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ACTIVE REDUCTION OF VIBRATION OF MECHATRONIC SYSTEMS

AKTYWNA REDUKCJA DRGAŃ UKŁADÓW MECHATRONICZNYCH*

In the work presented methods of reduction of vibration of mechanical systems using active elements, as well as examples of the implementation of the active reduction of vibration. Also presents a structural-parametric synthesis, which is defined as the design of active mechanical systems with specific requirements. These requirements apply to the value of the frequency of vibration of these systems. Presented at work considerations relate to illustrate the possible implementation of the physical elements of active using electrical components. In the active subsystems can also be used elements in other environments. To examine their effectiveness should be obtained analysis and check what are the interactions subsystems on the primary system.

Keywords: *analysis, synthesis, reduction of vibration.*

W pracy zaprezentowano metody redukcji drgań układów mechanicznych przy użyciu elementów aktywnych, jak również przykłady realizacji aktywnej redukcji drgań. Przedstawiono również syntezę strukturalno-parametryczna, która rozumiana jest jako projektowanie aktywnych układów mechanicznych o żądanych wymaganiach. Wymagania te dotyczą wartości częstości drgań tych układów. Przedstawione w pracy rozważania dotyczą zilustrowania możliwych realizacji fizycznych elementów aktywnych przy użyciu elementów elektrycznych. W podukładach aktywnych można stosować również elementy z innych środowisk. Aby zbadać ich skuteczność należy dokonać analizy otrzymanych układów oraz sprawdzić jakie są wzajemne oddziaływania podukładów na układ podstawowy.

Słowa kluczowe: *analiza, synteza, redukcja drgań.*

1. Introduction

Vibration belongs to one of the most common phenomena occurring in everyday life. It is defined as a periodical movement of a particle or a system. Such a movement is induced by external factors. Vibration occurs when a system or its part is relocated from a position of balance. The system put out of balance tends to return to its original state. One of the divisions of vibration is done according to the ways of its creation (free, forced and self-excited vibrations).

Most vibrations occurring in machines and devices are harmful and have a negative impact on their technical condition. The harmful effect of vibration is caused by occurrence of the increased stress and the loss of energy, which results in faster wear of the machines. Vibration also has a negative influence on human organism, particularly in the case of low-frequency vibration. That is the reason why many scientists in various research centres carry out investigations relating to the reduction or complete elimination of vibration [5,8,10,14,15,18].

This paper aims to develop a method of searching for structure and parameters, i.e. the structural and parametric synthesis of a mechanical system model with an active reduction of vibration. The goal of such a task is to perform a synthesis understood as a modification – already in the designing phase – of machines' subsystems with reference to the desirable frequency spectrum of the system vibration. This paper applies a non-classical method, i.e. polar graphs and structural numbers. Application of this method makes it possible to perform an analysis without any limitations caused by the type and number of elements of the mechanical system.

2. Methods of reduction of mechanical systems vibration

There are many well-known methods of vibration reduction which can be divided as follows: passive methods, semi-active and active methods.

Passive reduction of vibration consists in the introduction of additional elements such as vibration dampers. The vibration dampers dissipate or store energy. The parameters of passive dampers are subject to no variation in time. In the passive reduction of vibration there is a strong connection between efficiency and vibration frequency as well as there is sensitivity to the changes of parameters.

Semi-active methods consist in the application of semi-active eliminators of vibration. They combine some features occurring in the passive and active elements. The structure of a semi-active subsystem is similar to an active subsystem. The difference between active and semi-active subsystems, however, lies in the fact that the semi-active subsystems demand very little energy. On the other hand, they differ from the passive subsystems in this respect that their parameters may be subject to variation in time. Such changes depend on the current condition of the primary system.

Active reduction of vibration is characterized by the necessity of existence of additional external sources of energy. The energy supplied from the outside counteracts the undesirable vibration. Active subsystems may reduce vibration of the selected parts of machines or devices. The value of their parameters varies in time and depends on the current state of the system. Active subsystems may be constructed from different types of elements: mechanical, electric, pneumatic and hydraulic. The application of active and semi-active methods enables the elimination of limitations occurring in passive methods [1-5, 11, 14, 15, 18].

