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A conceptual framework for risk assessment and risk management

Keywords

risk, probability, risk assessments, uncertainties

Abstract

This paper presents and discusses a conceptual framework for risk assessment and risk management where risk is based on the triplet events, consequences and uncertainties. In addition to risk, the framework highlights the concepts of vulnerability and resilience. An example of the analysis of an LNG (Liquefied Natural Gas) plant is included to demonstrate the applicability of the framework. The proposed framework is more general than existing frameworks, for example the traditional Kaplan & Garrick approach, and provides also new perspectives on how to understand and describe uncertainties in a risk assessment and risk management context.

1. Introduction

A common perspective on risk is the so-called triplet definition based on Kaplan and Garrick [20]:

> Risk is equal to the triplet (s_i, p_i, c_i), where s_i is the *i*th scenario, p_i is the probability of that scenario, and c_i is the consequence of the *i*th scenario, i = 1,2, …N.

For unique situations, the probabilities are interpreted as subjective probabilities whereas, if repeated similar situations can be generated, the probabilities p_i have to be understood as relative frequency-interpreted probabilities (also referred to as chances). These probabilities are unknown, and subjective probabilities are used to express the (epistemic) uncertainties about the true value of the relative frequencies. The framework then established is referred to as the probability of frequency approach to risk assessment.

Aven [4], [8] extends the framework by considering risk as the triplet (A,C,U), where A is the initiating events (e.g. a leakage), C the consequences of A, and U the associated uncertainties (will A occur, what will the consequences C be?). Based on this risk concept, risk is described by (A,C,P,U,K) where P is a subjective probability and K is the background knowledge (including assumptions) that the assessments P and the uncertainties U are based on.

In this paper we provide an example to show the main features of this risk perspective. These features also cover key concepts like vulnerability and resilience. Comparisons are made with the Kaplan and Garrick [20] perspective. Before we introduce the case, we outline the main pillars of the (A,C,U) risk perspective.

2. The (A,C,U) risk perspective

We define risk by two-dimensional combination of [4], [8]

i) events A and the consequences of these events C, and
ii) the associated uncertainties U (will A occur and what value will C take?). (I)

We refer to this as the (A,C,U) perspective, as already mentioned. We may rephrase this definition by saying that risk associated with an activity is to be understood as [9]:

Uncertainty about and severity of the consequences of an activity (I').

Here severity refers to intensity, size, extension, scope and other potential measures of magnitude, and is with respect to something that humans value (lives, the environment, money, etc.). Losses and gains, for example expressed by money or the number of fatalities, are ways of defining the severity of the consequences. The uncertainties relate to the events and consequences; the severity is just a way of characterising the consequences [9].

To describe the uncertainties, we use subjective (knowledge-based) probabilities. If the probability equals 0.1 (say), this means that the assessor compares his/her uncertainty (degree of belief) about the occurrence of the event with the standard of drawing at random a specific ball from an urn that contains 10 balls [21]- [22]. A risk description based on this perspective includes the following elements: (A,C,U,P,K), that is, risk is described by events and consequences, associated uncertainties (whether A will occur and what value C will take), knowledge-based probabilities P, and K the background knowledge that U and P are based on. The probability assignments are based on hard data, expert judgments and models.

This perspective acknowledges that risk extends beyond probabilities. Probability is just a tool used to express the uncertainties but is not a "perfect" tool. By restricting risk to the probability assignments alone, aspects of uncertainty and risk are "hidden": there could be a lack of understanding about the underlying phenomena, and assumptions can be made bounding the space of possible events and scenarios, but the probability assignments alone are not able to fully describe this status. To explain this in more detail, consider the following two examples:

Consider the risk, seen through the eyes of a risk analyst in the 1970s, related to future health problems for divers working on offshore petroleum projects. An assignment is to be made for the probability that a diver would experience health problems (properly defined) during the coming 30 years due to the diving activities. Let us assume that an assignment of 1% is made. This number is based on the available knowledge at that time. There are no strong indications that the divers will experience health problems. However, we know today that these probabilities led to poor predictions. Many divers have experienced severe health problems [10], p. 7. By restricting risk to the probability assignments alone, we see that aspects of uncertainty and risk are hidden. There is a lack of understanding about the underlying phenomena, but the probability assignments alone are not able to fully describe this status.

