf{U}_s$ is the matrix that contains the eigenvectors (the signal eigenvectors) corresponding to the d largest eigenvalues of $\mathbf{R}_{\mathbf{xx}}$, $\mathbf{U}_n$ is the matrix that contains eigenvectors (the noise eigenvectors) corresponding to the $M-d$ smallest eigenvalues of $\mathbf{R}_{\mathbf{xx}}$, the diagonal matrix Λ_s contains the M largest eigenvalues. Since the eigenvectors in $\mathbf{U}_n$, are orthogonal to $\mathbf{A}$, we have:

$$\mathbf{U}_n\mathbf{a}(\theta_i) = 0 \quad i = 1, \ldots, d . \quad (16)$$

Using this property, the power spectrum of MUSIC technique is:

$$\mathbf{P}_{MUSIC} = \frac{1}{\mathbf{a}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta)} . \quad (17)$$

3.6. Minimum Norm

The minimum norm technique can be seen as an enhancement of the MUSIC algorithm, it consists in finding the DOA estimate by searching for the peaks in the power spectrum:

$$\mathbf{P}_{MN} = \frac{1}{|\mathbf{w}^H\mathbf{a}(\theta)|^2} . \quad (18)$$

By determining the array weight $\mathbf{w}$, which is of minimum norm [18] we find the spectrum:

$$\mathbf{P}_{MN} = \frac{1}{|\mathbf{a}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{W}\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta)|} , \quad (19)$$

where the matrix $\mathbf{W} = \mathbf{e}_1\mathbf{e}_1^T$ ($\mathbf{e}_1$ is the first vector of a $M \times M$ matrix) is needed to make the matrix dimensions match mathematically.

3.7. The Propagator Method

To reduce the computational complexity of the methods that are based on the eigen-decomposition. The propagator method [12], [13], [19] exploits the partition of the data covariance matrix defined as:

$$\widehat{\mathbf{R}}_{\mathbf{xx}} = \begin{pmatrix} \widehat{\mathbf{R}}_1 \\ \widehat{\mathbf{R}}_2 \end{pmatrix} , \quad (20)$$

where $\widehat{\mathbf{R}}_1$ is a square matrix of size $d \times M$ and $\widehat{\mathbf{R}}_2$ is a matrix of size $(M-d) \times M$. The propagator operator is defined as:

$$\begin{cases} \widehat{\mathbf{R}}_2 = \Psi_{21}\widehat{\mathbf{R}}_1 \\ \Psi_{21} = \widehat{\mathbf{R}}_2\widehat{\mathbf{R}}_1^{\dagger} \end{cases} , \quad (21)$$

where $\widehat{\mathbf{R}}_1^{\dagger}$ is the pseudo-inverse of $\widehat{\mathbf{R}}_1$ defined as $\widehat{\mathbf{R}}_1^{\dagger} = \left(\widehat{\mathbf{R}}_1^H\widehat{\mathbf{R}}_1\right)^{-1}\widehat{\mathbf{R}}_1^H$. Then the noise subspace constructed by this operator is given by $\mathbf{U}_n = [\Psi_{21}, \mathbf{I}_{M-d}]$, and the power spectrum is:

$$\mathbf{P}_{Pr} = \frac{1}{\mathbf{a}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta)} . \quad (22)$$

3.8. The Partial Propagator

Unlike the propagator method, the partial propagator [14] only needs to use the partial covariance matrix and reduce the computation complexity. The partial propagator is based on partitioning the steering matrix into three blocks under the assumption $M > 2d$. The steering matrix is partitioned as:

