Marcin FRYCZ*

THE EFFECT OF THE CONCENTRATION OF MAGNETIC PARTICLES ON THE OPERATING PARAMETERS OF A SLIDE JOURNAL BEARING LUBRICATED WITH FERRO-OIL

WPŁYW STĘŻENIA CZĄSTEK MAGNETYCZNYCH NA WARTOŚCI PARAMETRÓW EKSPLOATACYJNYCH W POPRZECZNYM ŁOŻYSKU ŚLIZGOWYM SMAROWANYM FERRO-OLEJEM

Key words: ferro-oil, magnetic particle's concentration, load carrying capacity, friction force, coefficient of friction.

Abstract This paper presents an analysis of changes in basic operational parameters of a slide journal bearing in an aspect of a concentration of magnetic particles in ferro-oil as a lubricant. The first part of the article presents an analytical-numerical calculation model. This model is based on experimentally determined physical parameters describing the dependence of ferro-oil viscosity on changes at basic operating parameters. Moreover, dimensionless load carrying capacity, dimensionless friction force and dimensionless coefficient of friction numerical calculations have been obtained by solving the Reynolds type equations using the finite difference method in Mathcad 15 program and the author's own calculation procedures. The obtained results have been presented in the form of graphs taking into account the influence of the following factors: external magnetic field, corrections related to the influence of pressure changes, corrections related to the influence of temperature changes and corrections related to non-Newtonian properties of the ferro-oil. The analysis of the obtained characteristics in the paper has been carried out, observations have been made as well as conclusions have been drawn regarding the optimal concentration of magnetic particles in the ferro-oil lubricating the slide journal bearing.

Słowa kluczowe: ferro-olej, stężenie cząstek magnetycznych, siła nośna, siła tarcia, współczynnik tarcia.

Streszczenie W niniejszej pracy została przedstawiona analiza zmian podstawowych parametrów eksploatacyjnych poprzecznego łożyska ślizgowego w aspekcie stężenia cząstek magnetycznych w ferro-oleju będącego czynnikiem smarującym. W pierwszej części pracy został zaprezentowany analityczno-numeryczny model obliczeniowy. Model ten oparty został na fizycznych wielkościach opisujących zależność lepkości ferro-oleju od zmian podstawowych parametrów pracy wyznaczonych na drodze doświadczalnej. Z kolei obliczenia numeryczne rozkładu bezwymiarowej siły nośnej, bezwymiarowej siły tarcia oraz bezwymiarowego współczynnika tarcia wykonano, rozwiązując równania typu Reynoldsa metodą różnic skończonych przy wykorzystaniu program Mathcad 15 i własnych procedur obliczeniowych. Uzyskane wyniki przedstawione zostały w postaci wykresów uwzględniających kolejno wpływ: zewnętrznego pola magnetycznego, korekty związane z wpływem zmian ciśnienia, korekty związane z wpływem zmian temperatury oraz korekty związane z właściwościami nienewtonowskimi ferro-oleju. Dokonana została analiza uzyskanych charakterystyk, zostały poczynione obserwacje oraz wyciągnięte wnioski dotyczące optymalnej zawartości cząstek magnetycznych w ferro-oleju smarującym poprzeczne łożysko ślizgowe.

INTRODUCTION

The issue presented in the article is a summary of long-term research work carried out by the author on the study of the impact of magnetic particle concentration on the flow and operational parameters of a slide journal bearing lubricated with a ferro-oil.

The selection of the most suitable oil, in the aspect of proper lubrication of journal bearings, strictly depends on the expectations towards the lubricant and

* ORCID: 0000-0001-8603-0523. Gdynia Maritime University, 81-225 Gdynia, Morska Str. 81-87, Poland, e-mail: m.frycz@wm.umg.edu.pl

these, as a consequence, are derivatives of the operating conditions of a bearing. Precise identification of these conditions as well as the determination and qualitative and quantitative aspects of expectations towards oil make it possible to shape optimal physical and chemical properties of the lubricant at the stages of its selection or production **[L. 1–3]**, but unfortunately, only at these stages. In many cases, the operating conditions of the bearing can exhibit such a wide and variable range of load values that none, even the best-chosen, classic lubricants will be able to cope with such conditions. It seems that one of the possible solutions to this problem is the use of lubricating oil belonging to the category of 'intelligent' ones. This term applies to the lubricants that are susceptible to controlling their properties and adaptively adjusting them to the changing operating conditions of a bearing. One of the commonly known lubricants of this type is ferro-oil. The rheological properties of ferro-oil, in particular its viscosity, like any other oil, depend on the basic operating parameters, i.e. temperature, pressure, or shear rate **[L. 4–13]**. The internal structure and chemical composition of substrates are also important **[L. 14]**. In the case of ferro-oil, one of the key parameters shaping its physical properties is the concentration of magnetic particles n_{cs} **[L. 15–21]**. This is what determines the "sensitivity" of ferro-oil to the controlling effect of the external magnetic field, variable in type, direction or value of induction and, as a result, of the range of possible adaptation to the changing operating conditions of a bearing.

The dynamic parameters of lubricating oil depend directly on the flow parameters of slide bearings, i.e. hydrodynamic pressure distributions, temperature distributions, and component values of the lubricant velocity vector in the lubrication gap. These determine the operational parameters of a bearing, such as friction force, carrying load, or the coefficient of friction. The values of the above mentioned parameters indicate, among others, the quality of slide friction in the friction nodes. In this context, it seems crucial to conduct research focused on determining the influence, both quantitative and qualitative, of the concentration of magnetic particles on the change of physical properties of a ferro-oil, and thus of its tribological properties. It is necessary to specify, among others, the principles of selecting the optimal concentration of the above particles depending on the existing environmental conditions or special expectations towards the bearing. The results of this research presented in this paper provide a partial answer to this problem, indicating the effect of the concentration of magnetically susceptible particles in ferro-oil on the operational properties of a journal slide bearings lubricated with ferro-oil. However, this is not an optimization analysis of determining the concentration *sensu stricto*.

ANALYTICAL-NUMERICAL CALCULATION MODEL

The analytical model of magnetohydrodynamic lubrication of slide journal bearings has been presented so far in several publications by the author **[L. i.e. 22–25]**; therefore, due to limited volume of this paper, only the key assumptions and transformations of the model closely related to the topic of the article will be presented and discussed. It is based on a model created by the research team of K. Wierzcholski **[L. 26–30]** and A. Miszczak **[L. 31–33]**, and it has been adopted for the analysis of lubrication of slide bearings with ferro-oil after making the property transformations.

The model was derived from fundamental equations, i.e. equations of momentum conservation, equations of flow continuity, equations of energy conservation, and Maxwell's equations **[L. 34–38]**.

There was assumed a non-isothermal bearing lubrication model with a laminar and steady lubricant flow rate and an external magnetic field was adopted as stationary and transverse to the ferro-oil flow in the bearing gap **[L. 39]**.

As the constitutive equation for ferro-oil, a non-Newtonian viscoelastic model was adopted (1) of a Rivlin-Ericksen' fluid **[L. 27, 30, 31, 35]**:

$$\boldsymbol{S} = -p\,\boldsymbol{I} + \eta \boldsymbol{A}_1 + \alpha \boldsymbol{A}_1 \boldsymbol{A}_1 + \boldsymbol{A}_2 \tag{1}$$

The following are the relationships (2) describing shear rate tensors:

$$\boldsymbol{A}_1 \equiv \boldsymbol{L} + \boldsymbol{L}^{\mathrm{T}}, \quad \boldsymbol{A}_2 \equiv \mathrm{grad}\,\boldsymbol{a} + (\mathrm{grad}\,\boldsymbol{a})^{\mathrm{T}} + 2\boldsymbol{L}^{\mathrm{T}} \cdot \boldsymbol{L} \tag{2}$$

and the acceleration vector in a formula (7):

$$\boldsymbol{a} \equiv \boldsymbol{L} \cdot \boldsymbol{v},\ \boldsymbol{L} \equiv \mathrm{grad}\,\boldsymbol{v} \tag{3}$$

where
- $\boldsymbol{A}_1$ – first shear rate tensor [s^{-1}],
- $\boldsymbol{A}_2$ – second shear rate tensor [s^{-2}],
- $\boldsymbol{I}$ – unit tensor,
- $\boldsymbol{L}$ – gradient of the velocity vector tensor [s^{-1}],
- $\boldsymbol{a}$ – acceleration vector [$m \cdot s^{-2}$],
- p – hydrodynamic pressure [Pa],
- α, β – experimental factors determining viscoelastic properties of ferro-oil [$Pa \cdot s^2$],
- η – dynamic viscosity coefficient [$Pa \cdot s$].