As a second example, consider an offshore petroleum installation where the operations management is concerned about the deterioration of some critical equipment. The maintenance discipline ensures that the deterioration will not cause safety problems. It refers to a special maintenance programme that will be implemented, which will cope with the deterioration problem. So what is the risk associated with hydrocarbon leakages caused by operational problems? Given the background information of the maintenance discipline, a 10% leakage probability (for a defined leakage size) is assigned. This number is based on relevant historical data, and does not in any respect reflect the concern of the operation's management. The assignment assumes that the maintenance programme will be effective. But surprises could occur. Production of oil over time leads to changes in operating conditions, such as increased production of water, H_2S and CO_2 content, scaling, bacteria growth, emulsions, etc.: problems that to a large extent need to be solved by the addition of chemicals. These are all factors causing increased likelihood of corrosion, material brittleness and other conditions that may cause leakages. By the assignment of 10%, we hide an important element of uncertainty. In a risk analysis a number of such probability assignments are performed, and the hidden uncertainties could create surprising outcomes somewhere. You do not know where they will come, but they definitely could happen.

The risk description (A,C,U,P,K) covers probability distributions of A and C, as well as predictions of A and C, for example a predictor C*, given by the expected value of C, unconditionally or conditional on the occurrence of A, i.e. $C^* = EC$ or $C^* = E[C|A]$. Stochastic models (with parameters) expressing aleatory uncertainty, i.e. variation in populations of similar units, are used to ease the probability assignment. Probability models constitute the basis for statistical analysis, and are considered essential for assessing the uncertainties and drawing useful insights [17], [32]. The probability models coherently and mechanically facilitate the updating of probabilities in line with the Bayesian paradigm. However, such models need to be justified, and if introduced they are to be considered as tools for assessing the uncertainties about A and C. The estimation of the parameters of the models is not the end product of the analysis as in a traditional risk analysis [8].

The knowledge-based (subjective) probabilities express *epistemic uncertainties*, the assessor's (lack of) knowledge about an event or an unknown quantity. The stochastic models represent variation in the populations of similar units to the one (those) studied. To formalise the concept of "similar", we need to introduce the term "exchangeability", but for the purpose of the present paper it is sufficient to think of the situations as similar. Loosely speaking, a chance, which we denote P_f, is the Bayesian term for a frequentist probability (cf. the representation theorem of de Finetti [13]; see Bernardo and Smith [12], p. 172. The frequentist probability used in a traditional statistical setting also represents a "success fraction" of an infinite population. Think of the throw of a drawing pin. The frequentist probability that the pin is up is understood as the fraction of throws showing pin up if the experiment could be repeated infinitely under similar conditions. In theory these concepts, the chance and the frequentist probability, are not the same, but from a practical point of view it is difficult to see much difference [3].

Chances (frequentist probabilities) P_f are in fact not measures of uncertainty. They do not represent the analyst's uncertainties. They are unknown quantities and need to be treated as such in the risk assessment. Bayesian analysis is a common tool used for this treatment. The idea is to firstly establish adequate probabilistic models representing the aleatory uncertainties, then to assess epistemic uncertainties about unknown parameters of these models by assigning prior distributions, next to use Bayes' formula to update the uncertainties in light of new data to obtain the posterior distributions, and finally to obtain the predictive distribution of the quantities of interest, i.e. A and C (this predictive distribution is epistemic, but it also reflects the aleatory uncertainties).

A risk assessment is a methodology designed to determine the nature and extent of risk, i.e. assess the risk (A,C,U). It comprises the following main steps:

1. Identification of hazards/threats/ opportunities (sources)
2. Cause and consequence analysis, including analysis of vulnerabilities
3. Risk description, using probabilities and expected values
4. Risk evaluations, i.e. comparisons with possible risk tolerability (acceptance criteria)

Risk management comprises all co-ordinated activities to direct and control an organization with regard to risk, i.e. manage risk. Two main purposes of the risk management are to ensure that adequate measures are taken to protect people, the environment and assets from undesirable consequences of the activities being undertaken, and to balance different concerns, for example safety and costs. Risk management covers both measures to avoid the occurrence of hazards/threats, and measures to reduce their potential consequences.

Next we introduce the concepts "vulnerability" and "resilience" [5], [29]:

Vulnerability (antonym robustness) = (C, U| A), in other words, the vulnerability is the two-dimensional combination of consequences C and associated uncertainties U, given the occurrence of an initiating event A. For example, the vulnerability of a person with respect to a certain virus is the potential consequences of this virus and associated uncertainties (what will the consequences be?). The definition of vulnerability follows the same logic as that of risk. The uncertainty of various consequences can be described by means of probabilities, for example for the probability that the person will die from the virus attack. A description of vulnerability thus covers the following elements:

(C, U, P, K | A)

i.e. the possible consequences C, uncertainty U, probability P, and the background knowledge K, given that the initiating event A takes place. In line with Aven and Renn [9], we may interpret vulnerability in relation to the event, A, as uncertainty about and severity of the consequences of an activity given the occurrence of A.

