

Determination of the Surface and Volume Porosity, on the Basis of the Main Fraction of the Polifractional Matrix of Moulding and Core Sands

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Received 12.04.2016; accepted in revised form 03.06.2016

Abstract

The aim of the hereby paper is to present the developed model of determining the volume and surface porosity based on the main fraction of polifractional materials, its experimental verification and utilisation for the interpretation of effects accompanying the formation of a moulding sand apparent density, porosity and permeability in the blowing processes of the core and moulds technology.

Keywords: Sand grains, Porosity, Moulding sand

1. Introduction

The porosity is a characteristic feature of products made of granular materials, since in their structure occur - either natural or artificially produced - discontinuities of the solid phase of the determined spatial characteristics, usually fulfilling certain roles in the realised physico-chemical, technological or functional processes. The porosity of materials and products is considered in the two-dimensional system (surface porosity) and three-dimensional system (volume porosity). Knowledge of this feature of materials is taken into account in describing processes generally related to physico-chemical effects determining mass and heat transfers that occur in several technological processes. As examples can be used: capillary transportation of water from intergranular spaces during sands cooling, effects occurring in fluidising devices, pneumatic transport and moulding sands compaction in the moulds production, setting of moulding sands

with resins by means of blowing them through by active gases, forming of technological properties of sand moulds and cores (permeability, strength, thermophysical properties) [1-9].

2. Basic notions

The necessity of the development of the theoretical model for determining the volume and surface porosity based on the main fraction of polifractional materials, occurred during the theoretical calculations of moulding sands tensile strength. It concerned, first of all, taking into account the real granulometric composition of polifractional quartz sands represented by fractions of grains of individual sizes in the main fraction as well as the packing structure of grains of various diameters. In the own model, the hypothetic moulding sand matrix is represented by grains of three different diameters, corresponding to sieve mesh clearances

within the main fraction of the given moulding sand, determined during the standard sieve analysis, according to PN-83/H-11077. A numerical participation of grains of the given diameter within the fraction, results from their mass participation in the moulding sand matrix. As a justification of such approach, it is worth to quote the fact that the total mass of matrix grains gathered on three successive sieves, called the main fraction, constitutes usually - in the applied moulding sands - more than 85% of the mass of all grains.

3. Determination of the packing density of grains, surface and volume porosities for the system of ideal matrix

It can be assumed in case of the hypothetical intergrain bonding, that within the given fraction there are grains of identical diameters, and their compaction degree can be expressed by means of its relation to the corresponding intergranular porosity, expressed as flat - two-dimensional (2D), as well as spatial - three-dimensional (3D). Assuming, that the determined porosities are related to idealised matrix models represented by smooth globular grains of the same diameters contacting with each other without a bonding material, and the surface ε_p and volume ε_v porosity values are related to the surface $A_c=1 \text{ cm}^2$ and volume $V_c=1 \text{ cm}^3$, respectively, the analysed relations can be interpreted as follows:

- Relative surface density of grains packing - 2D system

The relative surface density of grains packing δ_p , can be expressed by means of the decimal fraction of the surface taken by the grains projection on the cross-section plane A_c , (considered in the elementary reference surface [1,5]):

$$\delta_p = \frac{i \cdot F_z}{A_c} = i \cdot F_z \quad \text{cm}^2 / \text{cm}^2 \quad (1)$$

Expression in denominator represents the elementary reference surface: $i \cdot F_z + \varepsilon_p = 1 \text{ cm}^2$, and thus it can be written that the relative surface porosity (2D system) is expressed by equation:

$$\varepsilon_p = 1 - \delta_p = 1 - i \cdot F_z \quad (2)$$

where:

i - number of individual grains falling to the assumed cross-section surface unit A_c ;

