Masking of ferromagnetic ellipsoidal shell in Earth's magnetic field

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A ferromagnetic object, located in the Earth's magnetic field, changes the distribution of that field. By measuring such disturbances it is possible to detect the object and destroy it. To conceal the object, a special winding is placed inside its ferromagnetic shell, which function is to eliminate the disturbances in the distribution of the Earth's magnetic field. A thin walled ellipsoidal shell made of ferromagnetic material are examined as the object model. Coils are placed inside the shell and their function is to generate a magnetic field, which eliminates the effect the shell makes on the distribution of the Earth's magnetic field in the surrounding area. Such a procedure is called magnetic masking and the winding used for this purpose is called the masking winding. The possibility of building the masking windings for the ferromagnetic ellipsoidal shell, situated in a magnetic field transverse in relation to its major axis, is also examined. Masking of a thin-walled ellipsoidal ferromagnetic object located in the longitudinal magnetic field is described in the article [1]. Investigating the possibility of masking of objects in a transverse magnetic field presented in this article will allow for a comprehensive assessment of a possibility of masking thin walled ferromagnetic objects of elongated ellipsoidal shape. The solution of Maxwell's equations, which describe the magnetic field distribution caused by the ferromagnetic shell presence in the Earth's magnetic field, are applied. Furthermore, the ability of selecting coils, which fully eliminate the perturbations of the magnetic field outside the shell are proven.

KEYWORDS: mathematical models of magnetic field, magnetization of ferromagnetic object, masking of ferromagnetic object, windings for demagnetization of the ship

1. Introduction

Ship hulls are most frequently constructed of ferromagnetic steel. The ferromagnetic hull causes significant disturbances in the Earth's magnetic field distribution. These disturbances are detected at a considerable distance from the ship and they allow for its detection and an eventual attempt aimed at destroying it. In order to avoid the influence of the ferromagnetic hull on the Earth's magnetic field distribution, specially shaped windings are placed inside the hull. The windings are mounted on the internal side of the ferromagnetic hull wall. They are power supplied from controlled current sources, which current efficiency is changed automatically depending on the measurement results for the magnetic field outside the object. The magnetic field generated by these windings significantly reduces the changes in the magnetic field distribution outside the ship. This paper investigates the ship hull model of the shape of an elongated thin walled ellipsoidal shell with a circular cross section in the plane z = const and the shell thickness δ (Fig. 1). Due to the fact that the Earth's magnetic induction does not exceed the value of about 50μT, it is assumed that the shell material has the linear magnetization characteristics. The adoption of the linear magnetization characteristics allows considering the masking winding for each of the three components of the magnetic induction separately. A mathematical model and the results of calculation of the masking coil for the component in the z axis (Fig.1) are described in the paper [1]. This paper presents a calculation of masking windings for the magnetic induction B_0 directed along the x axis (Fig. 1).

2. Mathematical model

In this model a system of elliptical coordinates η, θ, φ is used. These coordinates are related to the rectangular coordinates x, y, z associated with the ship (Fig. 1) with the following dependencies:

$$\begin{aligned} x &= a\sinh\eta\sin\theta\cos\varphi \\ y &= a\sinh\eta\sin\theta\sin\varphi \\ z &= a\cosh\eta\cos\theta \end{aligned} \tag{1}$$

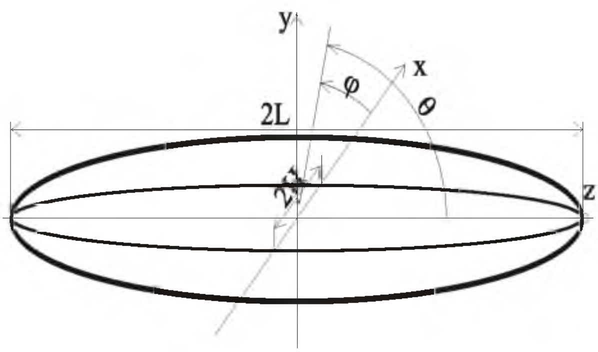

Fig. 1. The ship model and its own rectangular coordinates system x, y, z and the elliptical ones θ, φ

where the constant a is described by the formula:

$$a = \sqrt{L^2 - H^2} \tag{2}$$

and 2L and 2H are the length and the width of the ship model respectively.