When we say that a system is vulnerable, we mean that the vulnerability is considered high. The point is that we assess the combination of consequences and uncertainty to be high should the initiating event A occur. If we know that the person is already in a weakened state of health prior to the virus attack, we can say that the vulnerability is high. There is a high probability that the patient will die.

Vulnerability is an aspect of risk. Because of this, the vulnerability analysis is a part of the risk analysis. If vulnerability is highlighted in the analysis, we often talk about risk and vulnerability analyses.

Resilience is closely related to the concept of robustness. The key difference is the initiating event A. Robustness and vulnerability relate to the consequences and uncertainties given a fixed A, whereas resilience is open for any type of A, also surprising events. We may get ill due to different types of virus attacks; also new types of viruses may be created. From this idea we define resilience as

Resilience: (C,U|any A, including new types of A)

and the resilience description:

(C,U,P,K | any A, including new types of A).

Hence the resilience is considered high if the person has a low probability of dying due to any type of virus attack, also including new types of viruses. Resilience is about the consequences in the case of

any "attack" (virus attack) and associated uncertainties. We say that the system is resilient if the resilience is considered high. Of course, in practice we always have to define some boundaries for which A's to include.
For all these definitions, the consequences C depend on the performance of barriers (denoted B) [15], and to explicitly show this we write C = (B,C), resulting in a resilience description (B,C,U,P,K| any A, including new types of A).
The performance of the barrier can be expressed through the capacity of the barrier (and associated uncertainty, probability), for example the strength of a wall. The barriers and the system performance in general are influenced by a number of performance influencing factors (PIFs), for example resources, level of competence, management attitude, etc.
Analogous to risk assessment and risk management, we define vulnerability assessment, vulnerability management, resilience assessment and resilience management (engineering), for example:

Resilience engineering (management) comprises all measures and activities carried out to manage resilience (normally increase resilience).

These measures and activities are based on the PIFs. For example, we may add resources or increase the competence to obtain a higher level of resilience. We may exercise and avoid smoking to increase the resilience in case of an illness.
The above definition of resilience is in line with the one given by Hollnagel [18]: the intrinsic ability of a system to adjust its functioning prior to or following changes and disturbances, so that it can sustain operations even after a major mishap or in the presence of continuous stress.

A risk analysis following the (A,C,U) perspective describes risk by (A,C,U,P,K), as explained above. To ensure that the risk analysis includes the vulnerability and resilience dimensions, we may add that the risk assessment should also highlight the descriptions

(C,U,P,K|A) and (C,U,P,K | any A, including new types of A).

3. Application. An LNG (Liquefied Natural Gas) plant in an urban area

This case study concerns the risk related to a new LNG plant to be located on the west coast of Norway, in an urban area (Tananger) outside the city of Stavanger, about 4 km from the Stavanger Airport. Despite the formal approval according to the SEVESO II Directive, there is considerable resistance against the plant from the neighbours living less than one kilometre from the plant. The LNG plant is located only a few hundred metres from a ferry terminal and this also creates concern.
The LNG plant is now under construction by the energy supplier Lyse. The necessary approval from local and central authorities has been obtained. The plan is that natural gas from the North Sea is transported through pipelines to shore, and then liquefied at the plant before it is stored in a huge tank. The LNG is then distributed from the plant to local consumers by LNG tankers and LNG lorries [30]. The annual production is 300,000 tons of LNG per year, but the capacity may be increased to 600,000 tons if market conditions allow such an increase. The LNG plant has the following main components [31]: Pipeline landfall, Gas reception facilities, Pre treatment, LNG production, LNG tank and Export facilities.
In the following we will look at two potential ways of conducting the risk assessments for this plant, in line with the risk perspective described in the previous section. Some comments on how the assessments were in fact carried out in real life are given throughout the discussion.

3.1 An analysis approach with no chances introduced

The risk assessment will be based on uncertainty assessments of the "observables" A and C, so we first have to identify these. In the study two interesting observables are:

> N: the number of fatalities (3^{rd} parties)
> D: the occurrence of an accident leading to a fatality of person z (arbitrarily chosen).

The aim of the risk assessment is to predict these quantities and to describe uncertainties. In this case the predictions would be straightforward: there would be no fatalities and the event D would not occur. However, there are uncertainties, and accidents could occur, leading to deaths. To describe these uncertainties we use event tree models and subjective probabilities. We may introduce chances (relative frequencies), as in Section 3.2, but let us first assume that the analysts decide not to do so. They find that

- it is difficult to give meaningful interpretations of the chances.
- the introduction of the chances makes the analysis more complex, and no added value is identified.

