

MODELLING BATHYMETRY CHANGES WITHIN A WATERWAY VERSUS A LABORATORY EXPERIMENT

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Abstract

A three-layer theoretical model for transport of graded sediments was used in our analysis of the silting-up of waterways tested under laboratory conditions. The experiment was conducted in a laboratory basin, in which waves and a current were generated. The current interacted with the waves propagating perpendicularly to the direction in which it was flowing. It was assumed in the calculations that the sediment was entrained from the bed and suspended due to the impact of waves on the bed, after which it was transported by the current along the cross-shore profile of the navigation channel. Thus, it was assumed that the bathymetry changes occur only as a result of changes in the suspended load transport rate. The bed topographic modifications modelling results are in good agreement to the laboratory experimental results, including the rate of silting up the waterway and the bed reconfiguration. The key factor in the calculations concerning the waterway bed reconfiguration proved to be the inclusion of the effect of sediment size sorting on predicting the rate and character of bathymetry changes.

Introduction

One of the major problems in exploitation of waterways is the maintenance of the required navigation depth. Correct prediction of bathymetry changes and the silting-up rate is of key importance. This paper is an attempt at

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determination of morphological changes in a waterway bed analyzed during a laboratory experiment and compared to the results of computations. Afterwards, our results were confronted with the data reported by other researchers.

One of the conducted tests has been chosen in the analysis (T10.20.90; wave height about 0.1 m, current velocity about 0.2 m/s, wave propagation angle 90° , perpendicular to the channel – to the current flow direction). The choice was not haphazard but made in view of the fact that the results of this measurement were also analyzed in other renowned research centres (VAN RIJN 2005) under the European project SANDPIT (2002–2005) and, recently, by SANCHEZ and WU (2011). Thus, in order to be able to compare the results of our modelling test to other data, the same test was chosen.

For determination of changes in the bathymetry, a linear dependence is used that defines the relationship between a sediment transport rate and the thickness of a sediment layer composed of grains closely adhering to one another and in motion, described in greater detail by KACZMAREK et al. (2011). With this dependence, it is possible to solve an equation which describes bathymetry changes in time and space using the first-order upwind scheme. In this paper, a three-layer model of the transport of graded sediment was used for determination of the sediment transport rate and concentration of suspended sediment (KACZMAREK 1999, KACZMAREK, OSTROWSKI 2002, KACZMAREK et al. 2004).

The calculations were carried out in order to verify the model tested experimentally in a laboratory basin, where it was possible to create wave and current flow conditions with a simultaneous interaction of the current and waves propagating at an angle to the current flow direction.

Theoretical analysis, general assumptions of the model

Changes in the bed ordinate z_b after time dt can be presented as the equation:

$$z_b(x, t + dt) = z_b(x, t) + \frac{\partial z_m}{\partial t} dt \quad (1)$$

where

$$z_m^{+/-} = \frac{1}{(1 - n_p)} \frac{q^{+/-} dt}{dx^{+/-}} \quad (2)$$

and

$$z_m = z_m^+ + z_m^- \quad (3)$$

where z_m is defined as the thickness of densely packed sediment grains which is in motion (cf. KACZMAREK et al. 2011), while q stands for the volumetric rate of

sediment transport. Besides, parameters connected with the wave crest phase are designated as (...⁺), whereas those associated with the wave trough phase are marked with (...⁻).

Dependence (2) can be converted to the form describing the sediment transport rate:

$$q^{+/-} = (1 - n_p) z_m^{+/-} \frac{dx^{+/-}}{dt} \quad (4)$$

Equation (4) under hydrodynamic equilibrium conditions (verified experimentally – KACZMAREK et al. 2011) is a linear function versus the thickness z_m but non-linear versus the ordinate z_b .

The hydrodynamic equilibrium conditions means that the flux of sediment lifted up from the bed is balanced by the flux of sediment falling down on the bottom.

Next, taking advantage of dependences (2) and (3), an equation describing bathymetry changes is obtainable, which for a two-dimensional (2-D) case can be presented as follows (SAWCZYŃSKI et al. 2011):

$$\frac{\partial z_b}{\partial t} + \frac{1}{(1 - n_p)} \left(\frac{\partial q_x^+}{\partial x^+} + \frac{\partial q_x^-}{\partial x^-} + \frac{\partial q_y}{\partial y} \right) = 0 \quad (5)$$

where:

z_b – bed level elevation,

n_p – porosity of bed sediment,

q_y – alongshore sediment transport rate being under hydrodynamic equilibrium conditions,

q_x^+ – mean value during the wave phase T of onshore sediment transport rate being under hydrodynamic equilibrium conditions,

q_x^- – mean value during the wave phase T of offshore sediment transport rate being under hydrodynamic equilibrium conditions.

In area (j,k) of the cross-shore profile, z_m layer changes within the time period Δt . Erosion or accumulation occurs due to changes in the rates of transport q_x^+ and q_x^- over the distance Δx and q_y over the distance Δy . Using the numerical upwind scheme, equation (5) can be written down with the finite difference method in a form describing the thickness of an eroded or accumulated layer for each i^{th} fraction of the bed-forming sediment (cf. SAWCZYŃSKI et al. 2011):

$$\begin{aligned} \Delta(z_m)_{j,k}^i = & \frac{\Delta t}{\Delta x} [(q_{bx}^+)_{j,k}^i + (q_{cx}^+)_{j,k}^i + |(q_{bx}^-)_{j,k}^i| + |(q_{cx}^-)_{j,k}^i| + |(q_{0x}^-)_{j,k}^i| - (q_{bx}^+)_{j-1,k}^i] \\ & (q_{cx}^+)_{j,k}^i - |(q_{bx}^-)_{j+1,k}^i| - |(q_{cx}^-)_{j+1,k}^i| - |(q_{0x}^-)_{j+1,k}^i|] \\ & + \frac{\Delta t}{\Delta y} [(q_{0y}^-)_{j,k}^i - |(q_{0y}^-)_{j,k-1}^i|] \end{aligned} \quad (6)$$

where:

$$\Delta(z_m)_{j,k} = \sum_{i=1}^N \Delta(z_m)_{j,k}^i \quad (7)$$

The terms of the expressions marked with the indices b , c and 0 refer, respectively, to the bedload layer, contact load layer and outer flow region.

Within time Δt , grain-size distribution changes from $n_{j,k}^i$ to $m_{j,k}^i$ ($\Sigma m_{j,k}^i = 1$) in the control volume according to the formula (cf. KACZMAREK et al. 2004):

$$\Delta m_{j,k}^i = \frac{n_{j,k}^i (h_m)_{j,k} - \Delta(z_m)_{j,k}^i}{(h_m)_{j,k} - \Delta(z_m)_{j,k}} \quad (8)$$

In equation (8), value $(h_m)_{j,k}$ described thickness of the mixing layer, in which the grain-size distribution actually changes from $n_{j,k}^i$ to $m_{j,k}^i$ in time Δt . This change is caused by the difference in the rates of transport of particular fractions over the distances Δx and Δy .