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \end{pmatrix}, \tag{23}$$

where $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ are matrices of size: $d \times d$, $d \times d$, $(M-2d) \times d$ respectively. Using this partition, the partial correlation matrix are defined as:

$$\mathbf{R}_{12} = E\left[\mathbf{X}(t)(1:d,:)\mathbf{X}^H(t)((d+1):2d,:)\right] = \\ = \mathbf{A}_1\mathbf{R}_{ss}\mathbf{A}_2^H, \tag{24}$$

$$\mathbf{R}_{31} = E\left[\mathbf{X}(t)((2d+1):M,:)\mathbf{X}^H(t)(1:d,:)\right] = \\ = \mathbf{A}_3\mathbf{R}_{ss}\mathbf{A}_1^H, \tag{25}$$

$$\mathbf{R}_{32} = E\left[\mathbf{X}(t)((2d+1):M,:)\mathbf{X}^H(t)(d+1:2d,:)\right] = \\ = \mathbf{A}_3\mathbf{R}_{ss}\mathbf{A}_2^H, \tag{26}$$

where $\mathbf{X}$ is the matrix defined in Eq. 3. Based on these partitions, we define a matrix $\mathbf{U}_n$ as:

$$\mathbf{U}_n = \left[\mathbf{R}_{32}\mathbf{R}_{12}^{-1} \quad \mathbf{R}_{31}\mathbf{R}_{21}^{-1} \quad -2\mathbf{I}_{M-2d}\right] \tag{27}$$

for which we have: $\mathbf{U}_n\mathbf{A} = 0$. So, similarly to MUSIC and the propagator methods, we can form the power spectrum as follows:

$$\mathbf{P}_{Partial} = \frac{1}{\mathbf{a}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta)}. \tag{28}$$

4. Experimental Results

This section focuses on testing the resolution capability of each algorithm mentioned in Section 3. We determine the minimum number of the array antennas required to separate two far field sources that are spaced with an angular distance of 5°. The simulation is done by taking $d = 4$ sources impinging on a ULA of identical antennas with element spacing equaling to the half of the input signal wavelength, the number of snapshots is fixed at $N = 200$. Since the SNR highly influences on the resolution, four different noise level environments are considered in this study, which are $SNR_1 = -10$ dB, $SNR_2 = 0$ dB, $SNR_3 = 10$ dB, and $SNR_4 = 20$ dB. The number of array elements is thus varied until we find the minimum satisfying the resolution of the second and the third sources which are closely separated (5°). All the simulations are made using Matlab R2016b, the noise is a random process generated using a Matlab function and the signals are assumed to be snapshots of demodulated electromagnetic sources. Differentiation between the different sources is detected by vision. The degree of sensitivity to the number of array elements is different for the individual methods. This is why we notice, for some methods, that there's a small valley and a big one for others.

In the following figures, we show some of the simulations that we have performed. We present the response of each method for two values of the number of antennas, before and after resolution, at the noise level of $SNR_2 = 0$ dB.

We start with the conventional beamformer. Figure 2 represents the spectrum before and after resolution and the number of array elements used.

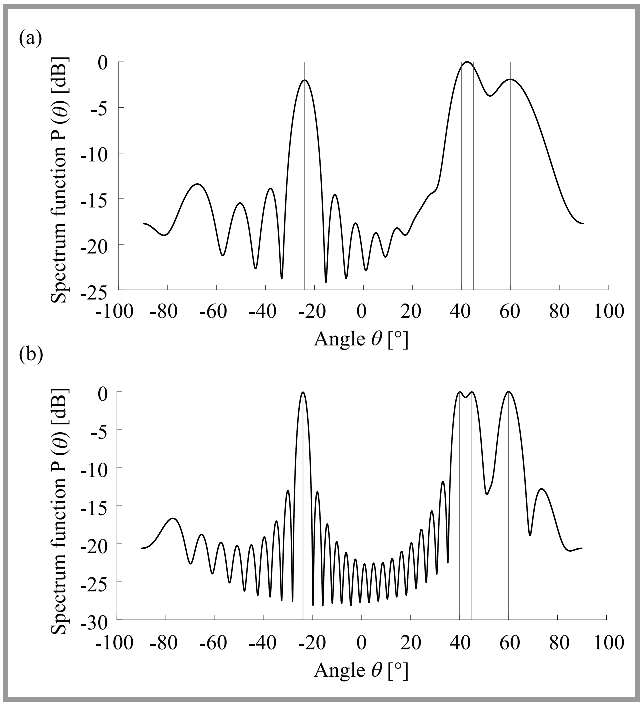

Fig. 2. Bartlett's spectrum: (a) before – 14 elements and (b) after – 30 elements.