In the following we restrict attention to N. To ease the assignment of knowledge-based (subjective) probabilities of N, we introduce a model g, an event tree model; see Figure 1. Here

X = number of releases (which is approximately equal to 1 if a release occurs and 0 otherwise as we ignore the probability of two releases in the period studied)
Z_1 = I(A) (I is the indicator function which is equal to 1 if the argument is true and 0 otherwise)
Z_2 = I(B)
Z_3 = I(pool fire).
$Z = (Z_1, Z_2, Z_3)$

We see that if a release occurs, it can either result in a pool fire, an explosion or no effect, depending on the results of the branching events, immediate ignition, and delayed ignition.
The model provides four scenarios:

s_1: release - A - pool fire
s_2: release - not A – B - flash (pool) fire
s_3: release - not A – B - explosion
s_4: release - not A - not B - no effect.

Assume that the number of people exposed to scenario, s_i is v_i, where $v_1 = 0$, $v_2 = 50$ and $v_3 = 100$. Furthermore, assume that the fraction of fatalities is d_i, where $d_2 = d_3 = 0.1$.
The model expresses that

$$N = g(X,Z) = 5\,X\,(1- Z_1)\,Z_2\,Z_3 + 10(1- Z_1)\,Z_2\,(1-Z_3),$$

as the number of fatalities is 5 in case of scenario 2, and this scenario occurs if $(1- Z_1) Z_2 Z_3 = 1$, and the number of fatalities is 10 in case of scenario 3, and this scenario occurs if $(1- Z_1) Z_2 (1-Z_3) = 1$.

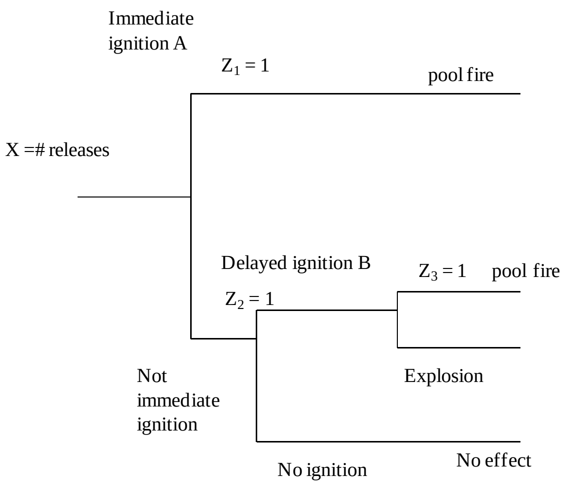

Figure 1. Event tree for the LNG plant case

The quantities, X and Z, are unknown and knowledge-based (subjective) probabilities are used to express the uncertainties (degree of belief). Suppose the following assignments have been made given the background knowledge K of the analysts:

$P(X= 1) = EX = 0.005$
$P(Z_1 = 1) = P(A) = 0.3$
$P(Z_2 =1|\, Z_1 = 0) = P(B|\text{ not } A) = 0.2$
$P(Z_3 =1|\, Z_1 = 0, Z_2 = 1) = P(\text{pool fire }|\text{ not } A,B) = 0.4$.

To interpret these numbers, consider for example, $P(Z_1 = 1)$. We have $P(Z_1 = 1) = P(A|K) = 0.3$, which means that the analysts consider the uncertainty of immediate ignition occurring (given a release) to be the same as drawing a red ball out of an urn which comprises ten balls of which three are red.
The Z probabilities are all reflecting the vulnerabilities (robustness) of the system given a leakage. Resilience is not relevant to consider when specifically addressing the consequences of leakages. However, focus on resilience is of course important in order to be able to sustain operation in case of any type of changes or disturbances in the process that could lead to leakages. The probability of a leakage is strongly dependent on the resilience management.

To compute the distribution of N given this input, we can follow the rules of probability as in the previous section. The results are shown in *Table 1.*

Table 1. Probability (subjective) distribution for the number of fatalities associated with the event tree of *Figure 1*

N: number of fatalities associated with release as defined by the event tree in Figure 1	Probability P	E contribution (E: Expected value)
0	0.99930	0
5	0.00028	0.0014
10	0.00042	0.0042

The probabilities assigned are based on background knowledge K, which includes a number of assumptions. Here are some few examples, evident from the above analysis:

- The event tree model
- A certain number of exposed people
- A fraction of fatalities in different scenarios

Examples of other assumptions made are:

- All vessels and piping are protected by the water application like monitors, hydrants.
- Release rates are constant throughout the release duration time.