The outer shell surface is defined by the constant elliptical coordinate η_2:

$$\eta_2 = 0.5\ln\left(\frac{L+H}{L-H}\right) \tag{3}$$

Maintaining a constant shell thickness δ is not possible in the same elliptic coordinates as they are assumed for the outer surface. The shell thickness was assumed as δ in the middle of the ship hull length for the coordinate η_1, which defines the location of the inner shell surface:

$$\eta_1 = \ln\left[\frac{H-\delta}{a} + \sqrt{\left(\frac{H-\delta}{a}\right)^2 + 1}\right] \tag{4}$$

Assuming that the Earth's magnetic induction in the space surrounding the ship is a uniform field B_0 directed along the x axis of the ship's own rectangular coordinate (Fig. 1), the Earth's magnetic induction has the following components in the elliptical coordinate system:

$$\begin{aligned} B_{0\eta} &= \frac{B_0 \cos\varphi \cosh\eta \sin\theta}{\sqrt{\sinh^2\eta + \sin^2\theta}} \\ B_{0\theta} &= \frac{B_0 \cos\varphi \sinh\eta \cos\theta}{\sqrt{\sinh^2\eta + \sin^2\theta}} \\ B_{0\varphi} &= -B_0 \sin\varphi \end{aligned} \tag{5}$$

The magnetic induction $\mathbf{B}_n$ (n = 1,2,3) in each of the areas is presented as the sum of the magnetic induction $\mathbf{B}_0$ and $\mathbf{B}_{fn}$, hereinafter referred to as the magnetic induction perturbation, which is caused by the presence of the ferromagnetic shell:

$$\mathbf{B}_n = \mathbf{B}_0 + \mathbf{B}_{fn} \tag{6}$$

The components of the magnetic induction $\mathbf{B}_0$ in the elliptical coordinates are determined by the relationships (5). The perturbation of magnetic induction, while assuming linear magnetization of the elliptical ferromagnetic shell, fulfils the Maxwell's equations in each of the three subareas:

$$\nabla \times \mathbf{B}_{fn} = 0 \qquad \nabla \cdot \mathbf{B}_{fn} = 0 \tag{7}$$

where: n = 1 – the area inside the ellipsoid, n = 2 – the inside of the ferromagnetic wall of the ellipsoid and n = 3 – the area outside the ellipsoid.

The continuity of the normal magnetic induction components occurs on the borders between particular areas:

$$B_{f1\eta} = B_{f2\eta}\Big|_{\eta=\eta_1} \qquad B_{f2\eta} = B_{f3\eta}\Big|_{\eta=\eta_2} \tag{8}$$

Whereas, in the case of the condition for the continuity of the tangential component of the magnetic induction, it is assumed that inside the ellipsoid there is a thin layer of two masking windings represented as linear currents I_θ and I_φ.

$$\begin{aligned}
&B_{f2\theta}-\mu_w B_{f1\theta}=\mu_0\mu_w I_\varphi+(\mu_w-1)\left.\frac{B_0\cos\varphi\sinh\eta\cos\theta}{\sqrt{\sinh^2\eta+\sin^2\theta}}\right|_{\eta=\eta_1}\\
&B_{f2\varphi}-\mu_w B_{f1\varphi}=\mu_0\mu_w I_\theta-(\mu_w-1)B_0\sin\varphi\Big|_{\eta=\eta_1}\\
&B_{f2\theta}-\mu_w B_{f3\theta}=(\mu_w-1)\left.\frac{B_0\cos\varphi\sinh\eta\cos\theta}{\sqrt{\sinh^2\eta+\sin^2\theta}}\right|_{\eta=\eta_2}\\
&B_{f2\varphi}-\mu_w B_{f3\varphi}=-(\mu_w-1)B_0\sin\varphi\Big|_{\eta=\eta_2}
\end{aligned}\tag{9}$$

where: μ_w – the relative ferromagnetic shell permeability, and $\mu_0=4\pi 10^{-7}$H/m – the magnetic permeability of vacuum.