As demonstrated by Kaczmarek et al. (2004), the mixing layer thickness can be described by the following dependence:

$$(h_m)_{j,k} = 2 \frac{\Delta t}{\Delta x} [(q_{bx}^+)_{j,k} + (q_{cx}^+)_{j,k} + |(q_{bx}^-)_{j,k}| + |(q_{cx}^-)_{j,k}| + |(q_{0x}^-)_{j,k}|] + 2 \frac{\Delta t}{\Delta t} |(q_{0y})_{j,k}| \quad (9)$$

Laboratory tests have proven that the mixing layer thickness ranges from 2 cm (SISTERMANS 2001) to 5 cm (CHATELUS et al. 1998).

If we assume that the change which occurs in the analyzed control volume of the thickness $(h_m)_{j,k}$ is erosion (value of $\Delta(z_m)_{j,k}$ is positive), then, after Δt time, this erosion will form “a carpet of sediment” $a_{j,k}$ thick:

$$a_{j,k} = (h_m)_{j,k} - \Delta(z_m)_{j,k} \quad (10)$$

Thus, after Δt time, i.e. at a time moment $(t + \Delta t)$, a new grain-size distribution in a new mixing layer of the thickness $(h_m)_{j,k}$, will consist of the distribution $m_{j,k}^i$ found in an $a_{j,k}$ – thick carpet of sediment and the distribution $(n_s)_{j,k}^i$ characteristic of the sediment in the parent bed:

$$[n(t + \Delta t)]_{j,k}^i = \frac{a_{j,k} m_{j,k}^i + [(h_m)_{j,k} - a_{j,k}](n_s)_{j,k}^i}{(h_m)_{j,k}} \quad (11)$$

However, if we assume that within the analyzed area sediment accumulation rather than erosion takes place (value $\Delta(z_m)_{j,k}$ is negative), the new grain-

size distribution after Δt time at a time moment $(t + \Delta t)$ in a new mixing layer of the thickness $(h_m)_{j,k}$ will consist exclusively of the distribution $m_{j,k}^i$ found in a carpet of sediments of the thickness $a_{j,k}$.

In a situation where, for example, a wave propagates along the waterway (parallel to its longest axis) without causing any changes in the bathymetry in that direction (when waves are formed over a shallow bottom, no bathymetry changes take place), and the direction of the current flow is perpendicular to the direction of waves, then the problem is reduced to a two-dimensional one. The question formulated as above means that the evolving considerations will concern exclusively the effects produced by the alongshore current, which means that we will have to leave out the terms connected with wave formation and the offshore current from equations (5), (6), and (9).

The rate of the alongshore sediment transport which takes place in the outer flow region can be presented as (SAWCZYŃSKI et al. 2011):

$$q_{0y} = \int_{\delta_{cr}}^h U_{0y}(z) \langle C_0(z) \rangle dz \quad (12)$$

where:

h is the depth of water, δ_{cr} is the upper boundary of contact load layer – shared by all fractions (KACZMAREK 1999) and velocity $U_{0y}(z)$ is the velocity of the alongshore current.

In order to achieve the averaged over time T value of the concentration $\langle C_0(z) \rangle$ of suspended sediment mixture, the following dependence is used (cf. e.g. RIBBERINK, AL-SALEM 1994):

$$\langle C_0(z) \rangle = \langle C_0(z = \delta_{cr}) \rangle \left(\frac{\delta_{cr}}{z} \right)^{\alpha_1} \quad (13)$$

where concentration on the reference level (δ_{cr}) is determined by averaging over the wave phase the results obtained for the contact load layer (c_c):

$$\langle C_0(z = \delta_{cr}) \rangle = \sum_{i=1}^N \langle c_{ci}(z = \delta_{cr}, t) \rangle n_i \quad (14)$$

BIEGOWSKI (2006) established the value of the exponent at $\alpha_1 = 0.6$.

The values of concentration c_{ci} in the contact load layer for all fractions of the sediment, averaged over the wave phase, are established according to the three-layer model of graded sediment (KACZMAREK 1999, KACZMAREK, OSTROWSKI 2002, KACZMAREK et al. 2004).

Description and conditions of the laboratory experiment Test 1

The laboratory measurements (HAVINGA 1992, WALSTRA et al. 1999) were completed at WL | Delft Hydraulics (current name Deltares) located in the north of the Netherlands.

Directional waves, in the form of the JONSWAP spectrum of the frequency peak 0.4 Hz, were generated. A system of pumps was used to create a current in the basin. The water depth was about 0.4 m. The canal was filled with sediment in its terminal part, and the bed consisted of fine sand ($d_{50} = 100 \mu\text{m}$; $d_{90} = 130 \mu\text{m}$), lying on the same level as the concrete floor of the basin around the channel. A detailed description of the Test 1 experiment was given by HAVINGA (1992).

The bed was shaped in the sand layer so as to resemble a navigation channel. Changes in the ordinates of the bed were monitored by regular soundings of the bed for over 25 hours in three measurement sections (Fig. 1). The measurements of the initial profile were as follows: depth about 0.2 m, bed width about 0.5 m and side slopes about 1 : 8. The trench (the longest axis) was situated normal to the current and parallel to the direction of wave propagation, cf. Figure 1.

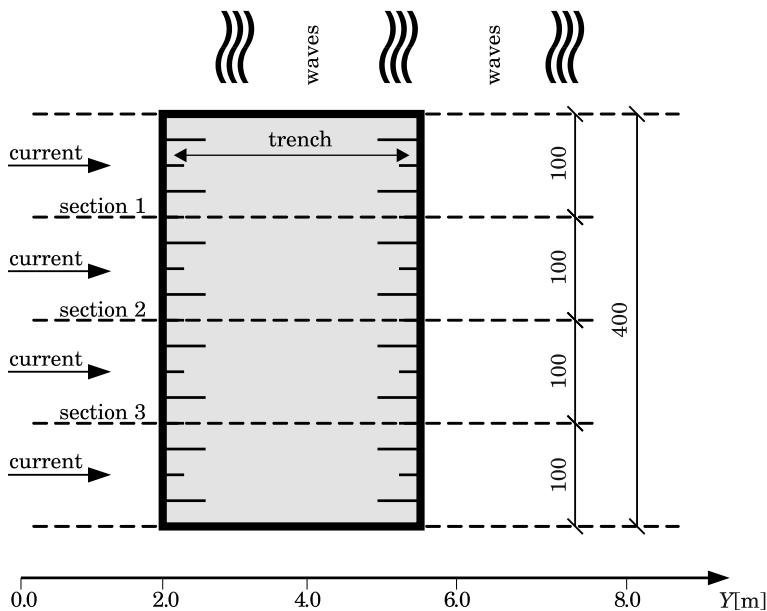


Fig. 1. Design of the Test 1 experiment

Source: HAVINGA (1992)

The basic parameters measured on the inflow side during the experiment are presented in Table 1.