The understanding of the physical phenomena and the computer codes used also strongly affect the results.
Vinnem [31] illustrates the dependencies of the assumptions made by pointing out that the frequency of accidents with at least 100 fatalities increased by a factor of 56 when compared to the results from the initial risk assessment performed for the operator of the plant. The initial assessment was performed before engineering studies had started, whereas the updated study made by the engineering contractor reflected all the engineering details. Nonetheless, the example clearly shows the large difference in background knowledge.
Vinnem [31] also points to the assumption made in the actual analysis of the plant that, in the event of impact of a passing vessel on an LNG tanker loading at the quay, the gas release would be ignited immediately, presumably by sparks generated by the collision itself. However, according to Vinnem [31], no explanation was provided of how such ignition of a very heavy and cold gas could occur physically. He concludes that it is very hard to foresee how it could be caused in this way. The implications of the assumption are important for the further analysis [31]:

> However, the implication of this assumption was that it was unnecessary to consider in the studies any spreading of the gas cloud due to wind and heating of the liquefied gas, with obvious consequences for the scenarios the public might be exposed to. Such a very critical assumption should at least have been subjected to a sensitivity study in order to illustrate how changes in the assumption would affect the results, and the robustness of the assumption discussed. None of this, however, has been provided in any of the studies.

3.2 An alternative analysis approach based on chances

Now let us assume that the analysts choose to introduce chances:

p = individual risk for a specific person in the group having the highest risk, i.e. the probability that a specific person (arbitrarily chosen) shall be killed due to the activity during a period of one year

and the f-n curve, G(n), expressing the frequency (i.e. the expected number) f of accidents that lead to minimum n number of fatalities, which can also be interpreted as the chance (frequentist) probability of an accident with at least n fatalities, i.e.

$$G(n) = E_f[\, Y(n)],$$

where Y(n) denotes the number of accidents with at least n fatalities during a period of one year.
These parameters (p and G(n)) are unknown and need to be estimated and uncertainties assessed. We start by addressing the problem of assessing the uncertainties about the true value of these parameters. The tool for this purpose is knowledge-based (subjective) probabilities. The approach is referred to as the probability of frequency approach, as was noted in Section 1.

The analysis can be viewed as an application of the Bayesian framework, which comprises the following steps:

1. Establish a probabilistic model
2. Assign a prior distribution on the parameters of interest
3. Use Bayes' theorem to establish the posterior distribution of the parameters.

To illustrate the analysis, we will use an event tree, as presented in Figure 2. The following parameters are introduced:

$q_0 = E_f[X]$
$q_1 = P_f(A)$
$q_2 = P_f(B|\text{ not } A)$
$q_3 = P_f(\text{pool fire}|\text{ not } A,B)$

For q_1 and q_2, it is tacitly assumed that the chances are conditional on the occurrence of a release. To interpret the parameters we need to construct infinite populations of similar situations to the one studied. For example, q_1 represents the fraction of times immediate ignition occurs in the case of a release if the situation is repeated over and over again.

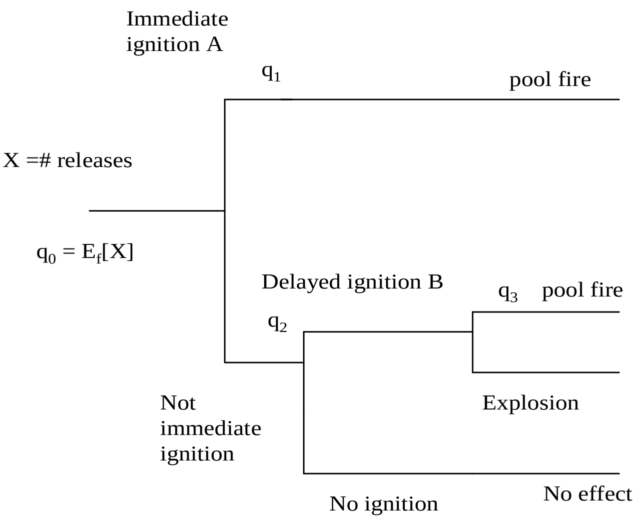

Figure 2. Event tree for the LNG plant case

If we know all parameter values we can calculate the contributions to p and G(n), using standard probability calculus. However, all parameters are unknown and we use knowledge-based (subjective) probabilities to express the analysts' uncertainties about the true value of these parameters. This analysis is normally carried out using a Monte Carlo simulation.