We remark that as mentioned in Section 3, Bartlet's method has a poor resolution. Indeed, it requires about 30 elements as a minimum to slightly separate our two close sources.

Figure 3 shows the result obtained by using the Capon's beamformer technique.

With the Capon's beamformer, we start having a low number of elements needed to separate the two close sources (14 elements). It performs much better than the conventional beamformer at the resolution level, but as illustrated in Fig. 3, the separation is not complete. To achieve better resolution while using this method, we should add more elements.

Figure 4 shows the spectrum obtained by using the linear prediction method and choosing the selected element for $\mathbf{u}$ in Eq. (12) as the element in the center.

The linear prediction method performs well. As can be seen clearly in the Fig. 4, the two close sources are well separated once we use 12 array elements.

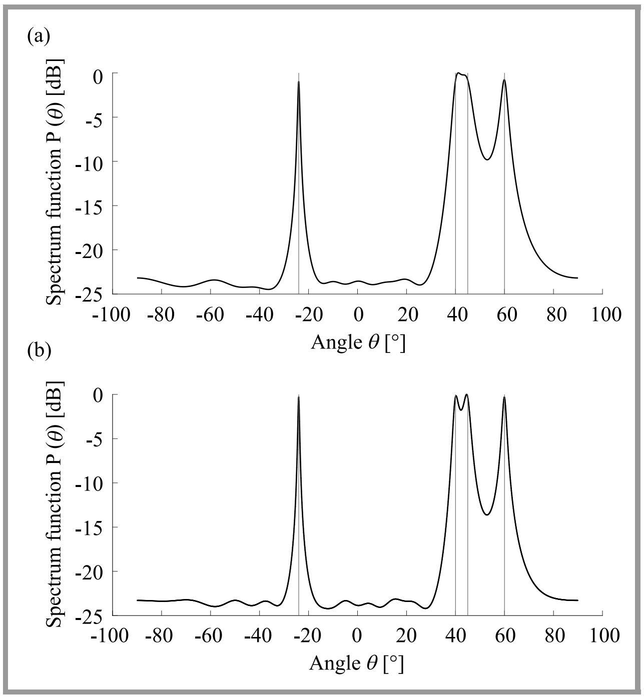

Fig. 3. Capon's spectrum: (a) before – 12 elements and (b) after – 14 elements.

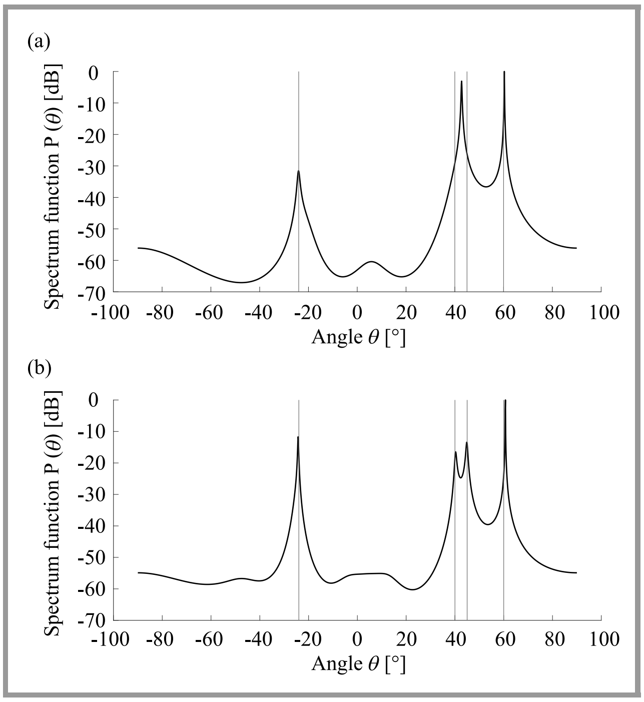

Fig. 4. Linear prediction spectrum: (a) before – 11 elements and (b) after – 12 elements.