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

3. Synthesis of active mechanical systems

The synthesis presented in this paper is a non-classical method of designing discrete vibrating mechanical systems. As a result of the synthesis the structure and parameters of a system of required properties may be obtained [5,6,8]. The synthesis may also be used for the modification of the existing systems in order to achieve an intended goal. The synthesis consists of two basic stages. In the first stage, which is a designing phase of a new system, the researchers determine requirements concerning the free vibration frequency of the system as well as obtain the structure and parameters of the system constructed only from passive elements (inertial and elastic elements). In the second stage, the reduction of vibration is selected, either passive or active, to fit the existing or obtained system (Fig. 1). The presented method of designing active mechanical systems with vibration reduction by means of polar graphs and structural numbers enables full automation and algorithmisation of calculations during the determination process of dynamic characteristics of the system as well as enables direct tracking of the introduced structural changes irrespective of the complexity of a given system.

In order to obtain the structure and parameters of inertial and elastic elements of a dynamic system two basic methods are applied [8]:

– distribution of characteristic function into continued fraction (3),
– distribution of characteristic function into vulgar fractions (4).

Characteristic functions may include functions in the form of mobility (1) or slowness (2):

$$V(s) = H\frac{c_k s^k + c_{k-1}s^{k-2} + \ldots + c_1 s}{d_l s^l + d_{l-1}s^{l-2} + \ldots + d_0} \qquad (1)$$

$$U(s) = H\frac{d_l s^l + d_{l-1}s^{l-2} + \ldots + d_0}{c_k s^k + c_{k-1}s^{k-2} + \ldots + c_1 s} \qquad (2)$$

where:

k, l – natural numbers,
c, d – real numbers,
H – any positive real number.

$$V(s) = \frac{c_1}{s} + m_1 s + \cfrac{1}{\cfrac{s}{c_2} + \cfrac{1}{m_2 s + \ldots + \cfrac{1}{\cfrac{s}{c_n} + \cfrac{1}{m_n s}}}} \qquad (3)$$

$$U(s) = \frac{c_1}{s} + m_1 s + \frac{1}{\frac{s}{c_2} + \frac{1}{m_2 s}} + \ldots + \frac{1}{\frac{s}{c_n} + \frac{1}{m_n s}} \qquad (4)$$

where:

c – elastic elements,
m – inert elements.

In order to design a system with a passive reduction of vibration one should follow the scheme presented in Fig 1. After performing the synthesis consisting in the distribution into continued fraction or vulgar fractions, it is necessary to define the type and value of external excitation affecting the system.

By choosing the passive reduction of vibration a designer determines if passive elements in the form of viscous dampers will be proportional to inertial elements (as illustrated in Fig. 2) or proportional

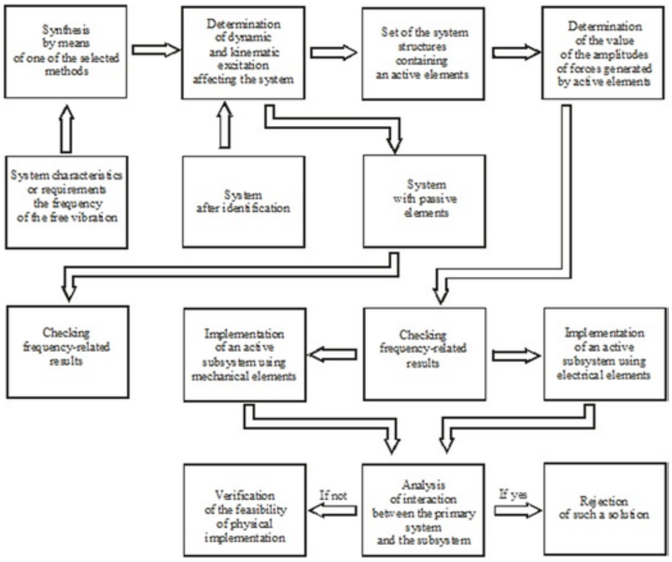

Fig. 1. Idea of synthesis of mechanical systems with reduction of vibration

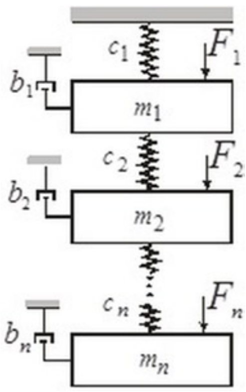

Fig. 2. Model of mechanical system with passive elements proportional to inertial elements