An important contribution of the author to this model is its complementation with mathematical models of changes in the dynamic viscosity of ferro-oil with the temperature, pressure, and intensity of the external magnetic field $\eta = \eta(T,p,B)$ and the determination of the real coefficients of viscosity changes due to the following parameters: δT, δp, δB. In previous publications, experimental studies on the effect of magnetic particle concentration on ferro-oil viscosity properties were presented both in the presence of **[L. 40, 41]** and in the absence **[L. 42–44]** of an external magnetic field in the aspect of changes in the basic parameters of the slide bearing. Also determined were the values of magnetic susceptibility ferro-oil factors χ on the experimental path, depending on the concentration of magnetic particles **[L. 45–46]**. Other material factors for the ferro-oil as α and β, resulting from the assumed model of the viscoelastic Rivlin-Ericksen's fluid, were adapted from **[L. 47]** and equated as constant values. In the paper **[L. 48]**, an analysis of the relationship $\eta = \eta(B)$ was presented as well as the determination of viscosity parameters δB of ferro-oils depending on the intensity of the external magnetic field changes. Furthermore, in papers **[L. 49, 50]**, analogous analysis of the relation $\eta = \eta(T)$ and the determination of δT viscosity parameters in the context of viscosity changes from temperature were presented. Finally, in paper **[L. 51]**, a similar analysis concerned the relation $\eta = \eta(p)$ between the dynamic viscosity of the tested ferro-oil with selected concentrations of magnetic particles and the pressure changes together with the determination of the parameters δp of these functions was shown. As a result of these studies, a comprehensive model of changes in ferro-oil viscosity from the temperature, pressure, and intensity of the external magnetic field was proposed $\eta = \eta(T,p,B)$ **[L. 52]**:

$$\eta = \eta_o \eta_1; \quad \eta_1 = \eta_{1B}\eta_{1p}\eta_{1T} \tag{4}$$

$$\eta_{1p}(\phi,z) = a_p e^{\zeta \cdot p_o \cdot p_1} = a_p e^{\zeta_p p_1} \tag{5}$$

$$\eta_{1T}(\phi,z,r) \equiv a_T e^{-\delta_T (T-T_o)} = a_T e^{-Q_{Br} T_1} \tag{6}$$

$$\eta_{1B}(\phi,z) = 1 + a_B (B_0 B_1)^{\delta_{1B}} = 1 + a_{B1}(B_1)^{\delta_{1B}} \tag{7}$$

where

- η_1 – total dimensionless dynamic viscosity,
- η_o – characteristic dimensional value of dynamic viscosity [Pa·s],
- η_{1p} – dimensionless dynamic viscosity depending on a pressure,
- η_{1T} – dimensionless dynamic viscosity depending on a temperature,
- η_{1B} – dimensionless dynamic viscosity depends on magnetic field induction,
- δ_{1B} – dimensionless material factor including changes in viscosity from a magnetic field,
- ζ, ζ_p – dimensional [Pa^{-1}] and dimensionless material factor including changes in viscosity depended on hydrodynamic pressure,
- δ_T, Q_{Br} – dimensional [K^{-1}] and dimensionless material factor including changes in viscosity depended on temperature,
- a_B – proportionality factor [$T^{-\delta_{B1}}$],
- a_{B1} – dimensionless proportionality coefficient,
- a_p – proportionality factor of pressure,
- a_T – proportionality factor of temperature,
- B_0 – dimensional value of magnetic field induction [T],
- B_1 – dimensionless value of magnetic field induction,
- p_1 – dimensionless hydrodynamic pressure,
- p_0 – dimensional value of hydrodynamic pressure.

The equations of motion were substituted with constitutive relationships (1) between stress tensor coordinates and shear rate tensor coordinates **[L. 31, 37, 38]**. The non-stationary units and units of inertia forces in equations of momentum were omitted. The full set of equations of motion for the classical, steady flow of lubricating oil was obtained in this way **[L. 31, 37, 38]**.

The next step in solving the system of equations was equalization and estimation of the order of values of the unit members. For this purpose, dimensional and dimensionless marks and numbers known in the literature were assumed **[L. 31, 35]**. A system of equations in the dimensionless form contains units of the order of a unity and members negligibly small order of radial relative clearance $\psi \approx 10^{-3}$. By neglecting the members of the member of radial relative clearance, which are about a thousand times smaller than the values of the other members, a new simplified system of equations could be obtained **[L. 31]**. For further analysis of the basic equations, it was assumed that the dimensionless density $\rho_1 = 1$ of the lubricant was constant and independent of both temperature and pressure.

The Reynolds boundary conditions, in order to determine the hydrodynamic pressure in the ferro-oil, were taken **[L. 31, 52]**.

Using the continuity equation and previously calculated peripheral and longitudinal components, after integrating the equation and applying appropriate boundary conditions, a velocity vector radial component and a Reynolds type equation were obtained for four cases which take into account the following: **(a)** both Newtonian properties and influence of a magnetic field, (**b)** an effect of a temperature on the change in viscosity, (**c)** an effect of a pressure on the change in viscosity, and **(d)** and impact of non-Newtonian properties on the change of viscosity. These are as follows:

a) For the first set of equations which takes into account the Newtonian properties and the influence of the magnetic field **[L. 52]**:

$$\frac{\partial}{\partial \phi}\left[\frac{h_{p1}^3}{\eta_{1B}}\left(\frac{\partial p_1^{(0)}}{\partial \phi} - M_1\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\frac{h_{p1}^3}{\eta_{1B}}\left(\frac{\partial p_1^{(0)}}{\partial z_1} - M_3\right)\right] = 6\frac{\partial h_{p1}}{\partial \phi} \tag{8}$$

b) For the second set of equations which takes into account the influence of temperature on viscosity **[L. 52]**:

$$\frac{\partial}{\partial\phi}\left[\frac{h_{p1}^3}{\eta_{1B}}\left(\frac{\partial p_{10}^{(1)}}{\partial\phi}\right)\right]+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\frac{h_{p1}^3}{\eta_{1B}}\left(\frac{\partial p_{10}^{(1)}}{\partial z_1}\right)\right]=$$
$$=12\left\{\frac{\partial}{\partial\phi}\left[\left(\int_0^{h_{p1}}\left(\int_0^{r_1}T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial r_1}dr_1\right)dr_1-\int_0^{h_{p1}}\frac{r_1}{h_{c1}}\left(\int_0^{h_{p1}}T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial r_1}dr_1\right)dr_1\right)\right]+\right. \tag{9}$$
$$\left.+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\int_0^{h_{p1}}\left(\int_0^{r_1}T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial r_1}dr_1\right)dr_1-\int_0^{h_{p1}}\frac{r_1}{h_{c1}}\left(\int_0^{h_{p1}}T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial r_1}dr_1\right)dr_1\right]\right\}$$

c) For the third set of equations which takes into account the influence of pressure on viscosity **[L. 52]**:

$$\frac{\partial}{\partial\phi}\left[\frac{h_{p1}^3}{\eta_{1B}}\left(\frac{\partial p_{11}^{(1)}}{\partial\phi}\right)\right]+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\frac{h_{p1}^3}{\eta_{1B}}\left(\frac{\partial p_{11}^{(1)}}{\partial z_1}\right)\right]=$$
$$=12\left\{\frac{\partial}{\partial\phi}\left[\int_0^{h_{p1}}\frac{r_1}{h_{c1}}\left(\int_0^{h_{p1}}p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial r_1}dr_1\right)dr_1-\int_0^{h_{p1}}\left(\int_0^{r_1}p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial r_1}dr_1\right)dr_1\right]+\right. \tag{10}$$
$$\left.+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\int_0^{h_{p1}}\frac{r_1}{h_{c1}}\left(\int_0^{h_{p1}}p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial r_1}dr_1\right)dr_1-\int_0^{h_{p1}}\left(\int_0^{r_1}p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial r_1}dr_1\right)dr_1\right]\right\}$$

d) For the fourth set of equations taking into account the influence of non-Newtonian properties on viscosity **[L. 52]**:

$$\frac{\partial}{\partial\phi}\left(\frac{h_{p1}^3}{\eta_{B1}}\frac{\partial p_{1p}^{(1)}}{\partial\phi}\right)+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left(\frac{h_{p1}^3}{\eta_{B1}}\frac{\partial p_{1p}^{(1)}}{\partial z_1}\right)=$$
$$=12\left\{\frac{\partial}{\partial\phi}\left[\frac{1}{\eta_{B1}}\left(\int_0^{h_{p1}}\int_0^{r_3}\int_0^{r_2}F(\varphi,r_1,z_1)dr_1dr_2dr_3-\frac{h_{p1}}{2\eta_{B1}}\int_0^{h_{p1}}\int_0^{r_2}F(\varphi,r_1,z_1)dr_1dr_2\right)\right]+\right. \tag{11}$$
$$\left.+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\frac{1}{\eta_{B1}}\left(\int_0^{h_{p1}}\int_0^{r_3}\int_0^{r_2}G(\varphi,r_1,z_1)dr_1dr_2dr_3-\frac{h_{p1}}{2\eta_{B1}}\int_0^{h_{p1}}\int_0^{r_2}G(\varphi,r_1,z_1)dr_1dr_2\right)\right]\right\}$$

where

$$v_1^{(0)}(r_1,\varphi,z_1)=\frac{1}{2\eta_{1B}}\left(\frac{\partial p_1^{(0)}}{\partial\phi}-M_1\right)\left(r_1^2-r_1h_{p1}\right)+1-\frac{r_1}{h_{p1}},\; v_3^{(0)}(r_1,\varphi,z_1)=\frac{1}{2\eta_{1B}}\left(\frac{\partial p_1^{(0)}}{\partial z_1}-M_3\right)\left(r_1^2-r_1h_{p1}\right),$$
$$T_1^{(0)}(r_1,\varphi,z_1)=1+\frac{1}{2}\eta_{1B}(1-2s)-q_{1c}^{(0)}h_{p1}s-\frac{1}{2}\Omega_1\left(h_{p1}s\right)^2-\frac{1}{6}h_{p1}^2\left(\frac{\partial p_1^{(0)}}{\partial\varphi}-M_1\right)s\,(3-3s+s^2)+$$
$$-\frac{1}{2}\eta_{1B}\left[\left(v_1^{(0)}\right)^2+\frac{1}{L_1^2}\left(v_3^{(0)}\right)^2\right]+\frac{1}{24\eta_{1B}}h_{p1}^4\left[\left(\frac{\partial p_1^{(0)}}{\partial\varphi}-M_1\right)^2+\frac{1}{L_1^2}\left(\frac{\partial p_1^{(0)}}{\partial z_1}-M_3\right)^2\right]s^3(s-2),$$

$$M_1 = R_f \chi \left[H_1 \frac{\partial H_1}{\partial \varphi} + \frac{1}{L_1} H_3 \frac{\partial H_1}{\partial z_1} \right], \quad M_3 = R_f L_1 \chi \left(H_1 \frac{\partial H_3}{\partial \varphi} + \frac{1}{L_1} H_3 \frac{\partial H_3}{\partial z_1} \right),$$

$$F(\varphi, r_1, z_1) \equiv \left(1 + 2\frac{\beta_o}{\alpha_o}\right)\left(\frac{\partial X_1}{\partial \varphi} + \frac{1}{L_1^2}\frac{\partial Z_1}{\partial \varphi}\right) - \frac{\partial X_1}{\partial \varphi} - \frac{1}{L_1^2}\left(\frac{\partial X_2}{\partial r_1} + \frac{\partial X_3}{\partial z_1}\right) - \frac{\beta_o}{\alpha_o}\left(\frac{\partial X_4}{\partial r_1} + 2\frac{\partial X_5}{\partial r_1}\right),$$

$$G(\varphi, r_1, z_1) \equiv \left(1 + 2\frac{\beta_o}{\alpha_o}\right)\left(\frac{\partial X_1}{\partial z_1} + \frac{1}{L_1^2}\frac{\partial Z_1}{\partial z_1}\right) - \frac{1}{L_1^2}\frac{\partial Z_1}{\partial z_1} - \left(\frac{\partial Z_2}{\partial r_1} + \frac{\partial Z_3}{\partial \varphi}\right) - \frac{\beta_o}{\alpha_o}\left(\frac{\partial Z_4}{\partial r_1} + 2\frac{\partial Z_5}{\partial r_1}\right),$$

$$\mathrm{X}_1 \equiv \left(\frac{\partial \mathrm{v}_1^{(0)}}{\partial \mathrm{r}_1}\right)^2, \ \mathrm{X}_2 \equiv \frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{r}_1}\mathrm{Y}_4 - 2\frac{\partial \mathrm{v}_1^{(0)}}{\partial \mathrm{r}_1}\frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{z}_1}, \ \mathrm{X}_3 \equiv \frac{\partial \mathrm{v}_1^{(0)}}{\partial \mathrm{r}_1}\frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{r}_1}, \ \mathrm{Y}_1 \equiv \frac{\partial \mathrm{v}_1^{(0)}}{\partial \phi}, \mathrm{Y}_2 \equiv \frac{\partial \mathrm{v}_2^{(0)}}{\partial \mathrm{r}_1} \ \mathrm{Y}_3 \equiv \frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{z}_1},$$

$$Y_4 \equiv \frac{\partial v_3^{(0)}}{\partial \varphi} + \frac{\partial v_1^{(0)}}{\partial z_1}, \mathrm{Z}_1 \equiv \left(\frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{r}_1}\right)^2, \ Z_2 \equiv \frac{\partial v_1^{(0)}}{\partial r_1}Y_4 - 2\frac{\partial v_3^{(0)}}{\partial r_1}\frac{\partial v_1^{(0)}}{\partial \varphi}, \quad \mathrm{Z}_3 \equiv \frac{\partial \mathrm{v}_1^{(0)}}{\partial \mathrm{r}_1}\frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{r}_1},$$

$$X_4 \equiv \frac{\partial}{\partial r_1}\left(v_1^{(0)}\frac{\partial v_1^{(0)}}{\partial \varphi} + v_2^{(0)}\frac{\partial v_1^{(0)}}{\partial r_1} + \frac{1}{L_1^2}v_3^{(0)}\frac{\partial v_1^{(0)}}{\partial z_1}\right), \ X_5 \equiv \frac{\partial v_1^{(0)}}{\partial \phi}\frac{\partial v_1^{(0)}}{\partial r_1} + \frac{1}{L_1^2}\frac{\partial v_3^{(0)}}{\partial \varphi}\frac{\partial v_3^{(0)}}{\partial r_1},$$

$$\mathrm{Z}_4 \equiv \frac{\partial}{\partial \mathrm{r}_1}\left(\mathrm{v}_1^{(0)}\frac{\partial \mathrm{v}_3^{(0)}}{\partial \phi} + \mathrm{v}_2^{(0)}\frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{r}_1} + \frac{1}{\mathrm{L}_1^2}\mathrm{v}_3^{(0)}\frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{z}_1}\right), \ \mathrm{Z}_5 \equiv \frac{\partial \mathrm{v}_1^{(0)}}{\partial \mathrm{r}_1}\frac{\partial \mathrm{v}_1^{(0)}}{\partial \mathrm{z}_1} + \frac{1}{\mathrm{L}_1^2}\frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{z}_1}\frac{\partial \mathrm{v}_3^{(0)}}{\partial \mathrm{r}_1},$$

$$h_{p1} = [1 + \lambda \cdot \cos\varphi + a_\gamma z_1 \cdot \cos(\varphi)], \quad a_\gamma = \frac{L_1}{\psi}\tan(\gamma),$$

$$0 \le r_1 \le h_{p1},\ 0 \le \varphi < \varphi_k,\ -1 \le z_1 < +1,\ s \equiv r_1/h_{p1},\ 0 \le s \le 1,\ 0 \le \mathrm{r}_1 \le \mathrm{r}_2 \le \mathrm{r}_3 \le \mathrm{h}_{c1}$$

a_γ – misalignment factor,
v_1, v_2, v_3 – dimensionless velocity vector components of ferro-oil,
r_1 – dimensionless radial coordinate,
$q_{1c}^{(0)}$ – dimensionless density of heat stream,
z_1 – dimensionless longitudinal coordinate,
h_{p1} – dimensionless height of lubrication gap,
ϕ – peripheral coordinate,
γ – angle of misalignment,
λ – relative eccentricity,
χ – magnetic susceptibility coefficient of ferro-oil,
ψ – dimensionless value of radial relative clearance,
Ω_1 – dimensionless heat supplied from outside sources to ferro-oil,
H_1, H_2, H_3 – dimensionless vector components of a magnetic field strength,
α_o, β_o – dimensional values of ferro-oil material coefficients,
L_1 – dimensionless length of bearing,
R_f – magnetic pressure number.