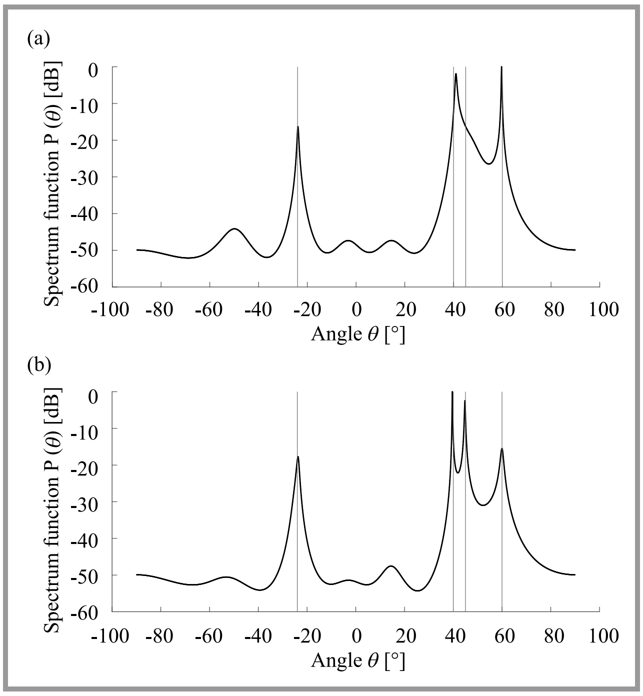

Fig. 5. Maximum entropy spectrum: (a) before – 13 elements and (b) after – 14 elements.

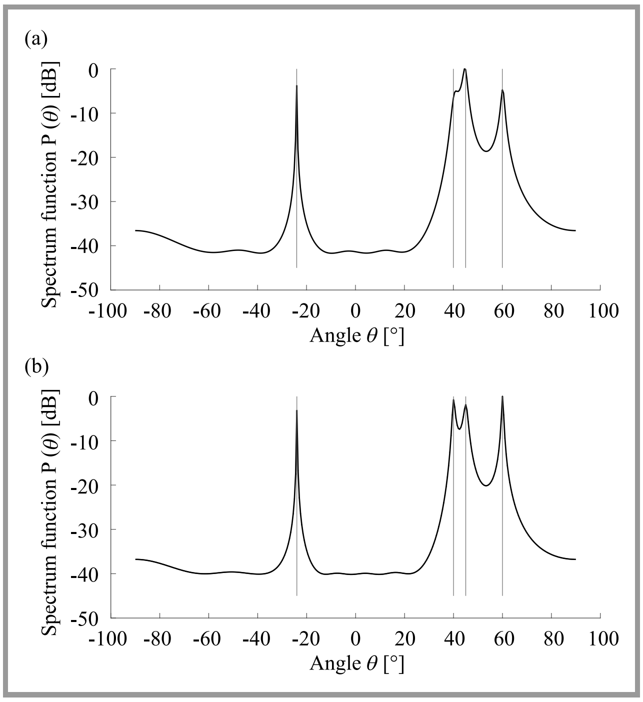

Fig. 6. MUSIC spectrum: (a) before – 9 elements and (b) after – 10 elements.

Figure 5 shows the result obtained by using the maximum entropy method by choosing $\mathbf{e}_i$ as the element in the center. As illustrated in Fig. 5, the maximum entropy method performs well too and allows to have a good resolution by using 14 array elements.

Figure 6 represents the result obtained by using the MUSIC method. With MUSIC, we could achieve a good resolution using only 10 array elements in this noise level. In addition, one can notice that the spectrum contains no secondary lobes which makes MUSIC be one of the most performing DOA estimation algorithms.

We now see the performance of minimum norm in Fig. 7. Minimum norm seems to be the best performing technique at this noise level among all the methods discussed earlier.