Basic parameters of the Test 1 experiment

Table 1

Parameter	Symbol	Value	Unit
Water depth	h_0	0.42	[m]
Significant wave height	H_s	0.105	[m]
Peak wave period	T_p	2.2	[s]
Depth – mean current velocity	u_0	0.245	[m/s]
Angle between current and waves	α	90	[°]
Representative particle size of bed	$d_{50}; d_{90}$	0.10; 0.13	[mm]
Fall velocity of suspended sediment	w_s	0.006	[m/s]
Suspended sand transport	$q_{s,o}$	0.018 – 0.024	[kg/s/m]
Ripple height	–	0.007	[m]
Ripple length	–	0.084	[m]
Sediment density	ρ_s	2650	[kg/m ³]
Fluid density	ρ_w	1000	[kg/m ³]
Porosity of bed material	n_p	0.4	[–]

Source: HAVINGA (1992)

Modelling distributions of the vertical concentration of suspended sediments

A three-layer model of the graded sediment transport (KACZMAREK 1999, KACZMAREK, OSTROWSKI 2002, KACZMAREK et al. 2004) is used for mathematical modelling of the vertical concentration of suspended sediment, which needs to be known in order to determine the sediment transport rate in the outflow region described by equation (12).

Two available parameters, i.e. representative diameters d_{50} and d_{90} , were taken as input data for sediment. By knowing these data, we were able to plot the input grain-size distribution curve, shown in Figure 2. The grain-size distribution shown in this paper was used in calculations for Test 1 experiment (HAVINGA 1992) and Test 2 experiment (VAN RIJN 1985), which were analyzed in a paper by SAWCZYŃSKI et al. (2011). Both experiments were also thoroughly discussed under the international project SANDPIT (VAN RIJN 2005).

Due to the narrow range of diameters of the fine sediment used in the experiment, three representative diameters, reflecting the lower values of the diameter interval ranges shown as a histogram in Figure 2, were taken for the calculations. The lower threshold values of these intervals correspond to the control sieve mesh sizes.

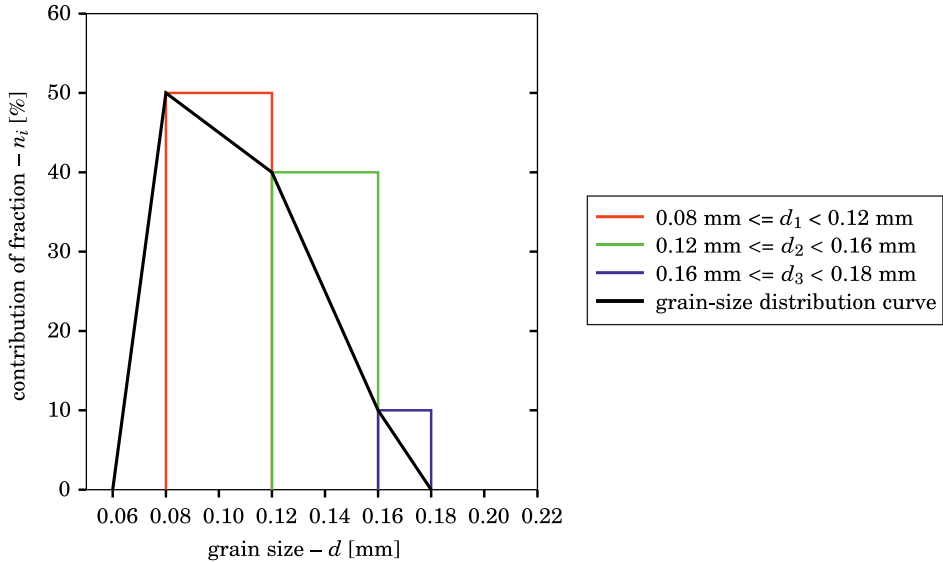


Fig. 2. Histogram of grain-size distribution according to d_{50} and d_{90} , used in the calculations

The mathematical modelling of the vertical concentration of suspended sediments was accomplished for a regular, symmetric (sinusoid) wave described by two parameters: root mean square wave height (H_{rms}) and zero-crossing wave period (T_z). The computed vertical concentration distributions were then compared with the results obtained during the experiment at a cross-section located at the beginning of the canal ($y=0$) before the model.

The measurement data from the experiment enabled us to analyze vertical distributions of the suspended sediment concentrations under weak hydrodynamic conditions. The dimensionless friction calculated only for the wave formation conditions (excluding the effect of a current), most often described by Shields' parameter $\Theta_{2.5}$ (calculated for $d_{50} = 0.10$ mm), assumed the values within $0.08 < \Theta_{2.5} < 0.12$. These values of Shields' parameter then indicate that the bed was rippled throughout the experiment (cf. Table 1).

Figure 3 shows a graphic comparison of the calculated vertical suspended sediment concentration profile versus the profile measured during the Test 1 experiment (HAVINGA 1992, WALSTRA et al. 1999). Calculations of the concentrations were completed based on the initial data contained in Table 2 and the bed grain-size distribution curve, as shown in Figure 2.

Table 2
Basic entry data used for modelling vertical concentration of suspended sediment and bathymetry changes

Parameter	Symbol	Value	Unit
Water depth	h_0	0.42	[m]
Root mean square wave height	H_{rms}	0.074	[m]
Zero – crossing wave period	T_z	1.86	[s]
Representative particle size of bed	d_{50}, d_{90}	0.10; 0.13	[mm]
Sediment density	ρ_s	2650	[kg/m ³]
Fluid density	ρ_w	1000	[kg/m ³]
Porosity of bed material	n_p	0.4	[-]

Figure 3 shows that the concentration of suspended sediments was not measured at a distance very close to the bed. The authors could not measure concentrations near the bed due to the varied and above all considerable height of ripples observed during the tests.