The total dimensional value of the carrying capacity coefficient C_Σ in the slide journal bearing has been determined from the commonly known relation **[L. 31]**:

$$C_\Sigma = C_{1\Sigma} \cdot bR\eta_o\omega / \psi^2 \tag{12}$$

The total dimensionless value of the carrying capacity coefficient $C_{1\Sigma}$ in the slide journal bearing lubricated with a ferromagnetic factor has been calculated from the dependence **[L. 31]**:

$$C_{1\Sigma} = C_1^{(0)} + Q_{Br}C_{10}^{(1)} + \zeta_p C_{11}^{(1)} + De_\alpha C_1^{(1)} + O(Q_{Br}^2) + O(\zeta_p^2) + O(De_\alpha^2) \tag{13}$$

The total dimensional friction force Fr_Σ and total dimensionless friction force Fr_1 in the journal slide bearing gap has been shown in the following relation **[L. 31]**:

$$Fr_\Sigma = Fr \cdot (bR\eta_o\omega)/\psi\ ;\ Fr_1 = Fr_1^{(0)} + Q_{BR}Fr_{10}^{(1)} + \varsigma_p \cdot Fr_{11}^{(1)} + De_\alpha \cdot Fr_1^{(1)} \tag{14}$$

Analogously, the total contractual coefficient of friction for ferro-oil taking into account the influence of magnetic field, pressure, temperature, and non-Newtonian properties on the change of dynamic viscosity has been determined from the following formula **[L. 52]**:

$$\left(\frac{\mu}{\psi}\right)_{\Sigma} = \frac{Fr_{\Sigma}}{\psi \cdot C_{\Sigma}} = \left(\frac{\mu}{\psi}\right)_{1}^{(0)} + Q_{Br}\left(\frac{\mu}{\psi}\right)_{10}^{(1)} + \zeta_{p}\left(\frac{\mu}{\psi}\right)_{11}^{(1)} + De_{\alpha}\left(\frac{\mu}{\psi}\right)_{1}^{(1)} \quad (15)$$

$$\left(\frac{\mu}{\psi}\right)_{1}^{(0)} = \frac{Fr_{1}^{(0)}}{C_{1}^{(0)}} \quad (16)$$

$$\left(\frac{\mu}{\psi}\right)_{10}^{(1)} = \frac{\dfrac{Fr_{1}^{(0)} + Q_{Br}Fr_{10}^{(1)}}{C_{1}^{(0)} + Q_{Br}C_{10}^{(1)}} - \left(\dfrac{\mu}{\psi}\right)_{1}^{(0)}}{Q_{Br}} \quad (17)$$

$$\left(\frac{\mu}{\psi}\right)_{11}^{(1)} = \frac{\dfrac{Fr_{1}^{(0)} + \zeta_{p}Fr_{11}^{(1)}}{C_{1}^{(0)} + \zeta_{p}C_{11}^{(1)}} - \left(\dfrac{\mu}{\psi}\right)_{1}^{(0)}}{\zeta_{p}} \quad (18)$$

$$\left(\frac{\mu}{\psi}\right)_{1}^{(1)} = \frac{\dfrac{Fr_{1}^{(0)} + De_{\alpha}Fr_{1}^{(1)}}{C_{1}^{(0)} + De_{\alpha}C_{1}^{(1)}} - \left(\dfrac{\mu}{\psi}\right)_{1}^{(0)}}{De_{\alpha}} \quad (19)$$

where
μ – magnetic permeability of ferro-oil,
b – half the length of the bearing,
De_{α} – Deborah's number.

THE CALCULATION METHODS ADOPTED IN THE MODEL

The aim of solving the obtained Reynolds type equations, which are substantially partial differential equations of the second order, the finite-difference numerical method has been applied. This method, in principle, consists in approximation of the partial derivatives with the finite differences **[L. 53–55]**. In paper **[L. 52]**, the derivatives of functions were replaced by progressive or central differences of the first or second degree of accuracy. Differential equations were converted into differential equations and numerical solutions of the latter were implemented on computational grids whose topology measuring 50x20 points corresponds to the model of the considered sliding bearing surface.

In order to determine the function of the expected operational parameters of slide bearings, such as load carrying capacity, friction force, and friction coefficient, the small parameter method has been used. This method consists in exchanging the wanted dimensionless quantities with a convergent series related to small parameters **[L. 31, 56–58]**. Essentially, by means of this method, a non-linear system of partial differential equations has been uncoupled into four systems of linear equations. The first one refers to classical Newton lubrication without taking into account pressure, temperature, or viscoelastic properties, and changes in viscosity of the lubricant, and allows one to determine the basic flow parameters. It is in this system that the influence of the external magnetic field on changes in ferro-oil viscosity is taken into account. The scheme of the other systems of equations allows one to determine three consecutive types of adjustments of flow and operating parameters. These corrections take into account the influence of pressure and temperature on changes in ferro-oil viscosity as well as impacts of viscoelastic ferro-oil properties.

As the small dimensionless parameters, the following values have been accepted: Deborah's number De_{α} which is responsible for determining the impact of non-Newtonian ferro-oil properties on changes in flow and performance parameters; and, dimensionless factor Q_{Br} describing the changes in viscosity on temperature. In turn, changes in dynamic viscosity under the influence of hydrodynamic pressure have been determined by changes in the dimensionless value of piezo-factor ζ_{p}. The appointments of the small parameters and the appropriate designations have been taken as applied in the monograph of A. Miszczak **[L. 31]**.

Table 1 below presents the determined and adopted values of the small parameters for particular concentrations of magnetic particles in ferro-oil.

Table 1. The small parameters Q_{Br}, ζ_{p} and De_{α} values adopted in the paper
Tabela 1. Przyjęte wartości małych parametrów: Q_{Br}, ζ_{p} oraz De_{α}

Values of the small parameters	Magnetic particles concentration n_{cs}					
	0%	1%	2%	4%	6%	8%
Q_{Br}	0.044603	0.047482	0.061210	0.083820	0.213011	0.318392
ζ_{p}	0.001579	0.002045	0.002815	0.003834	0.008493	0.012612
De_{α}	0.025856	0.025595	0.020626	0.015396	0.006106	0.004334

THE ANALYSIS OF THE OBTAINED RESULTS

The following dimensional and dimensionless quantities for all calculations of operational parameters have been adopted: a low-speed bearing with an angular velocity of the journal $\omega = 20 s^{-1}$ was assumed; the journal radius was R = 0.15 m and the dimensionless bearing length $L_1 = 1$; a constant dimensionless radial relative clearance value $\psi = 0.003$; the ferro-oil thermal conduction coefficient was established as unchangeable and was equal $\kappa = 0.15$; the material coefficients of a ferro-oil were, respectively, $\alpha = 0.000020$ and $\beta = -0.000010$;

and, the value of the magnetic field intensity vector was assumed at the level ensuring full magnetic saturation of a ferro-oil $H_o = 280000$ A·m^{-1} **[L. 45]**. In the calculations carried out, it was assumed that the bearing placement effect will not be taken into account in the model; hence, the misalignment angle was $\gamma = 0°$.

Calculations of operating parameters were carried out for the assumed concentrations of magnetic particles n_{cs} in ferro-oil of: 0%, 1%, 2%, 4%, 6%, and 8%, taking into account changes in the relative eccentricity of the slide bearing $\lambda = 0.1$ to $\lambda = 0.9$.

In addition, the characteristic dimensional values of dynamic viscosity η_o for $T = T_o = 90°C$ and $p = p_{at}$ as well as values of magnetic susceptibility coefficients of a ferro-oil have been determined experimentally **[L. 46]**, and their adopted values are shown in **Table 2**.