The winding represented by the linear current $I_\theta(\theta,\varphi)$ is directed along the axis θ, and the second one, with the linear current $I_\varphi(\theta,\varphi)$, is directed along the axis φ. The presence of the masking windings causes a discontinuity of the tangential components of magnetic induction. Taking into account the linear magnetization characteristics, the following boundary conditions for the tangential components are obtained (9). Taking into account the conditions (9), the linear currents distributions is assumed in the form of the following equations:

$$I_\varphi=\frac{I_{\varphi x}\cos\varphi\cos\theta}{\sqrt{\sinh^2\eta_1+\sin^2\theta}},\quad I_\theta=I_{\theta x}\sin\varphi \tag{10}$$

After taking into account the equations (10), the boundary conditions for the tangential components take the form of:

$$\begin{aligned}
&B_{f2\theta}-\mu_w B_{f10}=\left.\frac{\left[\mu_0\mu_w I_{\varphi x}+(\mu_w-1)B_0\sinh\eta_1\right]\cos\varphi\cos\theta}{\sqrt{\sinh^2\eta_1+\sin^2\theta}}\right|_{\eta=\eta_1}\\
&B_{f2\varphi}-\mu_w B_{f1\varphi}=\left[\mu_0\mu_w I_{0x}-(\mu_w-1)B_0\right]\sin\varphi\Big|_{\eta=\eta_1}\\
&B_{f2\theta}-\mu_w B_{f30}=(\mu_w-1)\left.\frac{B_0\cos\varphi\sinh\eta\cos\theta}{\sqrt{\sinh^2\eta+\sin^2\theta}}\right|_{\eta=\eta_2}\\
&B_{f2\varphi}-\mu_w B_{f3\varphi}=(\mu_w-1)B_0\sin\varphi\Big|_{\eta=\eta_2}
\end{aligned}\tag{11}$$

Due to the boundary conditions (11), the magnetic induction components are assumed as follows:

$$B_{f\eta k}=b_{\eta k}(\eta,\theta)\cos\varphi\quad B_{f\theta k}=b_{\theta k}(\eta,\theta)\cos\varphi\quad B_{f\varphi}=b_{\varphi k}(\eta,\theta)\sin\varphi \tag{12}$$

where k = 1, 2, 3.

Substituting the magnetic induction given by the formulas (12) to the rotation equations (7) recorded for the axis η, θ elliptical coordinate system respectively, we obtain:

$$b_{\eta k} = -\frac{\sin\theta}{\sqrt{\sinh^2\eta + \sin^2\theta}}\frac{\partial}{\partial\eta}\left(b_{\varphi k}\sinh\eta\right) \tag{13}$$

$$b_{\theta k} = -\frac{\sinh\eta}{\sqrt{\sinh^2\eta + \sin^2\theta}}\frac{\partial}{\partial\theta}\left(b_{\varphi k}\sin\theta\right) \tag{14}$$

The magnetic induction components expressed by (13), (14) satisfy the third rotation equation for φ axis. By substituting (13) and (14) into the divergence equation (7), we obtain the equation for the magnetic induction component $b_{\varphi k}$ in the following form:

$$\left(\sinh^2\eta + \sin^2\theta\right)b_{\varphi k} - \sin^2\theta\frac{\partial}{\partial\eta}\left[\sinh\eta\frac{\partial}{\partial\eta}\left(b_{\varphi k}\sinh\eta\right)\right] - \sinh^2\eta\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\left(b_{\varphi k}\sin\theta\right)\right] = 0 \tag{15}$$

Assuming the linear current $I_{\theta x}$ in the form:

$$I_{\theta x} = -\frac{I_{\varphi x}}{\sinh\eta_1} \tag{16}$$

and after taking into account (14), it can be proven that the boundary conditions (11) for the tangential components of magnetic induction of θ and φ axes are identical and they can be replaced by the boundary conditions for the surface η_1 and η_2 respectively:

$$\begin{aligned} -b_{2\varphi} + \mu_w b_{1\varphi} &= \left.\frac{\mu_0\mu_w I_{\varphi x} + (\mu_w - 1)B_0\sinh\eta_1}{\sinh\eta_1}\right|_{\eta=\eta_1} \\ b_{2\varphi} - \mu_w b_{3\varphi} &= -(\mu_w - 1)B_0\Big|_{\eta=\eta_2} \end{aligned} \tag{17}$$