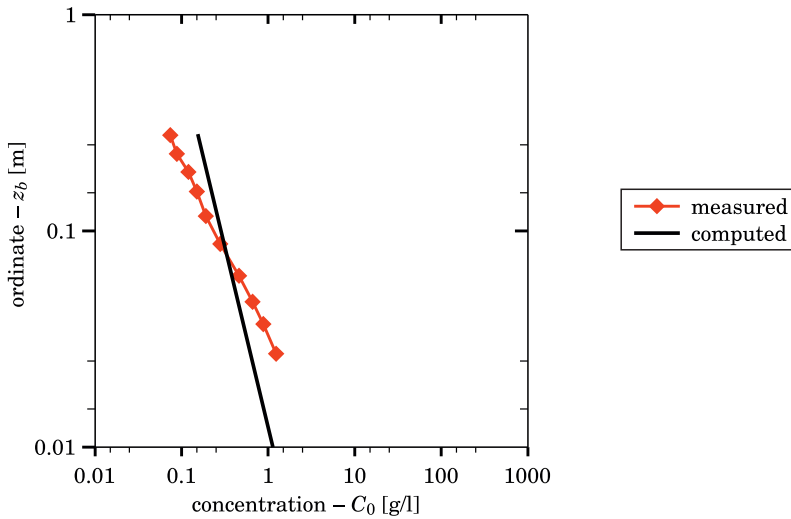


Fig. 3. Comparison of the computed (present model) and measured vertical profile of suspended sediment concentrations

Source: HAVINGA (1992)

In Figure 3, a slight difference is visible between the inclination angles of the graph drawn along the measurement points and the one obtained from the calculated sediment concentrations. At a height of about 7.5 cm (from the bed), the graphs intersect with each other, which obviously means that the cal-

culated and measured values are identical. At a distance closer to the bed, the measured concentration values are slightly higher than the ones calculated from the theoretical model, whereas at a larger distance, they are slightly smaller. This slight discrepancy of the results may be due to the fact that the analyzed hydrodynamic conditions were very weak and the bed was rippled.

Besides, as has been demonstrated by BOSMAN (1982, 1985), local and instantaneous measurements of concentration are typically burdened with high and random variability of results, ranging from 50 up to 100%. The main reason for such high variability of measurements is the strong influence of local conditions, especially near the bottom. In order to minimize such differences in measured concentrations of sediments, time- and space-averaged results are considered. Although the results are averaged, the differences may remain significant when the bed is rippled.

Considering the above, as well as any error possibly hidden in the measuring method used in the experiment, it is possible to claim quite confidentially that the coincidence between the measurements and the results calculated from the proposed theoretical model is satisfactory.

Modelling bathymetry changes and sediment grain size distributions

The process of modelling of bathymetry changes consisted of two major stages:

- excluding the effect of changes in the sediment grain-size distribution during the bed reconfiguration process,
- including the effect of size sorting on the bed reconfiguration.

A three-layer model of graded sediments transport (KACZMAREK 1999, KACZMAREK, OSTROWSKI 2002, KACZMAREK et al. 2004) was used for determination of the sediment transport rates. Two available parameters, i.e. representative diameters d_{50} and d_{90} , were taken as entry data in the mathematical modelling of bathymetry changes and the sediment grain-size distribution. The grain-size distribution curve can be seen in Figure 2. The basic entry parameters used for calculations are presented in Table 2. Other data included in the calculations are the angle between the current flow direction and the wave propagation direction, $\alpha = 90^\circ$ (normal to the waves) and depth-averaged velocity of the current $u_0 = 0.245$ m/s, related to the water depth $h_0 = 0.42$ m (cf. Figure 5). This current is identifiable with an alongshore current, characteristic for coastal zones. The graph showing changes in the calculated mean velocity of the alongshore current along the initial profile of the bed (for time simulation $t = 0$), depending on the water depth, is given in Figure 4.

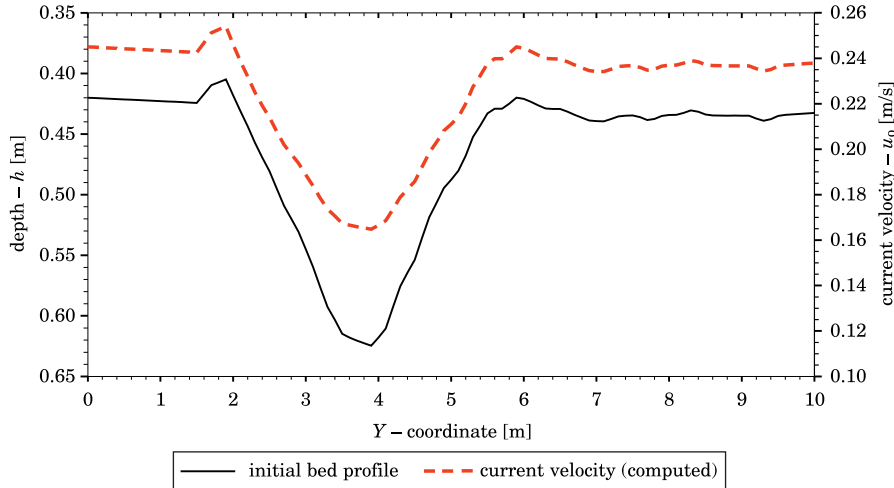


Fig. 4. Variability of the calculated alongshore current velocity in the initial bed profile depending on the water depth – experiment Test 1

Moreover, in our calculations of changes in the bed morphology and changes in the sediment grain-size distribution, we assumed that sediment is entrained from the bed into the suspension state only by the impact of waves onto the bed. Next (considering a two-dimensional case), the load lifted from the bed is transported by the current along the cross-shore profile of the navigation channel. Thus, it was assumed that modifications of the bed occur as a result of changes in the rate of suspended sediment transport.

Calculation procedure

It was decided that the spatial simulation would begin at the coordinate $y = 0.0$ m, and finish at $y = 10.0$ m, cf. Figure 5. The computational step in space was taken as 0.1 m, as a result of which 101 measuring points were achieved, including two extreme ones on the boundaries of the modelling area. In order to determine the sediment transport rate, the vertical interval was set up from the depth of $h = 0.35$ m to $h = 0.65$ m. This interval was divided into minute layers, each of the thickness equal the computational step in space, i.e. $\Delta h = 0.001$ m, and then, for each of the layers, the sediment transport rate was established.

The determination of the computational step in time Δt was accomplished based on the calculations derived from the assumption that the mixing layer thickness described with equation (9), determined in locations where the water depth was the smallest (i.e. the sites where the hydrodynamic impact on the

bed was the strongest) was approximately 2.0 cm (including the porosity of the sediment, approximately 3.3 cm). From the transformation of equation (9), the computational step in time was calculated to equal $\Delta t = 90$ s.