Table 2. Values of viscosity coefficients and magnetic susceptibility coefficients adopted in the paper

Tabela 2. Przyjęte wartości lepkości, współczynników lepkości i współczynników podatności magnetycznej

Values of the parameters	Magnetic particles concentration n_{cs}					
	0%	1%	2%	4%	6%	8%
$\eta_o^{(90°C)}$ [Pa·s]	0.01547	0.01563	0.01939	0.02598	0.06550	0.09229
χ [-]	0	0.04752	0.06007	0.08227	0.11764	0.14388
a_B [$T^{-\delta 1B}$]	0	0.20706	0.57169	0.78382	1.09677	1.38077
δ_{1B} [-]	1	0.25139	0.24601	0.25433	0.21690	0.21035
a_T [-]	0.93535	0.79514	0.73721	0.72791	0.68873	0.67321
δ_T [K^{-1}]	0.04805	0.05064	0.05261	0.05377	0.05419	0.05749
a_p [-]	1.35221	1.51631	1.60595	1.68489	1.75598	1.92889
ζ [Pa^{-1}]	$4.59 \cdot 10^{-8}$	$5.89 \cdot 10^{-8}$	$6.53 \cdot 10^{-8}$	$6.64 \cdot 10^{-8}$	$5.83 \cdot 10^{-8}$	$6.15 \cdot 10^{-8}$

The calculations of operating parameter distributions for cases of ferro-oil bearing lubrication taking into account the influence of external magnetic field and subsequent corrections of these parameters responsible for the effect of temperature changes and the effect of pressure changes on changes in dynamic viscosity of ferro-oil as well as corrections taking into account the viscoelastic properties of ferro-oil have been realized. **Figures 1** through **3** show the results of distributions of a calculated operational parameter in the form of five successive characteristics of basic values, the abovementioned three corrections values, and total values. The following markings have been adopted on each of Figures 1 through 3. Designation (**a**) concerns results that take into account the Newtonian properties of ferro-oils subject to the influence of the external magnetic field; designations (**b**) refer to corrections of operating parameters derived from the impact of temperature changes on the dynamic viscosity of ferro-oil, designations (**c**) means corrections from the impact of pressure changes on dynamic viscosity, designation (**d**) means corrections from non-Newtonian properties, and (**e**) refers to a summary statement, i.e. particular corrections multiplied by appropriate small parameters and added to the basic value.

In **Fig. 1**, dimensionless values of the following operating parameters have been presented: load carrying capacity for basic assumptions $C_1^{(0)}$; corrections of these forces from the influence of temperature on viscosity $C_{10}^{(1)}$, from the effect of pressure on viscosity $C_{11}^{(1)}$, from non-Newtonian viscoelastic properties $C_1^{(1)}$; and total values of load carrying capacity C_1.

Figure 2 presents the following: dimensionless values of basic friction forces $Fr_1^{(0)}$; subsequent corrections of these forces from the influence of temperature on viscosity $Fr_{10}^{(1)}$, from the effect of pressure on viscosity $Fr_{11}^{(1)}$, and from non-Newtonian viscoelastic properties $Fr_1^{(1)}$; and total values of friction forces Fr_1.

Analogously, **Fig. 3** presents the dimensionless values of the following: conventional coefficient of friction $(\mu/\psi)_1^{(0)}$; subsequent corrections of this coefficient taking account the influence of temperature on viscosity $(\mu/\psi)_{10}^{(1)}$, the influence of pressure on viscosity $(\mu/\psi)_{11}^{(1)}$, non-Newtonian viscoelastic properties $(\mu/\psi)_1^{(1)}$; and total values of the coefficient $(\mu/\psi)_1$.

Based on the analysis of the results presented in **Fig. 1**, it can be concluded that the value of the bearing load carrying capacity depends on the concentration of magnetic particles in the ferro-oil and increases with its concentration in the ferro-oil. Relatively, the largest growth in load carrying capacity, of approx. 21%, concerns the increase of magnetic particles concentration between 1% and 2%. Furthermore, for

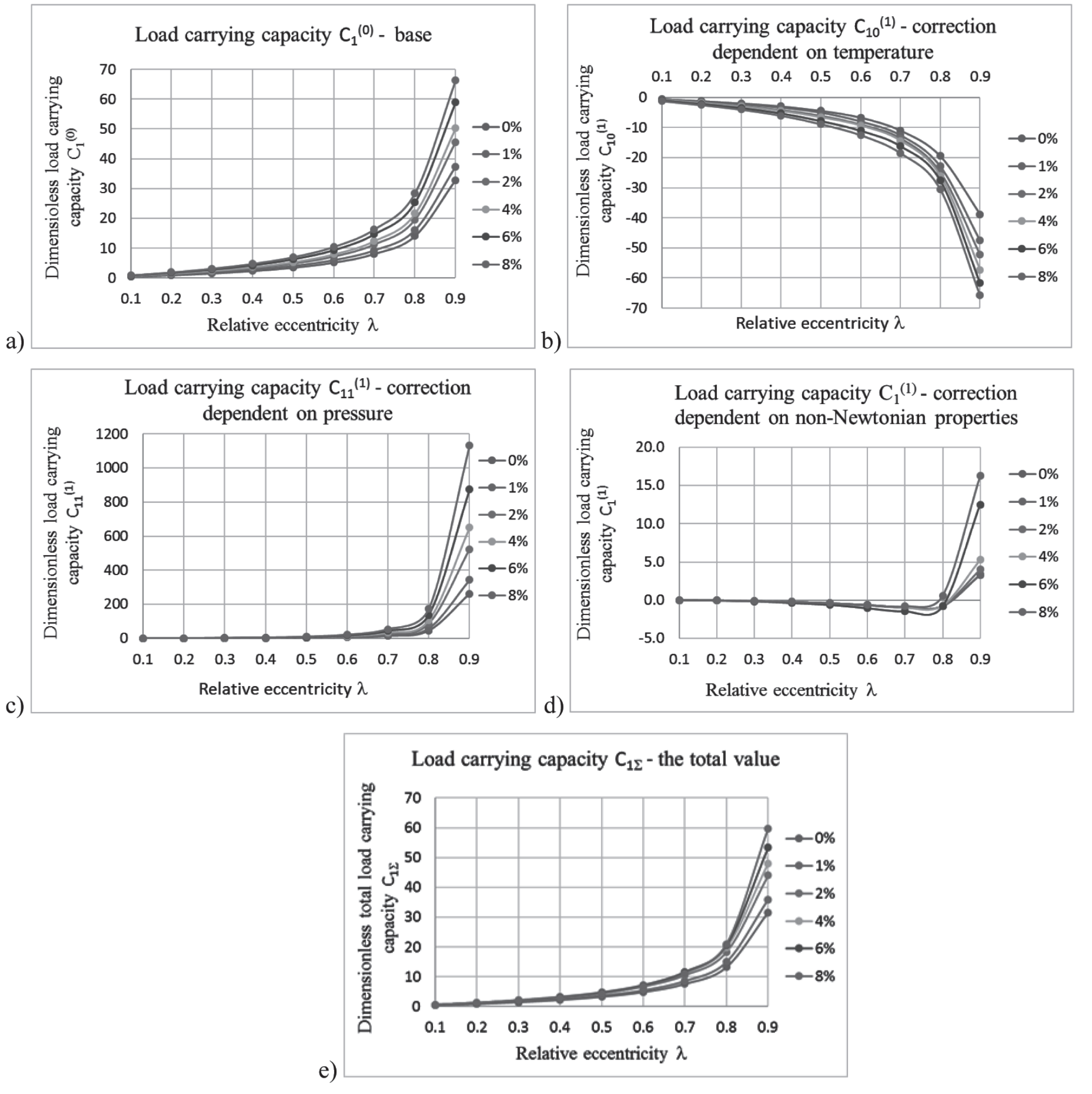

Fig. 1. The values of dimensionless bearing load carrying capacity, three corrections of dimensionless bearing load carrying capacity and dimensionless total bearing load carrying capacity for changes in relative eccentricity λ and concentration of magnetic particles in ferro-oil n_{cs}

Rys. 1. Wartości bezwymiarowych sił nośnych, trzech korekt bezwymiarowych sił nośnych i bezwymiarowej sumarycznej siły nośnej dla zmian mimośrodowości względnych λ i stężenia cząstek magnetycznych w ferro-oleju n_{cs}

large- and medium-sized lubrication gaps in the bearing, the influence of particular load-bearing force corrections on its total value is balanced. For small-sizes of the lubrication gap, the nature of the changes significantly becomes dominated by the influence of corrections resulting from changes in temperature and pressure, with the more significant effect of the correction coming from changes in temperature, which reaches even 25% of the base value and decreases it. The direction of changes in the correction against pressure changes is the opposite and influences the increase of the total value of this parameter. However, only for the highest concentrations, i.e. 6% and 8%, this increase is significant and reaches a value of about 20% of the base value. The influence of the load carrying capacity correction depends on non-Newtonian properties, and its total value remains negligibly small.