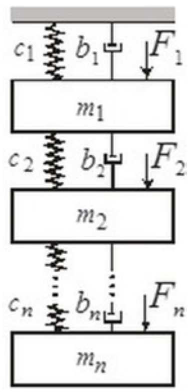

Fig. 3. Model of mechanical system with passive elements proportional to elastic elements

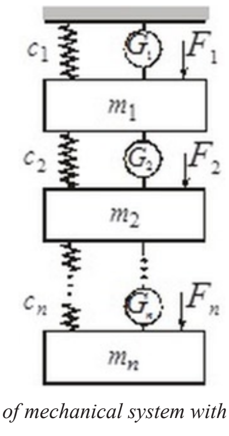

Fig. 4. Model of mechanical system with active elements

to elastic elements (Fig. 3). It is vital to check the efficiency of the application of passive elements by means of performing the analysis of the obtained system [5, 13].

Another possibility is the application of active elements in vibration reduction. The synthesis of the systems with the active reduction of vibration has been presented in Fig. 1. The first stage is analogical to the case of the application of passive elements. In the second stage, the structure with active elements and their parameters are selected.

The system with active elements is presented in Fig. 4. Active subsystems are located among inertial elements, which enables the reduction of the parts of the system pre-defined by the designer in the designing process.

4. System under investigation

The scope of this paper is limited to the description of the active reduction of vibration on the basis of a system with three degrees of freedom having a cascade structure. In order to obtain a system that would meet the requirements concerning the frequency of vibration, one should first define the values of such frequencies (5), then make a characteristic function (6) and perform its distribution into continued fraction (7):

$$\begin{cases}\omega_1 = 10\dfrac{rad}{s},\ \omega_3 = 30\dfrac{rad}{s},\ \omega_5 = 50\dfrac{rad}{s} - \text{resonant frequencies,}\\ \omega_0 = 0\dfrac{rad}{s},\ \omega_2 = 20\dfrac{rad}{s},\ \omega_4 = 40\dfrac{rad}{s} - \text{anti-resonant frequencies.}\end{cases} \quad (5)$$

$$U(s) = \frac{(s^2+\omega_1^2)(s^2+\omega_3^2)(s^2+\omega_5^2)}{s(s^2+\omega_2^2)(s^2+\omega_4^2)}. \quad (6)$$

Function distribution in the form of slowness into continued fraction results in the structure and values of the system with inertial and elastic elements:

$$U(s) = \frac{c_1}{s} + m_1 s + \cfrac{1}{\cfrac{s}{c_2} + \cfrac{1}{m_2 s + \cfrac{1}{\cfrac{s}{c_3} + \cfrac{1}{m_3 s} + \cfrac{1}{\cfrac{s}{c_4}}}}} = \frac{175}{s} + 1s + \cfrac{1}{\cfrac{s}{1325} + \cfrac{1}{1{,}7s + \cfrac{1}{\cfrac{s}{848{,}6} + \cfrac{1}{1{,}6s + \cfrac{s}{268{,}05}}}}}$$

(7)

On the grounds of the function distribution it is possible to obtain a polar graph and a mechanical system (Figs. 5, 6).

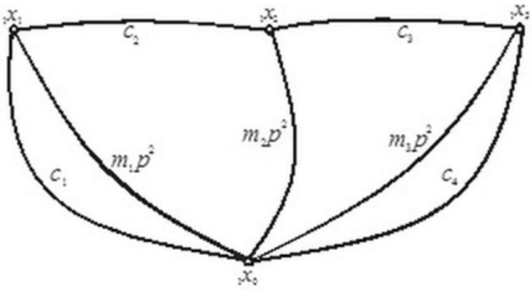

Fig. 5. Polar graph

Symbols in the Fig. 5 represent:

- inert elements

$m_1p^2 \to m_1 = 1\ [kg],\ m_2p^2 \to m_2 = 1{,}67\ [kg],\ m_3p^2 \to m_3 = 1{,}59\ [kg],$

- elastic elements

$c_1 \to c_1 = 175\left[\frac{N}{m}\right],\ c_2 \to c_2 = 1325\left[\frac{N}{m}\right],\ c_3 \to c_3 = 848{,}63\left[\frac{N}{m}\right],\ c_4 \to c_4 = 268{,}05\left[\frac{N}{m}\right].$

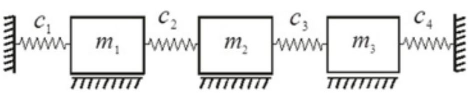

Fig. 6. Model of system obtained by synthesis

Following the diagram presented in Fig.1 it is necessary to define excitations influencing the system. In this case, force F(t) applied to inertial element 3 is exerted upon the system (Fig.7). The polar graph of the system has been presented in Fig.8.