F_z - surface of individual grain projected on the cross-section plane; cm^2 ,

- Relative volume density of grains packing - 3D system

The relative volume density of grains packing (3D system) can be expressed by means of the decimal fraction of the volume taken by grains in the elementary reference volume [1,2,8]:

$$\delta_v = \frac{i \cdot \sqrt{i} \cdot V_z}{V_c} = i \cdot \sqrt{i} \cdot V_z \quad \text{cm}^3 / \text{cm}^3 \quad (3)$$

The expression in the denominator represents the elementary reference volume, of the value:

$$i \cdot \sqrt{i} \cdot V_z + \varepsilon_v = V_c = 1 \text{ cm}^3$$

Thus, the relative volume density is given by equation:

$$\varepsilon_v = 1 - \delta_v = 1 - i \cdot \sqrt{i} \cdot V_z \quad (4)$$

where:

i - number of individual grains falling to the assumed volume V_c ; F_z - volume of individual grain; cm^2 ,

The dependence joining both values can be determined by rearranging equation (2) in order to describe „ i ”, followed by its substitution into equation (4). After rearrangements it can be written:

$$i = \frac{1 - \varepsilon_p}{F_z} \quad 1 / \text{cm}^2 \quad \varepsilon_v = 1 - \sqrt{\left(\frac{1 - \varepsilon_p}{F_z}\right)^3} \cdot V_z \quad (5)$$

Taking into account, in equation (5), that

$$\frac{V_z}{F_z} = \frac{2}{3} \cdot d_z \quad \left(\frac{V_z}{F_z}\right)^2 \cdot \frac{1}{F_z} = \frac{16}{9 \cdot \pi}$$

after rearrangements, the new form of this equation is obtained:

$$\varepsilon_v = 1 - \frac{4}{3} \sqrt{\frac{(1 - \varepsilon_p)^3}{\pi}} \cong 1 - 0,7522 \sqrt{(1 - \varepsilon_p)^3} \quad (6)$$

The dependence of the surface porosity ε_p on the known volume porosity ε_v , after taking into account substitutions analogous as in equation (5), can be written:

$$\varepsilon_p = 1 - \sqrt[3]{\frac{9\pi \cdot (1 - \varepsilon_v)^2}{16}} = 1 - 1,209 \cdot \sqrt[3]{(1 - \varepsilon_v)^2} \quad (7)$$

The value of the dependence given by equation (7) can be calculated directly on the bases of the direct measurement of the relative density of compacted moulding sand ρ_m and the known specific density of sand grains matrix ρ_f .

$$\varepsilon_p = 1 - \sqrt[3]{\frac{9\pi}{16} \left(\frac{\rho_m}{\rho_f}\right)^2} = 1 - 1,209 \cdot \sqrt[3]{\left(\frac{\rho_m}{\rho_f}\right)^2} \quad (8)$$

4. Porosity calculations for the system of grains of various sizes

It was shown in the study [hab.], that on the bases of existing models - taking into account idealised matrix grains - it was possible to calculate ε_p and ε_v values for characteristic ways of packing, while for polydisperse materials of various grain sizes such possibilities were not formally presented.

When 3 grains represented by radiuses: R_1, R_2, R_3 (Fig. 1), meeting the condition $R_1 \neq R_2 \neq R_3$, are in contact, their surface porosity is given by equation:

$$\varepsilon_p = 1 - \left(\frac{\pi}{180 \cdot (R_1 + R_2) \cdot (R_1 + R_3) \cdot \sin \alpha} \cdot \left[\begin{aligned} & \left(R_1^2 \cdot \arccos \frac{(R_1 + R_2)^2 + (R_1 + R_3)^2 - (R_2 + R_3)^2}{2 \cdot (R_1 + R_2) \cdot (R_1 + R_3)} \right) + \\ & \left(R_2^2 \cdot \arccos \frac{(R_2 + R_3)^2 + (R_1 + R_2)^2 - (R_1 + R_3)^2}{2 \cdot (R_1 + R_2) \cdot (R_1 + R_3)} \right) + \\ & \left(R_3^2 \cdot \arccos \frac{(R_1 + R_3)^2 + (R_2 + R_3)^2 - (R_1 + R_2)^2}{2 \cdot (R_1 + R_3) \cdot (R_2 + R_3)} \right) \end{aligned} \right] \right) \quad (9)$$