As the concentration of magnetic particles increases in the ferro-oil, the friction forces increase as well. The change in the friction force value between 0% and 8% is about 2.5 times for the whole range of relative bearing eccentricities. The largest differences, around 39%, in increments concern cases between 4% and 6% as well as 6% and 8%. The most important influence on the change of the friction force value is caused by the corrections

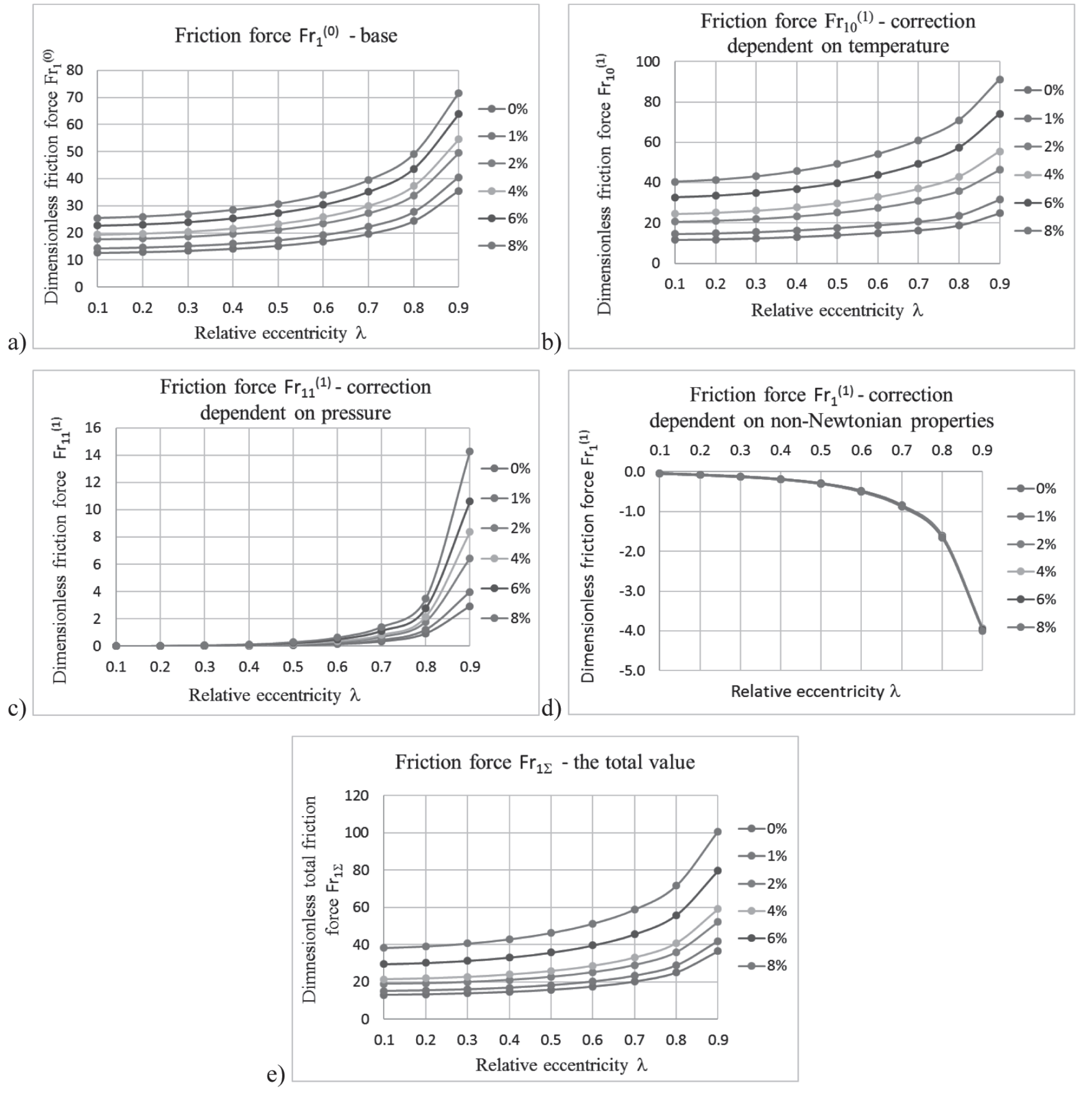

Fig. 2. The values of dimensionless bearing friction force, three corrections of dimensionless bearing friction force and dimensionless total bearing friction force for changes in relative eccentricity λ and concentration of magnetic particles in ferro-oil n_{cs}

Rys. 2. Wartości bezwymiarowych sił tarcia, korekt bezwymiarowych sił tarcia i bezwymiarowej sumarycznej siły tarcia dla zmian mimośrodowości względnych λ i stężenia cząstek magnetycznych w ferro-oleju n_{cs}

to the temperature changes of the ferro-oil viscosity in the bearing gap. The corrections of the friction force on pressure assume significant values only for the cases of low lubrication gap. The influence of corrections of friction forces depend on non-Newtonian properties and their total value remains negligible irrespective of the value of relative eccentricity value.

The dimensionless conventional coefficient of friction increases with increasing concentration of magnetic particles in the ferro-oil. The most important influence on the total value of the coefficient of friction is caused by corrections resulting from temperature changes, which locally reach up to 151%. The changes in the total value of the coefficient resulting from these corrections are strongly differentiated from the changes in the concentration of magnetic particles. The analysis of the characteristics shows that, in order to obtain conditions for optimal operation of a lubricated slide journal bearing lubricated with the ferro-oil, concentration values of magnetic particles should be between 2% and 4%. Above these values of the concentration of magnetic particles in ferro-oil, there is a strong increase in the coefficient of friction, in particular, for cases of small relative eccentricities in the slide journal bearing. The influence of the other two corrections of the coefficient of friction seems to be insignificant.