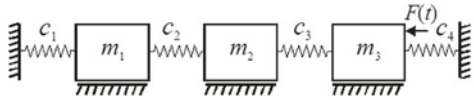

Fig. 7. Model of system (Fig. 6) with dynamic excitation

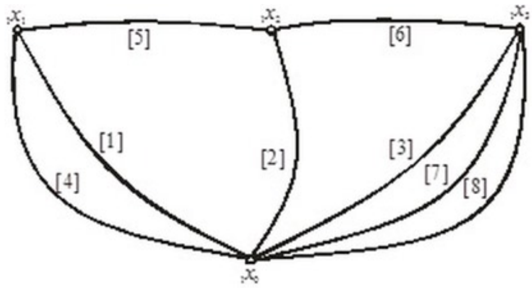

Fig. 8. Polar graph of the system from Fig. 7

The edges of the polar graph (Fig. 8) are numbered in the following way:

$$\left.\begin{matrix}[1] - m_1p^2,\\ [2] - m_2p^2,\\ [3] - m_3p^2,\end{matrix}\right\}\ \text{inert elements} \qquad \left.\begin{matrix}[4] - c_1,\\ [5] - c_2,\\ [6] - c_3,\\ [7] - c_4,\end{matrix}\right\}\ \text{elastic elements}$$

$$[8] - F(t) \to F(t) = 10\ \sin\omega t\ [N]\ \text{dynamic excitation}$$

In order to determine the amplitude of the system vibration (Fig.7), it is possible to use the algebra of structural numbers and its connection with polar graphs [2,4]. In this case the amplitudes will take the following forms (8-10):

$$A_1 = \left| \frac{\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[1]};\frac{\partial D(\omega)}{\partial[3]}\right)[8]}{D(\omega)} \right| =$$

(8)

$$= \left| \frac{c_2c_3\,F}{-m_1m_2m_3\omega^6 + \omega^4\left(m_1m_2c_3 + m_1m_2c_4 + m_1m_3c_2 + m_1m_3c_3 + m_2m_3c_1 + m_2m_3c_2\right) + -\omega^2\left(m_1c_2c_3 + m_1c_2c_4 + m_1c_3c_4 + m_2c_1c_3 + m_2c_1c_4 + m_2c_2c_3 + m_2c_2c_4 + +m_3c_1c_2 + m_3c_1c_3 + m_3c_2c_3\right) + c_1c_2c_3 + c_1c_2c_4 + c_1c_3c_4 + c_2c_3c_4} \right|$$

$$A_2 = \left| \frac{\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[2]};\frac{\partial D(\omega)}{\partial[3]}\right)[8]}{D(\omega)} \right| =$$

(9)

$$= \left| \frac{\left(-m_1c_3\omega^2 + c_1c_3 + c_2c_3\right)F}{-m_1m_2m_3\omega^6 + \omega^4\left(m_1m_2c_3 + m_1m_2c_4 + m_1m_3c_2 + m_1m_3c_3 + m_2m_3c_1 + m_2m_3c_2\right) + -\omega^2\left(m_1c_2c_3 + m_1c_2c_4 + m_1c_3c_4 + m_2c_1c_3 + m_2c_1c_4 + m_2c_2c_3 + m_2c_2c_4 + +m_3c_1c_2 + m_3c_1c_3 + m_3c_2c_3\right) + c_1c_2c_3 + c_1c_2c_4 + c_1c_3c_4 + c_2c_3c_4} \right|$$

$$A_3 = \left| \frac{\frac{\partial D(\omega)}{\partial [3]}[8]}{D(\omega)} \right| = \tag{10}$$