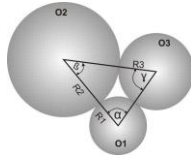


Fig. 1. Scheme for calculations the surface porosity of the system of 3 model grains with different diameters

In the situation when the fraction of grains of the given radius represents their relative amount in the main matrix fraction, it should be assumed that the total number of matrix grains within the main fraction consists of the quantitative participations of matrix grains meeting the condition: $n + k + l = 1$, where n, k, l denote the quantitative participations of grains (expressed in the unit decimal fraction) of radiuses: R_1, R_2, R_3 - respectively and are determined on the bases of their mass participation in the main fraction of the matrix.

$$L_n = \frac{3 \cdot u_{(R1)}}{4 \cdot \rho_f \cdot \pi \cdot R_1^3}; L_k = \frac{3 \cdot u_{(R2)}}{4 \cdot \rho_f \cdot \pi \cdot R_2^3}; L_l = \frac{3 \cdot u_{(R3)}}{4 \cdot \rho_f \cdot \pi \cdot R_3^3} \quad (10)$$

where:

L_n – number of grains of R_1 radius, of the quantitative participation n , L_k – number of grains of R_2 radius, of the quantitative participation k , L_l – number of grains of R_3 radius, of the quantitative participation l , $u_{(R1)}$ – mass participation of grains of R_1 radius; kg, $u_{(R2)}$ – mass participation of grains of R_2 radius; kg, $u_{(R3)}$ – mass participation of grains of R_3 radius; kg,

Total number of all grains (ΣL_c) equals:

$$\Sigma L_c = L_n + L_k + L_l = \frac{3 \cdot u_{(R1)}}{4 \cdot \rho_f \cdot \pi \cdot R_1^3} + \frac{3 \cdot u_{(R2)}}{4 \cdot \rho_f \cdot \pi \cdot R_2^3} + \frac{3 \cdot u_{(R3)}}{4 \cdot \rho_f \cdot \pi \cdot R_3^3} \quad (11)$$

therefore:

$$n = \frac{3 \cdot u_{(R1)}}{4 \cdot \rho_f \cdot \pi \cdot R_1^3 \cdot \Sigma L_c}; k = \frac{3 \cdot u_{(R2)}}{4 \cdot \rho_f \cdot \pi \cdot R_2^3 \cdot \Sigma L_c}; l = \frac{3 \cdot u_{(R3)}}{4 \cdot \rho_f \cdot \pi \cdot R_3^3 \cdot \Sigma L_c} \quad (12)$$

Knowing quantitative participations of grains of R_1, R_2, R_3 radiuses, it is possible to determine the expected value of the surface porosity, based on the calculus of probability. The probable occurrence of the determined combination of quantitative participations of grains of R_1, R_2 and R_3 radiuses in bonds O1, O2, O3 at O1 grain successively taking on radiuses: R_1, R_2 and R_3 can be found.

The number of possible variations Ω of radiuses of spheres calculated from the calculus of probability dependence (variation with repetitions for three elements) is expressed by equation:

$$\Omega = W_{(3)}^3 = 27 \quad (13)$$

The probability (P) of the occurrence of the determined combination of quantitative participation of grains of radiuses: R_1, R_2 and R_3 , for the assumed bonding systems, are illustrated in Figure 2 and in Table 1. In the individual bonding, grain O1 is the constant element, and its diameter can assume successively values: R_1 (Table 1), R_2 or R_3 , while diameters of grains O2 and O3 are subjected to variations with repetitions.