Taking into account the boundary conditions (17) and the limited values of magnetic induction for $\eta=0$ i $\eta\to\infty$, the solution is adopted in the following form:

$$b_\varphi = \begin{cases} C_1 & \text{dla } \eta < \eta_1 \\ C_2 + C_3\dfrac{Q_1^1(\cosh\eta)}{\sinh\eta} & \text{dla } \eta_1 < \eta < \eta_2 \\ C_4\dfrac{Q_1^1(\cosh\eta)}{\sinh\eta} & \text{dla } \eta > \eta_2 \end{cases} \tag{18}$$

where: $Q_1^1(\cosh\eta) = \frac{\sinh\eta}{2}\ln\left(\frac{\cosh\eta + I}{\cosh\eta - I}\right) - \mathrm{ctgh}\eta$ - the spherical function of the general type of the second kind [2,3]. Constants C_1, C_2, C_3 and C_4, by the boundary conditions (8) and (17) and taking into account (13), is determined from the system of equations:

$$
\begin{aligned}
C_2 + C_3 a_1 - \mu_w C_1 &= -\frac{\mu_0 \mu_w I_{\varphi x}}{\sinh \eta_1} - (\mu_w - 1) B_0 \\
C_2 + C_3 a_2 - \mu_w C_4 a_2 &= -(\mu_w - 1) B_0 \\
C_1 &= C_2 + a_3 C_3 \\
C_4 &= a_4 C_2 + C_3
\end{aligned}
\tag{19}
$$

where:

$$
a_1 = \frac{Q_1^1(\cosh \eta_1)}{\sinh \eta_1},\ a_2 = \frac{Q_1^1(\cosh \eta_2)}{\sinh \eta_2},\ a_3 = \frac{1}{\cosh \eta_1}\left[\frac{dQ_1^1(\cosh \eta)}{d\eta}\right]_{\eta=\eta_1},\ a_4 = \frac{\cosh \eta_2}{\left[\frac{dQ_1^1(\cosh \eta)}{d\eta}\right]_{\eta=\eta_2}}.
$$

Solving the system of equation (19), we obtain:

$$
C_2 = \frac{\mu_w - 1}{\Delta}\left\{\frac{\mu_0 \mu_w a_2 I_{\varphi x}}{\sinh \eta_1} + B_0\left[a_1 + (\mu_w - 1)a_2 - \mu_w a_3\right]\right\}
\tag{20}
$$

$$
C_3 = \frac{\mu_w - 1}{\Delta}\left[(1 - a_2 a_4)\mu_w B_0 - (\mu_w a_2 a_4 - 1)\frac{\mu_0 \mu_w I_{\varphi x}}{(\mu_w - 1)\sinh \eta_1}\right]
\tag{21}
$$

where: $\Delta = (\mu_w - 1)^2 a_2 - (\mu_w a_3 - a_1)(\mu_w a_2 a_4 - 1)$.

The condition, that perturbation magnetic induction equals zero outside the ellipsoid, means that $C_4=0$ and, hence, the linear current $I_{\varphi x}$ is:

$$
I_{\varphi x} = \frac{\mu_w - 1}{\mu_w}\frac{B_0}{\mu_0}\sinh \eta_1 \frac{a_4(a_1 - a_2) + \mu_w(1 - a_3 a_4)}{a_2 a_4 - 1}
\tag{22}
$$

and the linear current I_{0x} from (16) takes the value:

$$
I_{\theta x} = -\frac{\mu_w - 1}{\mu_w}\frac{B_0}{\mu_0}\frac{a_4(a_1 - a_2) + \mu_w(1 - a_3 a_4)}{a_2 a_4 - 1}
\tag{23}
$$