Let us concentrate our focus on G(1), the relative frequency probability of at least one fatality; to simplify the notation we refer to this quantity as r. From the above analysis, we have established a relationship (model) between this quantity and the underlying model parameters q_0, q_1, q_2 and q_3:

$$G(1) = r = P(s_1) + P(s_2) = q_0\,[(1- q_1)\,q_2\,q_3 + (1- q_1)\,q_2\,(1-q_3)] = q_0\,(1- q_1)\,q_2.$$

The aim of the analysis is now to establish uncertainty distributions on the q_i parameters and use the event tree model to propagate these uncertainties to an uncertainty distribution for G(1). A numerical example will explain the ideas.

Let us first consider q_0, the expected number of releases. As an estimate of q_0 we used 0.005. To reflect uncertainties we use a subjective probability distribution. This distribution may, for example, be a beta-distribution, a triangular distribution or a uniform distribution. For this case we will simply assume that the analyst specifies a uniform distribution on the interval [0.003, 0.007], which means that the analyst is confident that the true q_0 lies in this interval, and that his/her degree of belief that q_0 lies in the interval [0.003, 0.005] is the same as [0.005, 0.007] (50%). We make similar assumptions for the other parameters. See overview in *Table 2.*

Table 2. Knowledge-based probabilities for the parameters q_0, q_1, q_2 and q_3

Parameter	Distribution type	Interval
q_0	Uniform	[0.003,0007]
q_1	Uniform	[0.2,0.4]
q_2	Uniform	[0.1,0.3]
q_3	Uniform	[0.1,0.7]

Using these distributions and assuming "independent" distributions for the q_i parameters, we can calculate the knowledge-based distributions for r. Independence here means that if, for example, we know that q_2 is equal to 0.12 (say), this would not affect our uncertainty assessment of q_3 (say).

To establish the output distributions using analytical formulae is difficult. It is easier to use Monte Carlo simulation, and this is the common approach for performing this type of uncertainty assessment. In this case the analysis is simply carried out using an Excel sheet. Random numbers for each parameter are drawn from the sheet (1000 replications) and using the formula $r = q_0\,(1- q_1)\,q_2$, we obtain the associated uncertainty distribution of r, shown in *Table 3* and Figure 3. Note that these values are estimates of the probabilities given by the input of the Monte Carlo simulations: the uniform distributions and the formula $r = q_0\,(1- q_1)\,q_2$. The estimation error is rather small as the number of replications is large (1000), but not negligible. For example, for the category (0.0004, 0.0007] different runs of the simulation would give probabilities varying in the interval 0.39 to 0.45. Hence there is a knowledge-based probability of 44% that the chance of at least one fatality is in the interval (0.04%, 0.07%].

Table 3. Knowledge-based probabilities, P, for r = G(1)

Interval for r	Interval for r. Reformulated intervals (%) (x 10^{-2})	Simulated probability
≤0.0002	≤ 0.02	0.00
(0.0002, 0.0004]	(0.02, 0.04]	0.12
(0.0004, 0.0007]	(0.04, 0.07]	0.44
(0.0007, 0.0010]	(0.07, 0.10]	0.28
(0.0010,	(0.10, 0.13]	0.13

0.0013]		
(0.0013, 0.0016]	(0.13, 0.16]	0.03
> 0.0016	> 0.16	0.00

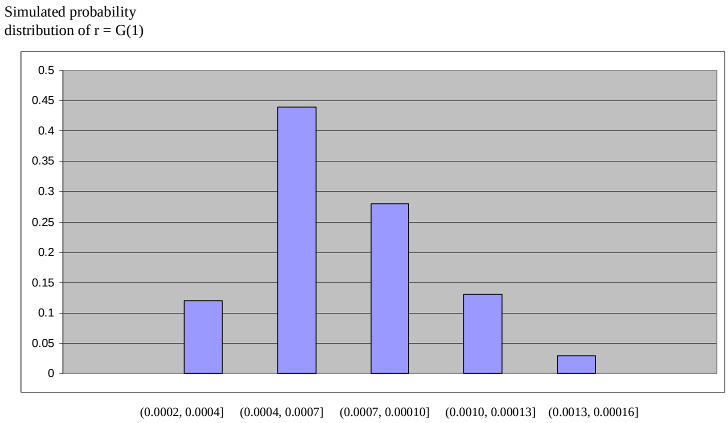

Figure 3. Knowledge-based probabilities, P, for r = G(1) based on *Table 3*

Uncertainty factors for the subjective probabilities can be defined as in Section 3.1.
The q_1-q_3 parameters reflect the vulnerabilities (robustness) of the system given a leakage. As was noted in Section 3.1, resilience is not relevant to consider when specifically addressing the consequences of leakages, but focus on resilience is of course important in order to be able to sustain operation in case of any type of changes or disturbances in the process that could lead to leakages.