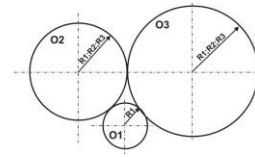


Fig. 2. Hypothetical bonding system of grains O1, O2, O3 for the calculation of the combination of the quantitative participation of grains of radiuses: R_1, R_2 and R_3 at the torn off grain O1 of radius R_1

Analogous bonding systems and tables of the probability of the occurrence of the determined combination of the quantitative participation (n, k, l) of grains with R_1, R_2 and R_3 radiuses, should be analysed for grains O1 - R_2 and O1 = R_3 .

On the bases of knowing the probability of the occurrence of every grain bonding combination (with different radiuses), the expected value (EV) can be determined. This value corresponds to the surface porosity ε_p of the grain set:

$$EV(\varepsilon_p) = \Sigma \varepsilon_p \cdot P_{I-XXVII} = \varepsilon_p(I) \cdot P(I) + \dots + \varepsilon_p(XXVII) \cdot P(XXVII) \quad (14)$$

Calculation of the volume porosity ε_v of the grain set linked, to their compaction degree, allowed to state that the most often occurring, in reality, compaction degree of grains - in moulding and core sands - corresponds to the model compaction matching the coordination number $LK=8$.

At calculations of the expected volume porosity values in the system corresponding to $LK = 8$, it is also possible to apply dependencies resulting from the probability theory, in a similar fashion as at calculations of the expected surface porosity values. However due to calculating reasons, performing the transformation of equation (6), in which in place of ε_p should be introduced $EV(\varepsilon_p)$ (from equation (14)), is more reasonable.

Table 1.

Probability (P) of the occurrence of the determined combination of the quantitative participation of grains (n, k, l) O1, O2, O3 of radiuses: R_1, R_2 and R_3 , at the torn off grain O1= R_1

Grain O1= R_1	Grain O2 of dimensions			
	R_1	R_2	R_3	
Grain O3 of dimm.	R_1	$P(I) = n^3$	$P(II) = n^2 \cdot k$	$P(III) = n^2 \cdot l$
	R_2	$P(IV) = n^2 \cdot k$	$P(V) = n \cdot k^2$	$P(VI) = n \cdot k \cdot l$
	R_3	$P(VII) = n^2 \cdot l$	$P(VIII) = n \cdot k \cdot l$	$P(IX) = n \cdot l^2$

The general equation determining the number of grains falling to the unit of the surface area (in case of grains arrangement as in Figure 3) equals:

$$i(LK = 8) = \left[\frac{1}{(R_1 + R_2) \cdot (R_1 + R_3) \cdot \sin \alpha} \right]; 1/m^2 \quad (15)$$

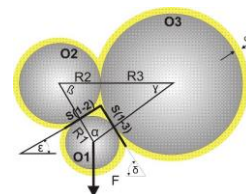


Fig. 3. Model system for calculating the surface area of bonding grains of various sizes in the system corresponding to the coordination number, $LK=8$

Taking into account participations of grains of individual sizes in the grain set, it is possible to determine the expected number of grains falling to the surface unit:

$$EV(i) = \sum i \cdot P_{r-xxvii} = i(I) \cdot P(I) + \dots + i(XXVII) \cdot P(XXVII) \quad (16)$$

5. Examples of calculations of the expected number of grains as well as the expected surface and volume porosity values

Calculations were performed for the selected mixtures of grains of grain fractions represented by radiuses $R_1, R_2,$ and $R_3,$ at their various quantitative participations in the mixture composition expressed in decimal fraction of the unit, in accordance with equation (12) related to values marked as n, k and $l.$ Two levels of $n, k,$ and l values, which are marked by the successive index numbers, were assumed for calculations. The first level: $n_1=33.33\%; k_1=33.33\%; l_1=33.33\%;$ the second level: $n_2=60\%; k_2=20\%; l_2=20\%;$

The calculation results of the expected values of the matrix surface and volume porosity $EV(\epsilon_p)$ and $EV(\epsilon_v),$ at various quantitative participations of these grains and taking into account the number of grains $EV(i)$ falling on this surface for R_1, R_2 and R_3 radiuses of hypothetical grains, are presented in Table 2.