$$= \left| \frac{\left(m_1 m_2 \omega^4 - \omega^2 (m_1 c_2 + m_1 c_3 + m_2 c_1 + m_2 c_2) + c_1 c_2 + c_1 c_3 + c_2 c_3\right) F}{-m_1 m_2 m_3 \omega^6 + \omega^4 (m_1 m_2 c_3 + m_1 m_2 c_4 + m_1 m_3 c_2 + m_1 m_3 c_3 + m_2 m_3 c_1 + m_2 m_3 c_2) + -\omega^2 (m_1 c_2 c_3 + m_1 c_2 c_4 + m_1 c_3 c_4 + m_2 c_1 c_3 + m_2 c_1 c_4 + m_2 c_2 c_3 + m_2 c_2 c_4 + +m_3 c_1 c_2 + m_3 c_1 c_3 + m_3 c_2 c_3) + c_1 c_2 c_3 + c_1 c_2 c_4 + c_1 c_3 c_4 + c_2 c_3 c_4} \right|$$

The symbols in equations 8-10 present:
$D(\varpi)$ – characteristic equation,
$\frac{\partial D(\omega)}{\partial [a]}$ – derivative of structural number in relation to the edge a,
$\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial [a]}; \frac{\partial D(\omega)}{\partial [b]}\right)$ – function of simultaneity of structural number, the inverse image of which contains two oriented edges a and b.

A graphic representation of the amplitudes of the analysed system (Fig.7) is shown in Figs. 9-11.

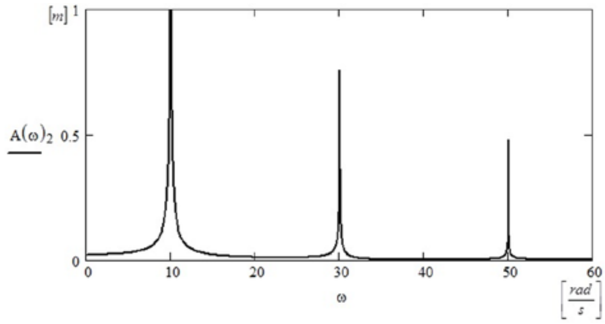

Fig. 9. Diagram of A_1 amplitude of a system with dynamic excitation (Fig. 7)

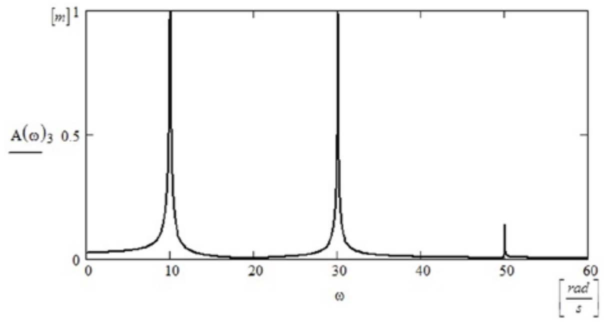

Fig. 10. Diagram of A_2 amplitude of a system with dynamic excitation (Fig. 7)

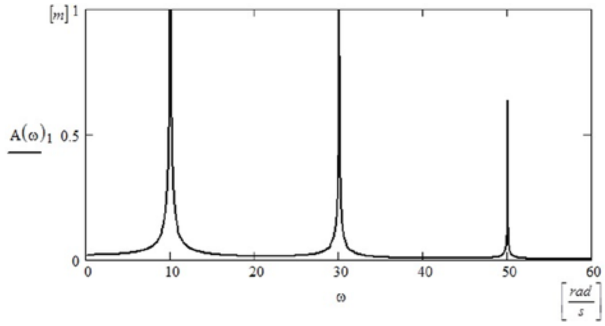

Fig. 11. Diagram of A_3 amplitude of a system with dynamic excitation (Fig. 7)

In order to reduce the system vibration active elements may be applied. The system with the active reduction of vibration has been presented in Fig. 12 and its polar graph in Fig. 13.