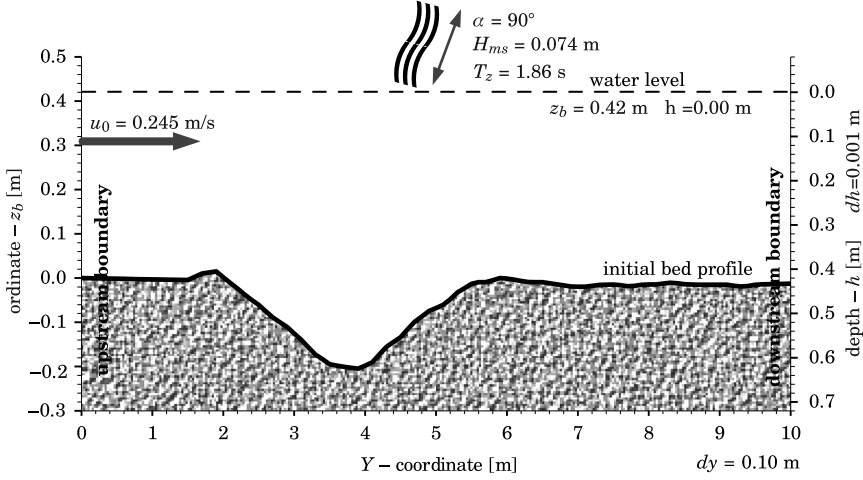


Fig. 5. Entry data for calculations of bathymetry and grain-size distribution changes

On the left-hand side margin of the solution area $y = 0.0$ m, for the sediment transport in the direction of the current flow, when $q_y > 0$, a marginal condition $z_b(y = 0, t) = z_{b(0)}(t) = 0.000$ m was set (corresponding to the water depth of $h_0 = 0.420$ m), for time $t \geq 0$.

As the navigation channel is silted up exclusively due to the impact of the current (a two-dimensional problem), equation (5) is simplified by zeroing out the wave-derived terms q_x^+ and q_x^- , which consequently leads to a one-stage solution of equation (5) describing bathymetry at each time level Δt .

Thus, at each time level, equation (5) is solved, simultaneously setting the initial condition: $z_b(y, t) = z_b^{(t=0)}$, with the function $z_b^{(t=0)}$ describing the initial bathymetric profile of the flume imitating a navigation channel, and the marginal condition on the left-hand side boundary: $z_b(y = 0, t) = z_{b(0)}(t) = 0.000$ m. Moreover, it has to be noticed that no condition is set for the right-hand side boundary.

For determination of a new grain-size distribution after Δt time, at a time moment $(t + \Delta t)$ in a new mixture layer of the thickness $(h_m)_{j,k}$, calculations are made based on some of the equations from (6) to (11) chosen depending on which process – accumulation or erosion – occurs at a given point. It should be underlined that the scope and course of the experiment (cf. Fig. 1 and Fig. 5.)

make us adopt a simplification in differential equations (6) and (9) consisting in the omission of the terms connected with waves and the offshore current.

Next, calculations are carried out at the subsequent time step ($t + \Delta t$) and the whole process is terminated once the final simulation time T is reached.

Comparison of the results of modelling bathymetry changes for uniform sediment with the experimental data

In the first stage of modelling the bathymetry evolution, calculations were carried out without taking into account changes in the bed grain-size distribution, which take place in response to changes in the sediment transport rate.

It was assumed that the bed was composed of uniform sediment, i.e. of grains of an identical diameter. The duration of the simulation event was 25.5 hours. Calculations were carried out for four situations, i.e. the bed was composed of grains of the diameter (i) $d = 0.08$ mm, (ii) $d = 0.10$ mm, (iii) $d = 0.13$ mm, (iv) $d = 0.16$ mm. Fig. 6 shows the results of the modelling of the sediment transport rate for the bed's initial profile, i.e. for time $t = 0$ and for uniform sediment and four grain diameters.

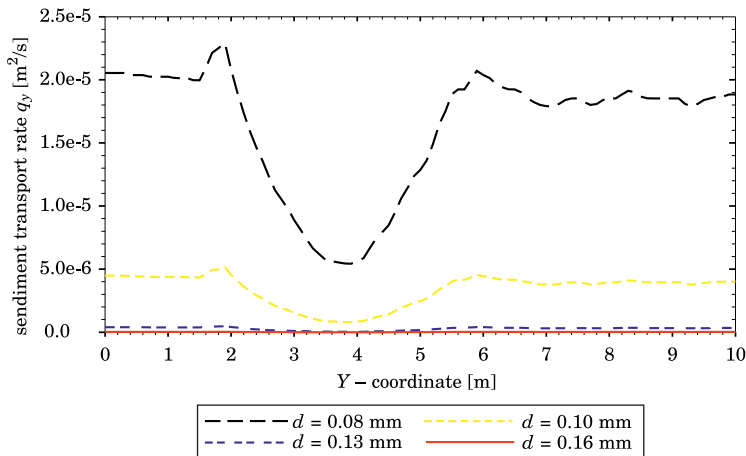


Fig. 6. Results of modelling of the sediment transport rate for the bed's initial profile, and for uniform sediment

The results of the modelling of changes in the bed's morphology for uniform sediment are presented in Figure 7. Because fine sediment fractions are transported much more intensively than coarser ones, the modelling of changes in the bed's morphology for the finest sediment ($d = 0.08$ mm) leads to the navigation channel being completely silted up. For the coarsest fraction ($d = 0.16$ mm), which remains immobile, that is does not participate in the

sediment transport under the effect of hydrodynamic forces at deeper water depths (cf. Fig. 6), no bathymetry change is practically observable. For grains of the diameter $d = 0.13$ mm, the sediment transport rate is slightly higher than calculated for grains measuring 0.16 mm in diameter, which also caused small changes in the reconfiguration of the waterway bed.