There is the constant radius value of grain O1 ($R_1=0.065$ mm) and the same two remaining grains O2 and O3 of radiuses $R_2 = R_3,$ being equal: 0.600; 0.358; 0.258; 0.180; 0.130; 0.090 and 0.065 mm, respectively.

Table 2.

Results of calculations of the expected values (EV) of the number of grains and the matrix surface $EV(\epsilon_p)$ and volume $EV(\epsilon_v)$ porosity of the mixture of grains O1, O2 and O3 with radius values: R_1, R_2 and $R_3,$ at different quantitative participations of grains (n, k, l)

R_1	R_2	R_3	$n = 33,3(3)\%;$ $k = 33,3(3)\%;$ $l = 33,3(3)\%;$			$n = 60\%;$ $k = 20\%;$ $l = 20\%;$		
			EV (i)	EV (ϵ_p)	EV (ϵ_v)	EV (i)	EV (ϵ_p)	EV (ϵ_v)
mm	mm	mm	%			%		
	0,600	0,600	1,6	4,94	35,39	3,7	5,71	36,94
	0,358	0,358	4,3	5,77	37,06	8,7	6,72	38,95
	0,258	0,258	7,7	6,54	38,59	14,3	7,42	40,31
0,065	0,180	0,180	14,4	7,75	40,95	23,5	8,17	41,75
	0,130	0,130	24,6	8,53	42,43	34,8	8,75	42,85
	0,090	0,090	43,2	9,13	43,56	51,2	9,18	43,65
	0,065	0,065	68,5	9,31	43,89	68,5	9,31	43,89

6. Verification of the model for calculating porosities of the system of grains of various sizes

The results of the theoretical calculations of porosities for the system of grains of different diameters and quantitative participations were subjected to the experimental verification.

One of the examples of comparing the experimental results with values calculated on the basis of the theoretical model is presented in the diagram in Figure 4, in which the average absolute deviation of the measurement result is also given.

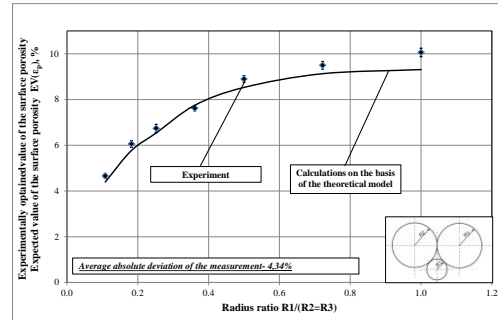


Fig. 4. Comparison of the expected value of the surface porosity $EV(\epsilon_p)$ and its value obtained experimentally ($\epsilon_{p(\text{exp})}$). Grains system: $R_1=0.065$ mm; $R_2=R_3=$ var; quantitative participation of components: $n = k = l = 33.33\%.$

7. Research conclusions

The presented own model of determining the surface and volume porosity, on the basis of the main fraction of polydisperse matrix and the analysis of the apparent density of moulding and core sands, satisfies to a higher degree - than up to the present - the demand of obtaining the calculation results being near the results obtainable under real conditions. In real conditions the sand matrix is composed of grains of various sizes, and simultaneously the cardinality of each of them in the set of grains and their known mass participation in the main fraction, is taken into account.

The experimental verification of the presented theoretical model for calculating the expected values of the surface porosity of the grains mixture O1, O2, O3 of R_1, R_2 and R_3 radiuses allows to notice, that it satisfactorily represents the real results of experiments, which usually are of higher values than the theoretically calculated.

Acknowledgements

This work was supported by Polish NCN project UMO-2014/15/B/ST8/00206.

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