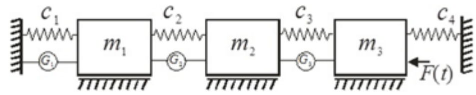

Fig. 12. System of three degrees of freedom with three active elements

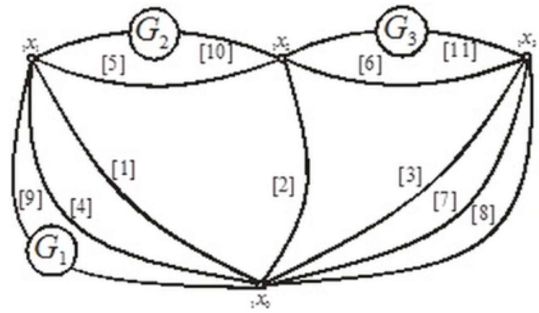

Fig. 13. Polar graph of the systems from Fig. 12

The edges of the graph in Fig. 13 are equivalent to Fig. 8 beyond edges 9-11 which mean the following:

$[9] - G_1 \Rightarrow$ active element 1,

$[10] - G_2 \Rightarrow$ active element 2,

$[11] - G_3 \Rightarrow$ active element 3.

In order to determine the values of forces generated by active elements G_1, G_2 and G_3, it is necessary to solve the following set of equations (11):

$$\begin{bmatrix} \left(\frac{\partial D(\omega)}{\partial([2][3])}\right) & -\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial([2][3])}; \frac{\partial D(\omega)}{\partial([1][3])}\right) & -\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial([2][1])}; \frac{\partial D(\omega)}{\partial([3][2])}\right) \\ -\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial([2][3])}; \frac{\partial D(\omega)}{\partial([1][3])}\right) & \left(\frac{\partial D(\omega)}{\partial([1][3])}\right) & -\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial([1][3])}; \frac{\partial D(\omega)}{\partial([1][2])}\right) \\ -\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial([2][1])}; \frac{\partial D(\omega)}{\partial([3][2])}\right) & -\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial([1][3])}; \frac{\partial D(\omega)}{\partial([1][2])}\right) & \left(\frac{\partial D(\omega)}{\partial([1][2])}\right) \end{bmatrix} \cdot \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ F \end{bmatrix} = \begin{bmatrix} -G_1 \\ -G_2 \\ -G_3 \end{bmatrix} \tag{11}$$

where:

$\frac{\partial D(\omega)}{\partial([2][3])} = -m_1\omega^2 + c_1 + c_2$, $\frac{\partial D(\omega)}{\partial([1][3])} = -m_2\omega^2 + c_2 + c_3$, $\frac{\partial D(\omega)}{\partial([1][2])} = -m_3\omega^2 + c_3 + c_4$,

$-\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial([2][3])}; \frac{\partial D(\omega)}{\partial([1][3])}\right) = -c_2$, $-\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial([1][3])}; \frac{\partial D(\omega)}{\partial([1][2])}\right)$,

$\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial([2][1])}; \frac{\partial D(\omega)}{\partial([3][2])}\right) = 0.$

After solving (11) the values G_1, G_2 and G_3 were obtained.

At $\omega = \omega_1 = 10 \left[\frac{rad}{s}\right]$, the values G_1, G_2 and G_3 are as follows:

$G_1 = 0,225 \sin\omega t \; [N]$, $G_2 = -0,501 \sin\omega t \; [N]$, $G_3 = -9,673 \sin\omega t \; [N]$.

At $\omega = \omega_3 = 30 \left[\frac{rad}{s}\right]$, the values G_1, G_2 and G_3 are as follows:

$G_1 = -2,175 \sin\omega t \; [N]$, $G_2 = -4,51 \sin\omega t \; [N]$, $G_3 = -13,489 \sin\omega t \; [N]$.

At $\omega = \omega_5 = 50 \left[\frac{rad}{s}\right]$, the values G_1, G_2 and G_3 are as follows:

$G_1 = -6,975 \sin\omega t \ [N], G_2 = -12,525 \sin\omega t \ [N], G_3 = -21,12 \sin\omega t \ [N].$

Having conducted the synthesis and defined the values of the forces generated by active subsystems, the researchers chose electric elements in the form of a coil with movable core as a physical realisation of such subsystems (Fig.14).

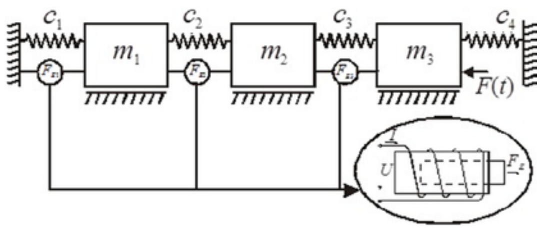

Fig. 14. A model of the system with electric elements

In order to determine the value of electrodynamic force, it is necessary to use the dependence presented below [12,16-18]:

$$F_E = BIL_C \tag{12}$$

where: F_E – electrodynamic force,
B – magnetic flux density,
I – current in the conductor,
L_C – length of the conductor.