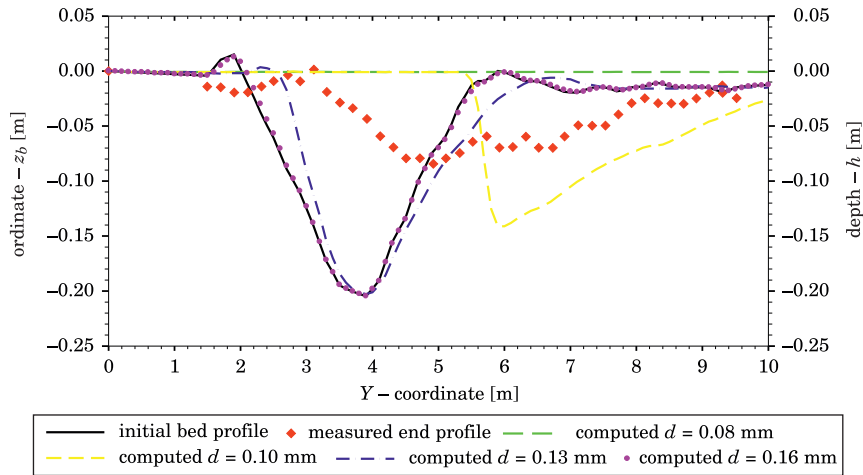


Fig. 7. Changes in the bed profile after $T = 25.5$ hours. Comparison with the results of the modelling for uniform sediment

Considering the above information and the data presented in Figure 7 corresponding to the changes in the bed profile after time $T = 25.5$ hrs for uniform sediment, it can be concluded that none of the results of calculations of the changes in the bed's morphology, at any of the tested situations, is a reflection of the results achieved during the measurements. Even if we take into consideration the representative diameter of sand grains, such as $d_{50} = 0.10$ mm, the results of modelling changes in the level of the bed of the navigation channel are far different from the results obtained during the laboratory experiment.

Effect of the sediment sorting on the bathymetry changes

During the second stage of modelling bathymetry changes, the calculations included the effect of sediment sorting on the reconfiguration of the bed. An additional analysis was performed for a case where sediment consisted of a mixture of three fractions (cf. Fig. 2) but changes in its grain-size distributions were not included into the calculations. Thus, it was assumed that at all points of calculations, located along the length of the cross-shore profile and during the

whole time of the simulation experiment, the bed was composed of the load whose grain-size distribution was constant and identifiable with the original sediment.

Figure 8 shows a comparison of the modelling results with the measurements of the final bathymetry (after time $T = 25.5$ hrs) of the navigation channel. The results of calculations for sorting and non-sorting sediments were distinguished. Figure 9 illustrates the final changes in grain-size distributions, where – additionally – the initial grain-size distribution (for time $t = 0$) was shown (thin, dotted, horizontal lines).

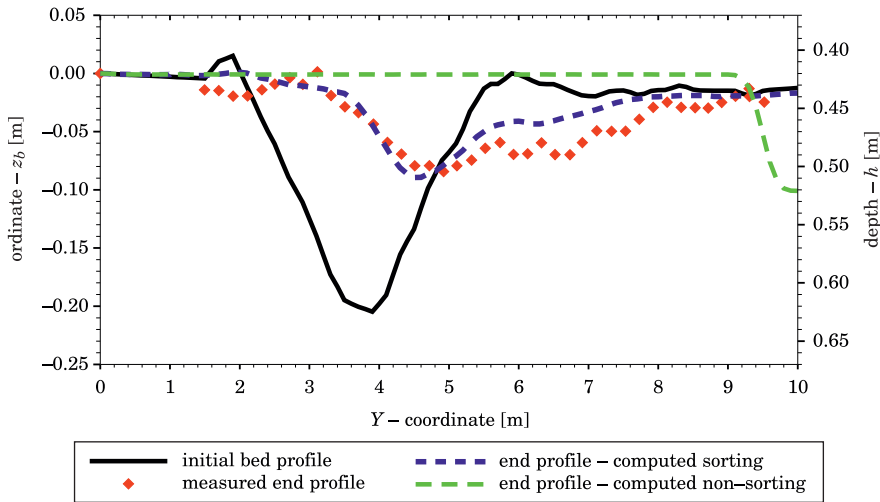


Fig. 8. Comparison of the results of modelling changes in the bed level with measurements after time $T = 25:30$ h

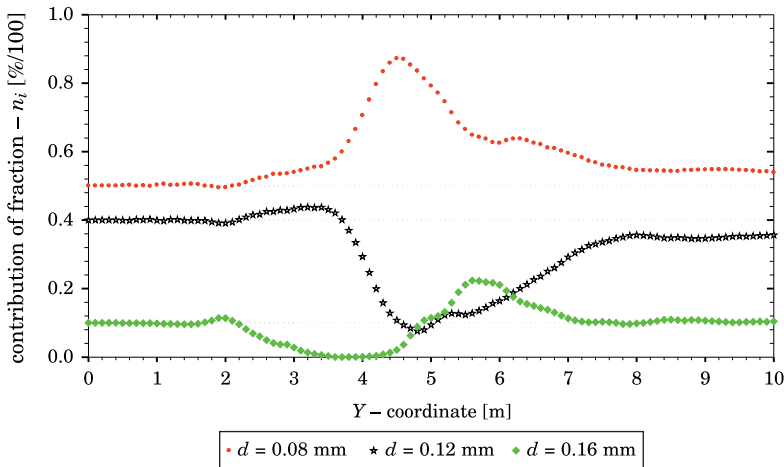


Fig. 9. Results of modelling changes in the grain-size distribution after time $T = 25:30$ h

The bed morphology modifications modelling results demonstrates that the navigation channel migrates towards the direction of the current. With the current flow conditions throughout the experiment, the resultant sediment transport (understood as a difference in the transports of all the sediment fractions at points j and $j-1$) assumes positive values on the downstream slope (cf. Fig. 6), thus increasing the depth, that is causing erosion of the bed. A reverse situation can be observed on the upstream slope. The resultant sediment transport achieves negative values and therefore the depth decreases, which means deposition of sediment.

The results of our modelling of the changes in the morphology of the bed over time show that when the process of sediment grading is not included in calculations then changes in the bed shape occur much faster than the measured results. Also, the character of bed reconfiguration is different. Noteworthy is an exceptionally large angle of the inclination of the upstream slope as well as a high rate of migration, which is so fast that eventually the navigation channel is far outside the scope of measurements.

Recapitulating, when the calculations do not account for changes in the grain-size distribution of the sediment, then the rate of migration of the canal is too fast and – at the same time – the rate of the accumulation process, which slows down the migration of the canal and causes the silting-up of the waterway, is too slow.

As Figure 6 demonstrates, at the biggest depths tested during the experiment, only the finest fractions were transported while the coarse ones did not participate in motion. In areas of the highest deposition, where practically only the finest fraction ($d = 0.08$ mm) is transported (Fig. 6), a considerable increase in its percentage in the total grain-size distribution is observed, until reaching the maximum value for the whole profile, and a simultaneous decrease in the share of fractions $d = 0.12$ mm and $d = 0.16$ mm occurs. In the areas subjected to maximum erosion, the material is becoming coarser, a development confirmed by a decreasing tendency in the contribution of the finest fraction $d = 0.08$ mm, a decrease in the percentage of fraction $d = 0.12$ mm and a large increase in the share of the coarsest fraction $d = 0.16$ mm, up to the highest value for the whole profile.