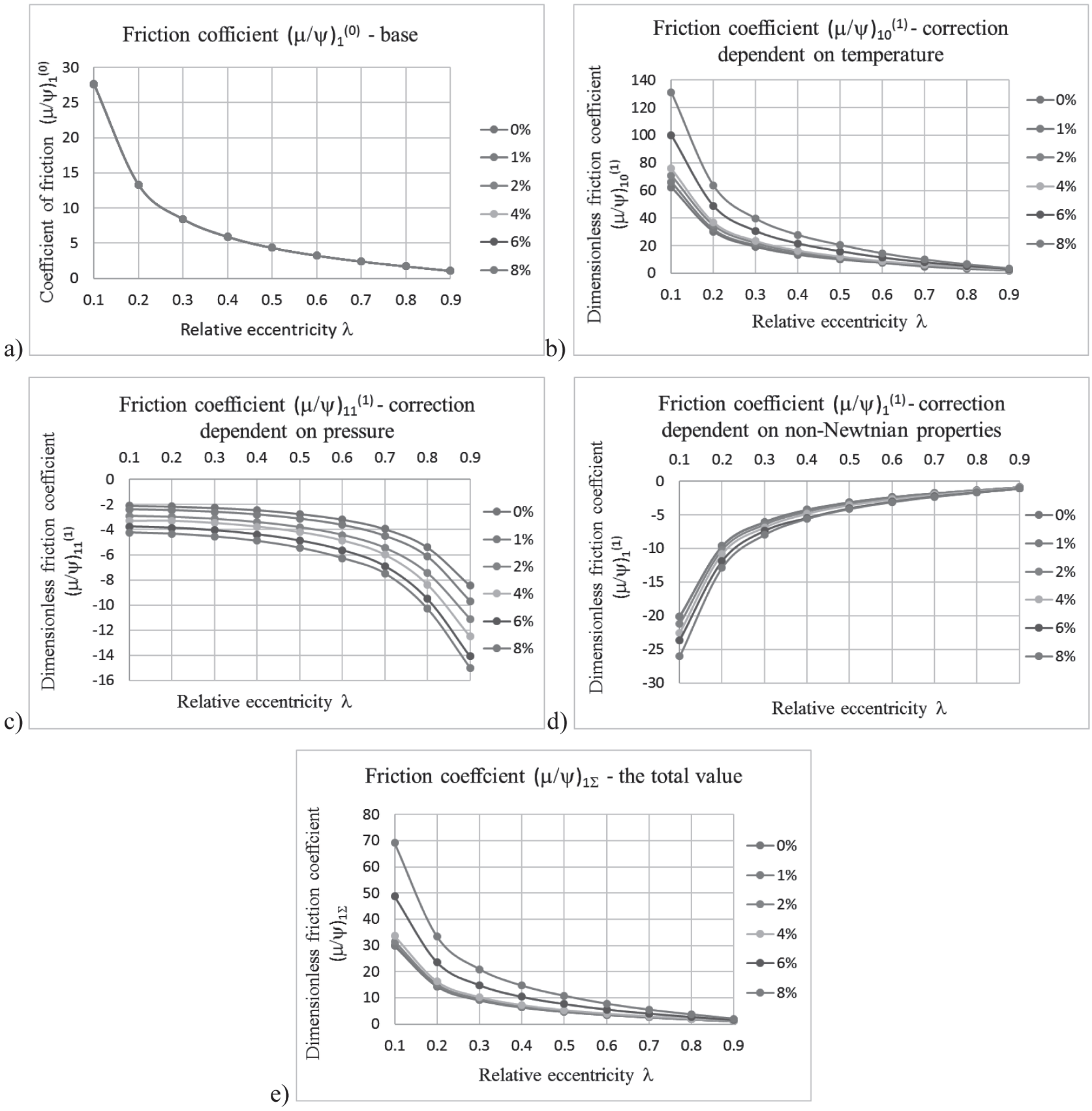

Fig. 3. The values of dimensionless bearing friction coefficient, three corrections of dimensionless bearing friction coefficient and dimensionless total bearing friction coefficient for changes in relative eccentricity λ and concentration of magnetic particles in ferro-oil n_{cs}

Rys. 3. Wartości bezwymiarowych współczynników tarcia, trzech korekt bezwymiarowych współczynników tarcia i bezwymiarowego sumarycznego umownego współczynnika tarcia dla zmian mimośrodowości względnych λ i stężenia cząstek magnetycznych w ferro-oleju n_{cs}

CONCLUSIONS

The analysis of the obtained results leads to the conclusion that, in order to obtain the best operating conditions for a slide bearing subjected to magnetohydrodynamic lubrication, it should be assumed that the concentration of magnetic particles in ferro-oil for bearings operating in low or medium load conditions should not exceed 4%. Above this concentration, there is a strong increase in the value of the conventional friction coefficient in a friction bearing lubricated with ferro-oil. What is more, due to technical aspects, as well as economic aspects, as long as the expectation towards a bearing allows it, the concentration of magnetic particles should be even lower and amounts to approx. 2%. The adoption of such a strategy to select the concentration of magnetic particles in the ferro-oil would allow maximizing the value of the load carrying capacity while minimizing the friction force value.

Additionally, in the case of a bearing operating in the range of large relative eccentricities, i.e. of high load conditions, it can be noticed that the coefficient of friction is no longer subject to such strong differentiation in the context of the concentration of magnetic particles in the ferro-oil; therefore, there is no legitimization for the use of ferro-oils with high concentration, i.e. higher than 2%.

REFERENCES

1. Hamrock, B.J., Schmid S.R., Jacobson B.O.: Fundamentals of Fluid Film Lubrications, Marcel Deker, Inc., New York 2004.
2. Hebda, M., Wachal, A.: Trybologia, Wydawnictwo Naukowo-Techniczne, Warszawa 1980.
3. Hori, Y.: Hydrodynamic Lubrication, Springer, Tokyo 2006.
4. Harada, M., Yang, W., Tsukazaki, J,. Yamamoto, H.: Characteristics of Journal Bearings Lubricated With Ferro-Fluid, Applied Mechanics and Engineering, Vol. 4, 1999, pp. 33–38.
5. Hsu, T.Ch., Chen, J.H., Chiang, H.L., Chou, T.L.: Lubrication performance of short journal bearings considering the effects of surface roughness and magnetic field, Tribology Letters, Vol. 61, 2013, pp. 169–176.
6. Ilg, P., Kroger, M., Siegfried, H.: Structure and rheology of model-ferrofluids under shear flows, Journal of Magnetism and Magnetic Materials, Vol. 289, 2005, pp. 325–327.
7. Ma, Y.-Y., Wang, W.-H., Cheng, X.-H.: A study of dynamically loaded journal bearings lubricated with non-Newtonian couple stress fluids, Springer Verlag, Tribology Letters, Vol. 17, No.1, 2004, pp. 69–74.
8. Osman, T.A., Nada, G.S., Safar, Z.S.: Static and dynamic characteristic of magnetized journal bearings lubricated with ferrofluid, Tribology Letters, Vol. 34, 2001, pp. 369–380.
9. Patel, N.S., Vakharia, D., Deheri, G.: Hydrodynamic journal bearing lubricated with a ferrofluids, Industial Lubrication and Technology, Vol. 69, 2017, pp.754–760.
10. Prabhu, B.S.: The Load Capacity of a Partial Journal Bearing with a Non–Newtonian Film, Wear, Vol. 40, No. 1, 1976, pp. 9–14.
11. Tipei, N.: Theory of lubrication with ferrofluids: application to short bearing, Transactions of the ASME, Journal of Lubrication Technology, Vol. 104, 1982, pp. 510–515.
12. Wang, J., Kang, J., Zhang, Y., Huang, X.: Viscosity monitoring and control on oil–film bearing lubrication with ferrofluids, Tribology International, Vol. 75, 2014, pp. 61–68.
13. Zhang, C., Yi, Z., Zhang, Z.: THD Analysis of High Speed Heavily Loaded Journal Bearings Including Thermal Deformation, Mass Conserving Cavitation, and Turbulent Effects, Transactions of the ASME, Journal of Tribology, Vol. 122, 2000, pp. 597–602.
14. Beran, E.: Wpływ budowy chemicznej bazowych olejów smarowych na ich biodegradowalność i wybrane właściwości eksploatacyjne, Prace Naukowe Wydziału Chemicznego Politechniki Wrocławskiej, Wrocław 2008.
15. Alam, J., Riaz, U., Sharif A.: Effect of ferrofluid concentration on electrical and magnetic properties of Fe_3O_4/PANI composites, Journal of Magnetism and Magnetic Materials, Vol. 314, 2007, pp. 93–99.
16. Cunha, F.R., Rosa, A., Dias N.J.: Rheology of very dilute magnetic suspension with micro-structures of nanoparticles, Journal of Magnetism and Magnetic Materials, Vol. 397, 2016, pp. 266–274.
17. Hosseini, M., Vafajoo, L., Ghasemi, E., Salman, B.: Experimental investigation the effect of nanoparticle concentration on rheological behavior of paraffin-based nickel ferrofluid, Int. Journal of Heat and Mass Transfer, Vol. 93, 2016, pp. 228–234.
18. Malekzadeh, A., Pouranfard, A.R., Hatami, N., Banari, A.K., Rahimi, M.R.: Experimental investigations on the viscosity of magnetic nanofluids under the influence of temperature, volume fractions of nanoparticles and external magnetic field, Journal of Applied Fluid Mechanic, Vol. 9, 2016, pp. 693–697.
19. Resiga, D.S., Vekas, L.: Yield stress and flow behavior of concentrated ferrfluid-based magnetorheological fluids: the influence of composition, Rheology Acta, Vol. 53, 2014, pp. 645–653.
20. Zubarev, A.Y., Fleischer, J., Odenbach S.: Toward a theory of dynamical properties of polydisperse magnetic fluids: Effects of chain like aggregates, Physica, Vol. 358, 2005, pp. 475–491.
21. Zubarev, A.Y., Iskakova, L.Y.: To the theory of rheological properties of ferrofluids: influence of drop-like aggregates, Physica, Vol. 343, 2004, pp. 65–80.
22. Frycz, M., Anioł, P.: Distribution of Temperature in the Gap of Sliding Journal Bearings Ferrofluid's Lubricated with Different Concentrations of Magnetic Particles, Zeszyty Naukowe Akademii Morskiej w Gdyni, No 81, Gdynia 2013, pp. 38–54.
23. Frycz, M., Miszczak, A.: Distribution of hydrodynamic pressure in the interspace of lateral sliding bearing lubricated by ferrofluid with different concentrations of magnetic particles, Journal of KONES. Powertrain and Transport, Vol. 18 No. 3, 2011, pp. 83–89.
24. Miszczak, A., Frycz, M.: Wpływ stężenia cząstek magnetycznych w ferrooleju na wartość siły nośnej w poprzecznym łożysku ślizgowym przy uwzględnieniu zmian lepkości od pola magnetycznego, ciśnienia i temperatury, Logistyka – nauka, Vol. 6, 2014.
25. Frycz, M., Miszczak, A.: Influence of magnetic particles concentration in ferro-oils on values of friction force and coefficient of friction of slide journal bearings, Journal of KONES Powertrain and Transport, Vol. 24, 2017, pp. 143–150.