The value of electrodynamic force is directly proportional to the intensity of the current flowing in the conductor and to the length of the conductor segment located in a given magnetic field. The values of "F_E" forces are equivalent to the previously determined values of "G" forces. In the above-presented dependence there is an element which can be altered in time, i.e. the current flowing through the conductor "I". Exemplary values of these elements are shown in Table 1.

The next step consisted in the analysis of the impact of the subsystem on the primary system. The analysis is presented in the form of diagrams comparing the amplitudes and deflections of the system without vibration reduction and the system with electric elements reducing vibration (Figs.15-23).

The conducted analysis has resulted in the diagrams showing that an active subsystem does not alter the primary system. The values of free vibration frequency of the system are subject to no alteration, therefore the initial requirements are met by the system.

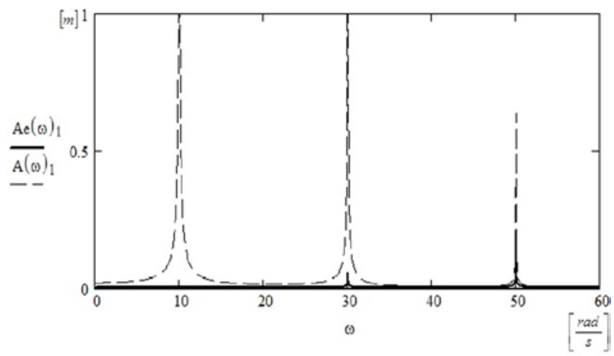

Fig. 15. Diagram of A_1 amplitude and Ae_1 displacement of system with electric elements (Fig. 14) at $\omega = \omega_1$= 10 rad/s

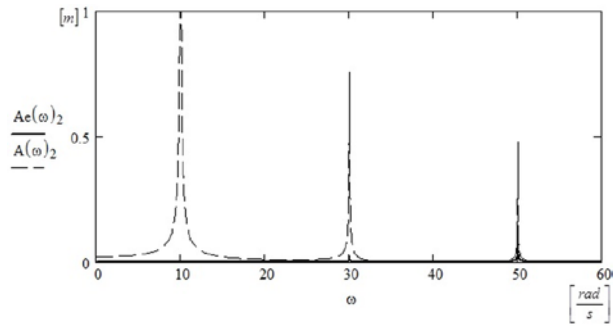

Fig. 16. Diagram of A_2 amplitude and Ae_2 displacement of system with electric elements (Fig. 14) at $\omega = \omega_1$= 10 rad/s

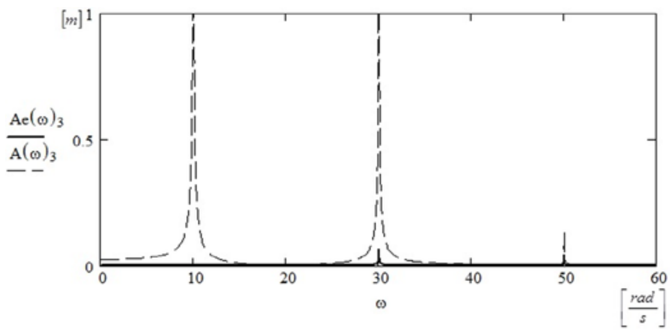

Fig. 17. Diagram of A_3 amplitude and Ae_3 displacement of system with electric elements (Fig. 14) at $\omega = \omega_1$= 10 rad/s