Bathymetry changes – comparison with the modelling results

Tests on calibration and improvement of own mathematical models has been undertaken by acclaimed research centres participating in the international project SANDPIT (VAN RIJN et al. 2005, WALSTRA et al. 2005). The models were scaled on two planes, i.e. ‘A’ and ‘B’, in order to ‘adjust’ the

results of modelling to the results achieved during a laboratory experiment (VAN RIJN et al. 2005, WALSTRA et al. 2005). The ‘A’ problem concerned the adjustment of results of calculated changes in the bed’s bathymetry according to the measured values of the sediment transport rate; the other approach (‘B’) dealt with observations of changes in the bed’s morphology over time. Comparison of the results of calculations of bathymetry changes based on the authors’ own mathematical model, versus some previous results achieved once the mathematical models suggested and tested by the SANDPIT project participants had been scaled, is illustrated in Figure 10 (approach ‘A’) and Figure 11 (approach ‘B’). More details on the process of scaling the mathematical models can be found in the articles by WALSTRA et al. (2005) and VAN RIJN et al. (2005).

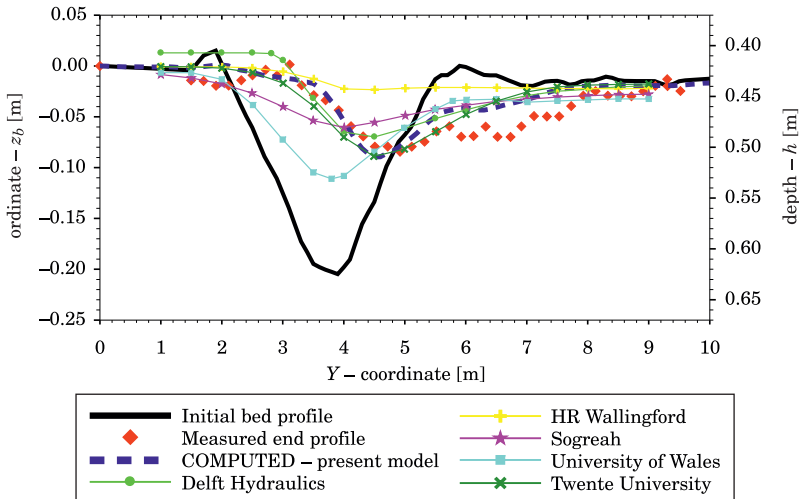


Fig. 10. Comparison of the results of modelling bathymetry changes after time $T = 25:30$ hours. Approach ‘A’ – calibration of models performed by the SANDPIT project participants based on measured sediment transport rates

Figure 10 and Figure 11 show that none of the models tested under the SANDPIT project was able to correctly predict erosion and flattening on the downstream slope or estimate the rate of migration and nature of changes of the downstream slope of the navigation channel. It was also interesting to notice that calibrations of the models carried out according to the observations of changes in the bed’s morphology in time and space did not result in attaining a better agreement between the results of calculations and measurements of the end morphology.

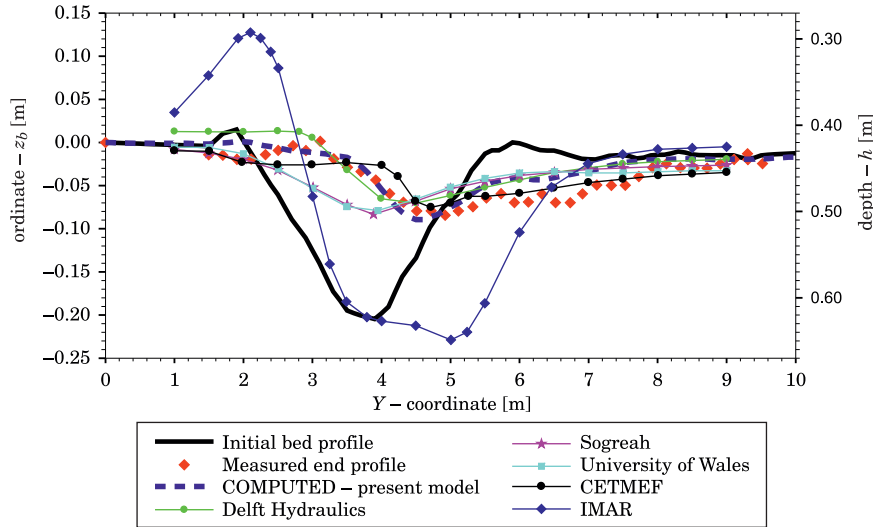


Fig. 11. Comparison of the results of modelling bathymetry changes after time $T = 25:30$ hours. Approach 'B' – calibration of models performed by the SANDPIT project participants based on observations of changes in the bed's morphology

Recently, SANCHEZ and WU (2011) showed some interest in the experiment called Test 1. Analogously to researchers participating the SANDPIT project, they decided to verify their own model with reference to the results obtained from the laboratory measurements. Some important conclusions can be found in their article (SANCHEZ and WU 2011). Comparison of the results of modelling bathymetry changes over time $T = 25:30$ hours and the results obtained by SANCHEZ and WU (2011) versus the results derived from the laboratory measurements are shown in Figure 12.

This comparison is impressive but we should bear in mind that the solution reported by SANCHEZ and WU (2011), analogously to the other models cited in this paper, does not provide us with any information regarding changes in bed grain-size distributions within the navigation channel or its environs.

Such information, however, is given by the mathematical model presented in this paper. Thus, it is worth noticing once more a much better agreement between the bathymetry changes computed when a higher number of fractions in the bed grain-size distribution is included versus the measured data than achieved with the model used in Test 1 experiment and presented by SAWCZYŃSKI et al. (2011) or the model in Test 2. Figure 13 shows two slightly different variants of bed grain-size distribution consisting of far more fraction intervals (seven instead of three in Fig. 2), but also preserving the indices characteristic for d_{50} and d_{90} , i.e. $d_{50} = 100 \mu\text{m}$ as well as $d_{90} = 130 \mu\text{m}$.