26. Wierzcholski, K., Miszczak, A.: Pressure distributions for turbulent lubrication in journal bearing gap, Zagadnienia Eksploatacji Maszyn, Vol. 37, zeszyt 3 (131), 2002, pp. 191–200.
27. Wierzcholski, K., Miszczak A.: Równania hydrodynamicznej teorii smarowania cieczą o cechach modelu Rivlina Ericksena, Zagadnienia Eksploatacji Maszyn, Vol. 31 zeszyt 3 (106), 1996, pp.l 7–18.
28. Wierzcholski, K., Miszczak, A.: The theory of turbulent lubrication with ferroil in variable magnetic field, Zeszyty Naukowe Katedry Mechaniki Stosowanej Politechnika Śląska, 18, 2002, pp. 445–450.
29. Wierzcholski, K., Pytko S.: Obliczenia walcowych łożysk ślizgowych smarowanych olejami o własnościach nienewtonowskich, Zagadnienia Eksploatacji Maszyn, kwartalnik PAN, zeszyt 4, (84), Vol. 25, 1990, pp. 383––391.
30. Wierzcholski, K., Wissussek, D.: The Rivlin Ericksen Lubricant Flow in Journal Bearing Gap, System Modelling Control 8, Vol. 2, 1995, pp. 388–393.
31. Miszczak, A.: Analiza hydrodynamicznego smarowania ferrocieczy poprzecznych łożysk ślizgowych, Akademia Morska w Gdyni, Gdynia 2006.
32. Miszczak, A.: Korekty parametrów eksploatacyjnych dla lepkosprężystych ferrosmarów, Zeszyty Naukowe Katedry Mechaniki Stosowanej Politechnika Śląska, 23, 2004, pp. 315–320.
33. Miszczak, A.: Lepkosprężyste smarowanie poprzecznych łożysk ślizgowych w nieustalonym ruchu, Tribologia, 5 (197), 2004, pp. 181–194.
34. Dudziewicz, J.: Podstawy elektromagnetyzmu, WNT, Warszawa 1972.
35. Walicka, A.: Rheology of fluids in mechanical engineering, Oficyna Wydawnicza Uniwersytetu Zielonogórskiego, Zielona Góra 2017.
36. Wierzcholski, K.: Teoria niekonwencjonalnego smarowania łożysk ślizgowych, Prace Naukowe Politechniki Szczecińskiej, Vol. 527, Szczecin 1995.
37. Wierzcholski, K.: Mathematical Theory of Lubrication, TU Press, Stettin 1992.
38. Wierzcholski, K.: Mathematical Methods of Hydrodynamic Theory of Lubrication, Politechnika Szczecińska, Monografia, nr 511, Szczecin 1993.
39. Frycz, M., Miszczak, A.: Longitudinal magnetic field in the journal sliding bearing gap, Tribologia. Teoria i Praktyka, Nr 6 (240), 2011, pp. 77–85.
40. Frycz, A., Horak, W.: Effect of the Magnetic Particles Concentration on the Ferro-Oil's Dynamic Viscosity in Presence of an External Magnetic Field in the Aspect of Shear Rate's Variations, Journal of Kones Powertrain and Transport, Journal of KONES. Powertrain and Transport, Vol. 20, No. 3, 2013, pp. 139–144.
41. Frycz, M., Horak W.: Effect of the concentration of the magnetic particles in the ferrooil on its dynamic's viscosity changes in an external magnetic field, Solid State Phenomena, Vol. 220–221, 2015, pp. 324–327.
42. Frycz, M.: Effect of Temperature and Deformation Rate on the Dynamic Viscosity of Ferrofluid, Solid State Phenomena, Vol. 199, 2013, pp. 137–142.
43. Czaban, A., Frycz M.: Influence of Pressure on Dynamic Viscosity of Oil, Journal of KONES. Powertrain and Transport, Vol. 20, No. 2, 2013, pp. 49–54.
44. Czaban, A., Frycz M., Horak, W.: Effect of the Magnetic Particles Concentration on the Ferro-Oil's Dynamic Viscosity in Presence of an External Magnetic Field in the Aspect of Temperature Changes, Journal of KONES. Powertrain and Transport, Vol. 20, No. 2, 2013, pp. 55–60.
45. Frycz, M., Anioł, P.: Impact of magnetic particles concentration in ferro-oil on its magnetic susceptibility coefficient χ, Journal of KONES. Powertrain and Transport, Vol. 21, No. 4, 2014, pp. 111–117.
46. Frycz, M.: Impact of temperature on magnetic susceptibility coefficient χ of ferro-oils with different concentration of magnetic particles, Tribologia. Teoria i Praktyka, Vol. 5, 2016, pp. 21–30.
47. Miszczak A., Determination of variable pseudo-viscosity coefficients for oils with Rivlin-Ericksen properties, Journal of KONES. Powertrain and Transport, Vol. 20, No.1/2013, pp. 201–208.
48. Frycz, M., Czaban, A.: Models of viscosity characteristics $\eta = \eta(B)$ of ferro-oil with different concentration of magnetic particles in the presence of external magnetic field, Journal of KONES. Powertrain and Transport, Vol. 21, No. 4, 2014, pp. 119–126.
49. Frycz, M.: The ferro-oils viscosity dependent simultaneously on the temperature and magnetic oil particles concentration $\eta = \eta(T,\varphi)$ – part I, Journal of KONES. Powertrain and Transport, Vol. 23, 2016, pp. 113–120.
50. Frycz M.: The ferro-oils viscosity dependent simultaneously on the temperature and magnetic oil particles concentration $\eta = \eta(T,\varphi)$ – part II, Journal of KONES. Powertrain and Transport, Vol. 23, 2016, pp. 127–134.
51. Frycz, M.: The research and modeling of dynamic viscosity of the ferro-oils with the different concentrations of magnetic particles in the aspect of pressure changes, Tribologia. Teoria i Praktyka, Vol. 2, 2017, pp. 39–48.
52. Frycz, M.: Wpływ stężenia cząstek magnetycznych w ferrooleju na parametry przepływowe i eksploatacyjne poprzecznych łożysk ślizgowych, rozprawa doktorska, Wydział Mechaniczny UMG, 2018.

53. Andrzejewski, B.: Metoda różnic skończonych jako narzędzie badawcze, Zeszyty Naukowe Mechanika politechnika Opolska, Vol. 78, 2004, pp. 305–308.
54. Kulas, Ł., Sypek, P., Mrozowski, M.: Makromodele w metodzie różnic skończonych, Zeszyty Naukowe EPT Politechniki Gdańskiej, Vol. 8, 2005, pp. 753–760.
55. Pokorska, I.: Algorytm rozwiązania sprzężonych równań przepływu ciepła i wilgoci w materiałach porowatych metodą różnic skończonych, Zeszyty naukowe Politechniki Częstochowskiej, Vol. 160, 2005, pp. 127–131.
56. Kalinin, A.I., Lavrinovich L.I.: Application of the small parameter method to the singularly perturbed linear-quadratic optimal control problem, Automation and Remote Control, Vol. 77, 2016, pp. 751–763.
57. Pawlak, W.: Practical methods of dynamic objects identification Part I. Small perturbations method, Prace Instytutu lotnictwa, Vol. 170–171, 1998, pp. 57–64.
58. Wnu, M.: O pewnej transformacji warunku brzegowego w zagadnieniu rozwiązywanym metodą małego parametru, Applicationes Mathematicae, Vol. 7, 1963, pp. 323–334.