4. Discussion

The actual risk assessments of the LNG plant were based on best-estimates. The scientific basis was not clarified. By reference to the calculated probabilities, the operator concluded that the risk was acceptable and this was communicated to all other parties, including the neighbours. Uncertainties were not reported. In fact the term “uncertainty” was not referred to at all in the main risk analyses report. The basic conclusion was that the risk is very low and the neighbours would acknowledge this if informed by the experts used by the operator. This approach cannot be justified, and the actual process has also been strongly criticized; see Vinnem [31] and Aven [6]. The two approaches presented in Section 3 have a stronger scientific basis and they both acknowledge the need for seeing beyond the probabilistic analysis. A risk assessment informs the decision maker -- the decision should not be risk-based [2].
To compare the two approaches in Section 3, let us firstly focus on the approach in Section 3.2, which extends the probability of frequency approach. Following this approach, the analysts are to express the epistemic uncertainties about the parameters of the probability models using subjective probabilities. In practice it is difficult to perform a complete uncertainty analysis within this setting. In theory an uncertainty distribution on the total model and parameter space should be established, which is impossible to do. So, in applications, only a few marginal distributions on some selected parameters are normally specified, and therefore the uncertainty distributions on the output probabilities are just reflecting some aspects of the uncertainty. This makes it difficult to interpret the produced uncertainties.
It is obviously a challenge in practice to establish the epistemic distributions, as indicated in Section 3.2. However, more important is the conceptual issues. Introducing the chances means two levels of uncertainty, and one may question what is gained by this second level. The standard answer would be that we need to establish the probability models with the associated parameters to be able to apply the Bayesian machinery for ensuring consistency in the probability assignments and in the updating of probabilities in the case that new information becomes available. For many types of applications, such updating is important, in particular for risk assessments in an operational phase. However, for the LNG case, such an updating is not considered essential, as the assessments are carried out at particular points in time to support specific decisions at these points. The assessment process is not of the form typically implemented when using Bayes’ formula. Our recommended approach in this case is, therefore, the former one. The assessment is simpler and we have not been able to point to decisive arguments for using the alternative and much more complex approach. This does not mean, however, that chances cannot be introduced in a case like this. The point made is that if such models and concepts are introduced, they need to be properly justified.

Both approaches studied in Section 3 are based on the use of subjective probabilities, and these reflect the uncertainties (degree of belief) of the assessors, conditional on the background knowledge K. These probabilities may camouflage uncertainties, as was discussed in Section 2. The assigned probabilities are conditioned on a number of assumptions and suppositions. Uncertainties are often hidden in the background knowledge, and we may consequently question whether the assigned subjective probabilities adequately describe the assessor's uncertainties of the unknown quantities considered. As an example, think of the assumption made in the actual LNG study that in the event of impact of a passing vessel on an LNG tanker loading at the quay, the gas release would be ignited immediately, presumably by sparks generated by the collision itself. This assumption could be wrong. Uncertainties are not revealed by not addressing uncertainties about this assumption.

This issue is discussed by, for example, Mosleh & Bier [23]. They refer to a subjective probability P(A|X), which expresses the probability of the event A, given a set of conditions X. As X is uncertain (it is a random variable), a probability distribution for the quantity h(X) = P(A|X) can be constructed. Thus there is uncertainty about the random probability, P(A|X). However, we will stress that the probability is not an unknown quantity (random variable) for the analyst. To make this clear, let us summarise the setting of subjective probabilities. A subjective probability P(A|K) is conditional on the background knowledge K, and some aspects of this K can be related to X, as described by Mosleh and Bier [23]. The analyst has determined to assign his/her probability based on K. If he/she finds that the uncertainty about X should be reflected, he/she would adjust the assigned probability using the law of total probability. This does not mean however that P(A|K) is uncertain, as such a statement would presume that a true probability value exists. The assessor needs to clarify what is uncertain and subject to the uncertainty assessment and what constitutes the background knowledge. From a theoretical point of view, one may think that it is possible (and desirable) to remove all such Xs from K, but in a practical risk assessment context that is impossible. We will always base our probabilities on some type of background knowledge, and often this knowledge would not be possible to specify using quantities such as X.

Model inaccuracies (uncertainties) are not incorporated in the analysis in Section 3. We will argue that the epistemic uncertainty analysis cannot and should not aim at quantifying the model inaccuracies.