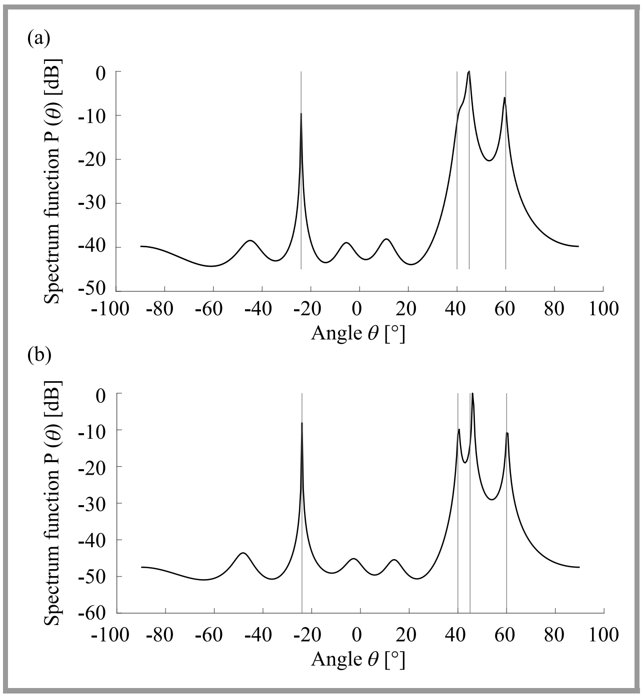

Fig. 7. Minimum norm spectrum: (a) before – 8 elements and (b) after – 9 elements.

It only needs 9 elements to give a good and clear resolution of the second and the third sources as illustrated in Fig. 7. We will see further the results found at other noise levels. The next spectrum is the propagator's one, it's represented in Fig. 8.

The propagator method requires 14 elements as a minimum to provide a clear resolution of the two close sources.

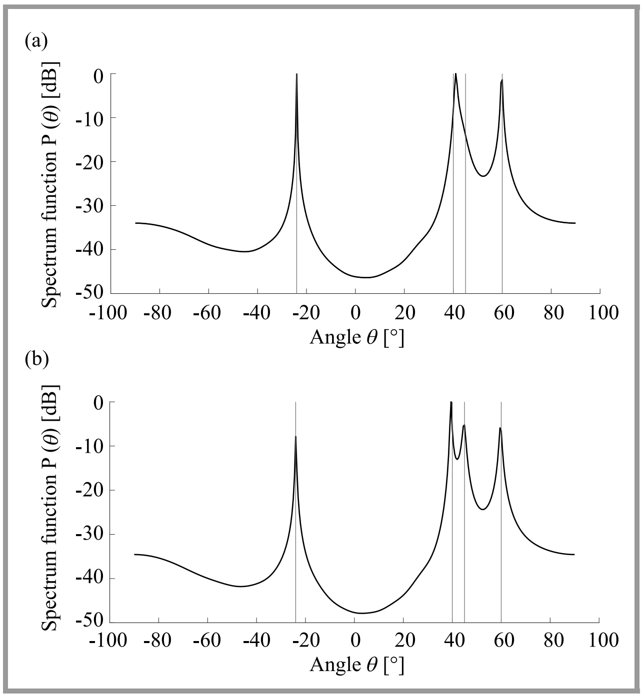

Fig. 8. Propagator spectrum: (a) before – 13 elements and (b) after – 14 elements.

Although the number of array elements required is higher than minimum norm and MUSIC, the big advantage of the propagator method is lower level of complexity compared with the eigen-decomposition-based methods [16].

We finally deal with the partial propagator method, its spectrum is illustrated in Fig. 9.