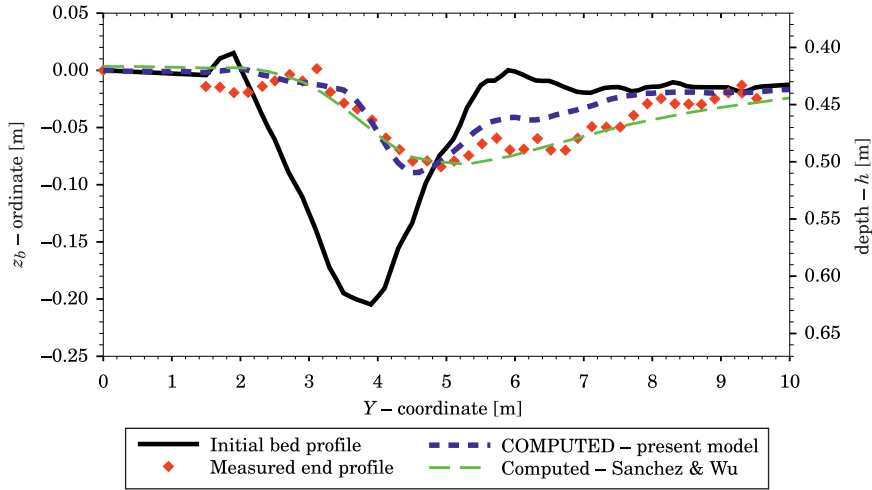


Fig. 12. Comparison of the results of modelling bathymetry changes after time $T = 25:30$ hours with the measurements and the results obtained by SANCHEZ and WU (2011)

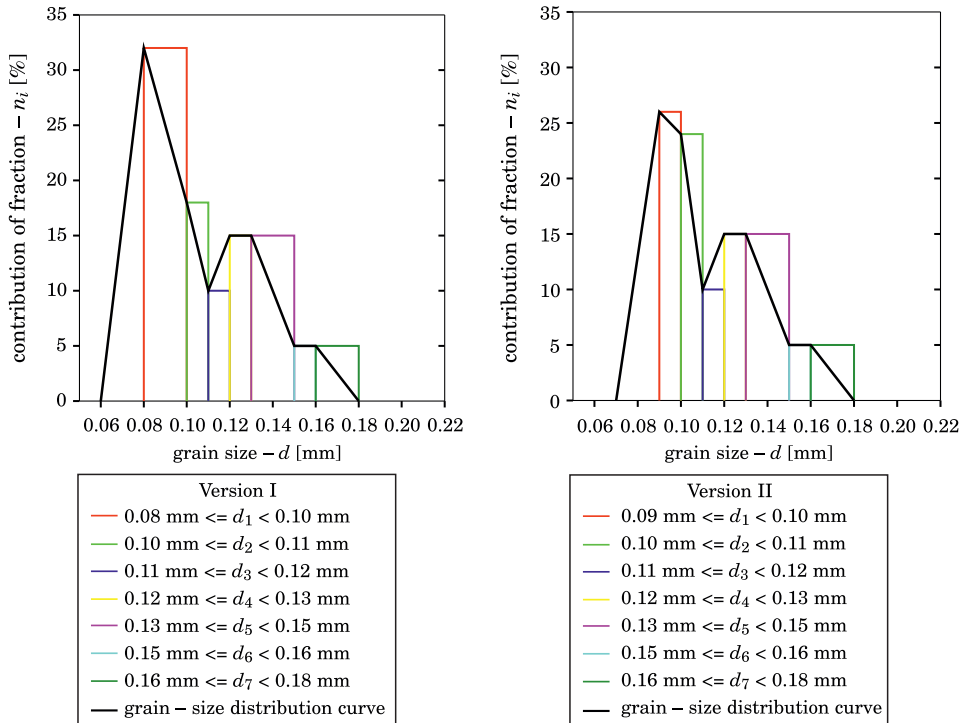


Fig. 13. Histograms of two variants of bed grain-size distribution according to indices d_{50} and d_{90} , included in calculations in order to attain the most precise reflection of the results of bathymetry changes obtained by taking measurements

Figure 14 illustrates results of modelling changes in the bed's morphology using these grain-size distribution patterns.

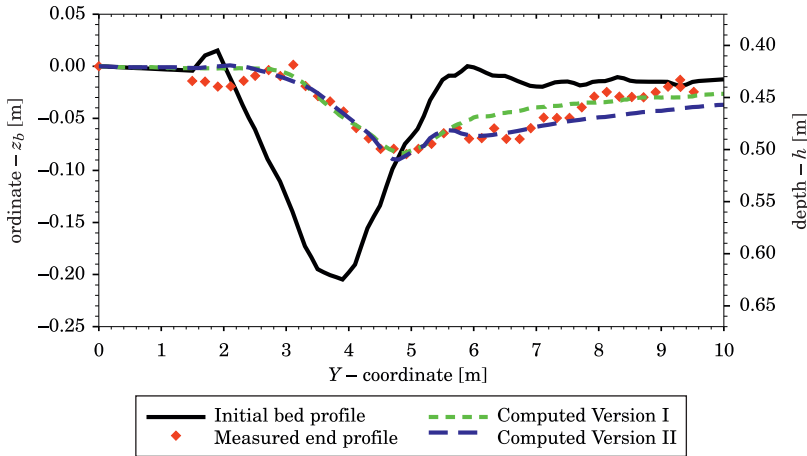


Fig. 14. Comparison of results of modelling bathymetry changes after time $T = 25:30$ hours with results of measurements. An attempt at the possibly most precise reflection of results of bathymetry changes obtained during measurements

By introducing into calculations a higher number of grain fractions, it was possible to improve considerably the results of modelling changes in the bed configuration on the downstream side, i.e. to increase its erosion. At the same time, it was observed that the results for the upstream side slightly improved. The results of calculations carried out for two variants of bed grain-size distribution are a kind of an envelope within which the experimental results are successfully fitted. It is therefore concluded that the precision of computed bathymetry changes improves as the number of sediment grain fractions included in a modelling effort is higher. The effect of sediment grain-size distribution on the rate and character of the bed profile reconfiguration remains very important.

Summary

When comparing results of computations of bathymetry changes of a waterway, excluding or including the effect of changes in the sediment grain-size distribution modifications on the bed reconfiguration, it can be noticed and should be emphasized that the latter are in a better agreement with the results obtained from laboratory measurements. The agreement was

further improved when the number of sediment grain fractions was higher. Thus, it is advisable to underline the significant effect played by the process of sediment grading on bathymetry changes, not only because of the rate at which a waterway is silted up but also because of the character of the bed reconfiguration.

While performing the present calculations, the authors did not calibrate their model, in contrast to reported applications of the other models, whose results of computed morphological changes within a waterway are likewise presented in this article. It is not necessary to scale the present model because of the proposed method (that does not require scaling) of a mathematical description of sediment transport as well as bathymetry changes and interrelated changes in bed grain-size distribution, presented in greater detail by KACZMAREK et al. (2011) and SAWCZYŃSKI et al. (2011).

Recapitulating, the mathematical model presented in this paper can be recommended as a useful tool, which could be applicable to predicting bathymetry changes within hydroengineering constructions (waterways) and a rate of silting-up of navigation channels to harbours, but also to determination of grain-size distributions of sediments lying on the bed of a navigation channel.

Translated by JOLANTA IDŹKOWSKA

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