We use the model g(X,Z) of the number of fatalities N in Section 3.1 as an illustrating example. Here g is given by:

$$g(X,Z) = 5\,X\,(1- Z_1)\,Z_2\,Z_3 + 10(1- Z_1)\,Z_2\,(1-Z_3).$$

The model inaccuracy is defined by the difference between the "true" N and the model output, i.e. N – g(X,Z). This difference is also referred to as model uncertainty; see e.g. [25], [19], [24]. It obviously needs to be addressed as the uncertainty assessments are conditional on the use of this model. But how should we deal with this "error" – should we quantify it?

"No" is our clear answer [7]. It is not meaningful to quantify the model inaccuracy. The point we make is that if the model is not considered good enough for its purpose, it should be improved. The uncertainty assessments are based on the model used. Of course, when observations of N (from similar plants) are available, we would compare the assessments of N which are conditional on the use of the model, g, with these observations. The result of such a comparison provides a basis for improving the model and accepting it for use. But at a certain stage we accept the model and apply it for comparing options and making judgments about, for example, risk acceptance (tolerability). Then it has no meaning in quantifying the model inaccuracy. The results are conditional on the model used. Instead of specifying $P(N \le y)$ directly, we compute $P(g(X,Z) \le y\,|K)$ and g is a part of the background knowledge K.

An important task for the scientific communities in different areas is to develop good models. The models are justified by reference to established theories and laws explaining the phenomena studied, and the results of extensive testing. The performance of a model must, however, always be seen in light of the purpose of the analysis. A crude model can be preferred instead of a more accurate model in some situations if the model is simpler and it is able to identify the essential features of the system performance.

In the literature, attempts have been made to explicitly incorporate the model inaccuracies (an example is given in Aven (2003) taken from the field of structural reliability analysis (SRA)). The use of g(X,Z) means a simplification, and the idea is then to introduce an error term, a (say), such that we obtain a new model

$$g_0(X,Z) = a\,g(X,Z) = a(5\,X\,(1- Z_1)\,Z_2\,Z_3 + 10(1- Z_1)\,Z_2\,(1-Z_3)).$$

Clearly, this may give a better model, a more accurate description of the world. However, it would probably not be chosen in a practical case as it may complicate the assessments. It may be much more difficult to specify a probability distribution for (a,X,Z) than for (X,Z). There might be lack of relevant data to support the uncertainty analysis of a and there could be dependencies between a and (X,Z). We have to balance the need for accuracy and simplicity.

In the literature, various methods have been suggested to reflect model uncertainties (see e.g. [1], [24], [14], [33]. Above we briefly looked into one typical approach (the standard SRA approach). As another typical approach we refer to Apostolakis [1], which addresses the issue of weighing different models: Let M_1 and M_2 be two alternative models to be used for assigning the probability, A. Conditional on M_i, we have an assignment $P(A|K_i)$. Unconditionally, this gives

$$P(A|K) = P(A|K_1)\, p_1 + P(A|K_2)\, p_2, \qquad (4.1)$$

where p_i is the analyst's subjective probability that the *i*th model, i.e. the set of associated assumptions, is true (here $p_1 + p_2 = 1$). In a practical decision making context, the analysts would most likely present separate assignments for the different models, i.e. $P(A|K_i)$, in addition to the weighed probability assignment (4.1). To specify the subjective probability $P(A|K)$, the analysts may choose to apply the assignment procedure given by (4.1) also when p_i cannot be interpreted as a probability that a specific assumption is true. In such a case, p_i must be interpreted as a weight reflecting the confidence in the model *i* for making accurate predictions.

Hence, model uncertainty quantification in the sense of model weighing can be covered by the uncertainty assessment. Model weighing is a completely different issue from quantification of model inaccuracy. As stressed above, when using the framework to compute $P(g(X,Z) \leq y)$, we accept the use of specific models and procedures for weighing the models. The models and procedures are part of the background knowledge K.

5. Conclusions

In this paper we have presented and discussed a conceptual framework for risk assessment and risk management where risk is based on the triplet events, consequences and uncertainties. Compared to other approaches, this framework provides broader uncertainty assessments, by seeing beyond the knowledge-based (subjective) probabilities. An example of an analysis of an LNG (Liquefied Natural Gas) plant is used to demonstrate the applicability of the framework. Two ways of approaching the LNG case are discussed: one where chances (frequentist probabilities) are introduced and one where such concepts are not introduced. The latter approach is simpler and is recommended. It does not allow for Bayesian updating procedures to be implemented, but this is not considered a problem in this particular case study. In other cases, especially in an operational phase, it could, however, be decisive for the choice of approach. Anyway, if chances are introduced they need to be justified and meaningful interpretations provided.

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