3. Results analysis

In order to investigate the practical feasibility of making the winding masking of a ship, the size of the necessary linear currents defined by the relations (20) and (21) is defined. The value of the Earth's magnetic induction is assumed as B_0 = 50 μT and the object of a moderate size, with the length of 2L = 30 m and the width of 2H = 6 m, is examined. Figure 2a shows the calculated dependence of the line current on the ferromagnetic ship wall thickness, and Fig. 2b shows the line current dependence on the relative permeability of the ferromagnetic hull. The presented calculations show that the magnitude of the linear current is, practically, a linear function of both: the shell thickness δ and the relative magnetic permeability μ_w. The value of the linear current I_{0x}, determined by the relation (16), will be bigger because $\sinh(\eta_1)$ is less than one. However, the required line current densities are in the range of 2 kA/m for $I_{\varphi x}$ and 10 kA/m for I_{0x}. It is worth mentioning that the increase of the object's geometric dimensions does not result in a significant manner in the increase of the masking currents density.

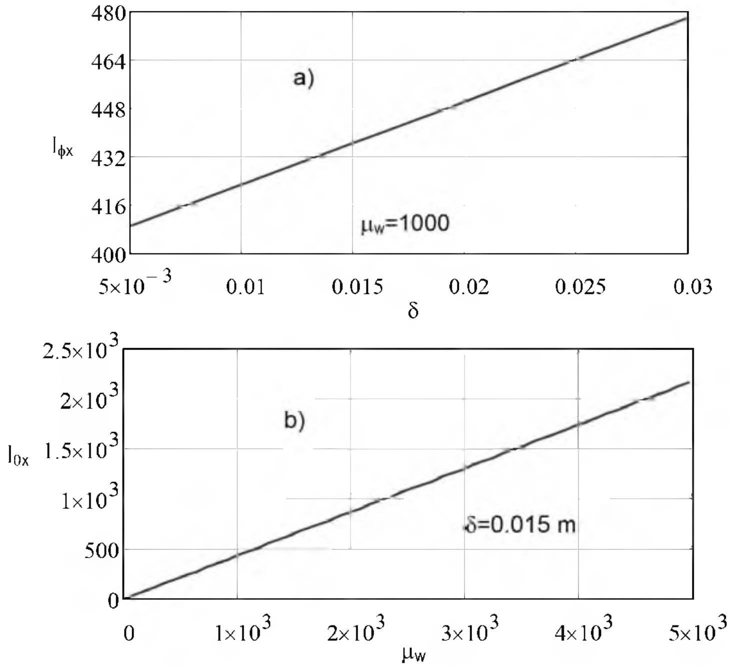

Fig. 2. The line current in A/m as a function of the shell thickness δ in m (a) and as a function of the relative magnetic permeability μ_w (b)

4. Conclusions

The following conclusions can be drawn from the performed theoretical investigations into the masking of the thin walled ferromagnetic ellipsoidal objects located in a transverse magnetic field:

– the assumed shape of the object is a good approximate representation of a real object such as a ship. This shape allows to formulate a mathematical model and develop an analytical solution, which in turn make it possible to draw general conclusions on masking,
– the process of masking of an object presented in this article concerns the so-called induced magnetization that is magnetization which results from placing of a ferromagnetic object in an external magnetic field, e.g. that of the Earth,
– it was assumed in the selected model that the coils for masking of an object are placed in line, therefore, linear current was assumed in the coils. The

actually installed coils will be hardly noticeable as they will be mounted in the form of segments on the inside wall of the hull shell,

- two groups of coils are necessary to mask an object in a transverse magnetic field: the first group placed along the coordinate φ and the other group along the coordinate θ (Fig. 1). The value of the linear currents in respective coil groups are defined by the dependencies (22) and (23),
- the value of the linear current required for masking depends mainly on the thickness of the object's wall (ship's hull) and the relative magnetic permeability of the ferromagnetic. Within the actual, achievable range of changes in the hull's thickness (several centimeters), the value of the linear current increases practically in a linear way as the thickness of the wall increases (Fig. 2). Moreover, the larger the value of the hull magnetic permeability, the larger linear current is required, with the dependence being approximately proportional (Fig. 2),
- in case when the external magnetic field, which acts as an enforcing source, has also a longitudinal component, the linear currents in the masking coils should be changed in such a way so as to take into account the results achieved under the investigations described in the article [1].

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