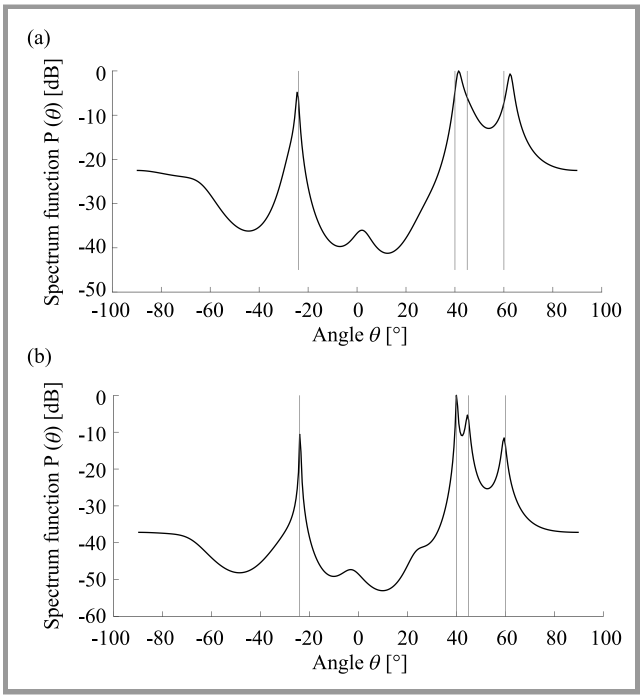

Fig. 9. Partial-propagator spectrum: (a) before – 10 elements and (b) after – 11 elements.

The partial propagator method needs at least 11 elements as a minimum to separate clearly the two close sources, which is also a good result of high resolution. In addition, the partial propagator performs well in the presence of a colored noise [14], and it also reduces the computational complexity compared to the propagator method.

We now discuss the resolution capability of these methods in four noise levels, namely $SNR_1 = -10$ dB, $SNR_2 = 0$ dB, $SNR_3 = 10$ dB, and $SNR_4 = 20$ dB.

Figure 10 illustrates the number of array elements needed for each method at the different noises levels, to resolve the two closely separated sources.

The first remark to be made here is that the Bartlett's spectrum is not influenced very much by the noise. Indeed, the minimum array elements remain stable for all the noise levels, and this can be explained by the fact that noise eigenvalues (the smallest ones) of the covariance matrix $\widehat{\mathbf{R}}_{\mathbf{xx}}$ do not have much influence in Eq. (7) because it's in the numerator of the equation, unlike the other methods which have the covariance matrix or some of its characteristics (like the noise subspace) in the denominator. As can be seen in Fig. 10, in noisy environments (low SNR), the minimum norm method performs better than all the others methods, by requiring fewer array elements for the resolution. On the other hand, one can see that in a high SNR environ-

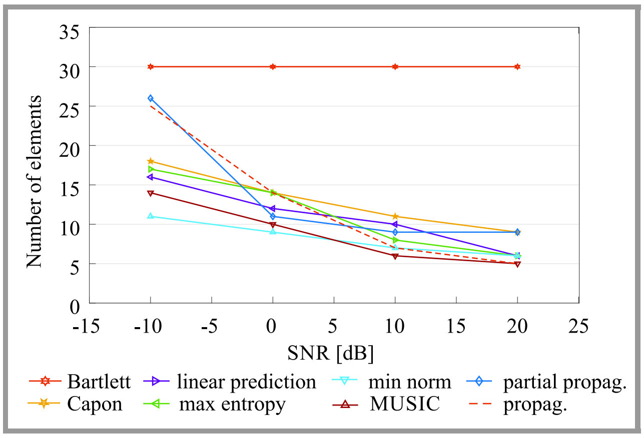

Fig. 10. Minimum elements needed for resolution in each SNR level.

ment, the MUSIC method is the best performing one. One can also note that in the noise-level SNR_4 the propagator requires the same number of elements as MUSIC.
However, an experimental verification of the proposed study in the research laboratory using physical materials is a direction for future work.

5. Conclusion

In this paper we have discussed the performance analysis related to the resolution capability of several DOA algorithms. The noise was assumed to be AWGN and the sources were narrowband and far-field impinging on a uniform linear array. The algorithms have been simulated under four different noise level environments. For each noise level, we have presented the performance of the resolution of the algorithms by searching the minimum array elements needed to separate two closely spaced sources. The results shown that in noisy environments, the minimum norm algorithm is the best performing one and requires fewer elements to separate the close sources. The minimum norm algorithm is more significant and in the same time the less sensitive to noise. Otherwise, in clean environments, MUSIC performs well and requires less array elements.

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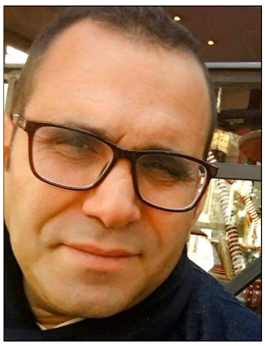

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