Table 1. The values of electric elements

Frequency	Magnetic flux density	Current in the conductor	Length of the conductor
$\omega = \omega_1 = 10 \left[\frac{rad}{s}\right]$	$B_1 = 5,825$	$I_1 = 0,386$	$L_{C1} = 0,1$
	$B_2 = 4,346$	$I_2 = 0,576$	$L_{C2} = 0,2$
	$B_3 = 12,731$	$I_3 = 2,533$	$L_{C3} = 0,3$
$\omega = \omega_3 = 30 \left[\frac{rad}{s}\right]$	$B_1 = 18,11$	$I_1 = 1,201$	$L_{C1} = 0,1$
	$B_2 = 13,04$	$I_2 = 1,729$	$L_{C2} = 0,2$
	$B_3 = 15,03$	$I_3 = 2,991$	$L_{C3} = 0,3$
$\omega = \omega_5 = 50 \left[\frac{rad}{s}\right]$	$B_1 = 32,43$	$I_1 = 2,151$	$L_{C1} = 0,1$
	$B_2 = 21,73$	$I_2 = 2,88$	$L_{C2} = 0,2$
	$B_3 = 18,81$	$I_3 = 3,742$	$L_{C3} = 0,3$

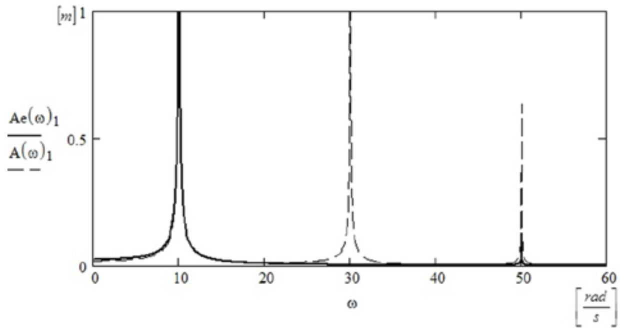

Fig. 18. Diagram of A_1 amplitude and Ae_1 displacement of system with electric elements (Fig. 14) at $\omega = \omega_3 = 30$ rad/s

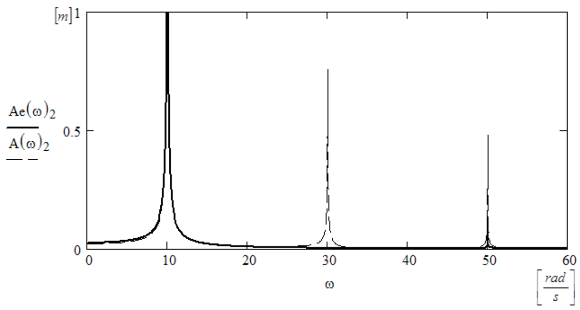

Fig. 19. Diagram of A_2 amplitude and Ae_2 displacement of system with electric elements (Fig. 14) at $\omega = \omega_3 = 30$ rad/s

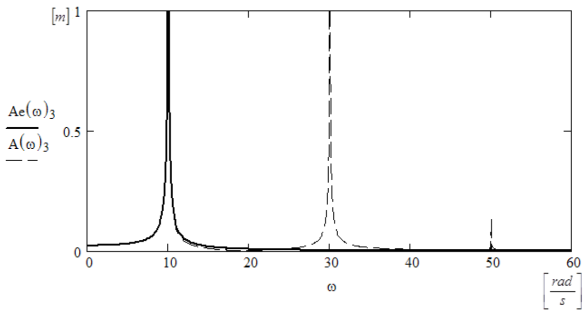

Fig. 20. Diagram of A_3 amplitude and Ae_3 displacement of system with electric elements (Fig. 14) at $\omega = \omega_3 = 30$ rad/s

Another possibility of the application of electric elements presented in the paper is the use of piezoelectric elements [7, 9, 14].

Summary

The paper presents a non-classical method of designing discrete vibrating mechanical and mechatronic systems. The designing consists in the structural and parametric synthesis. As a result of such a synthesis, one obtains a system having pre-defined required properties relating to the frequency values of the system free vibration. Such an approach makes it possible, already in the designing phase,

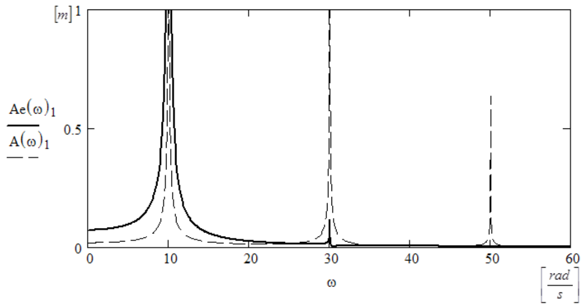

Fig. 21. Diagram of A_1 amplitude and Ae_1 displacement of system with electric elements (Fig. 14) at $\omega = \omega_5 = 50$ rad/s

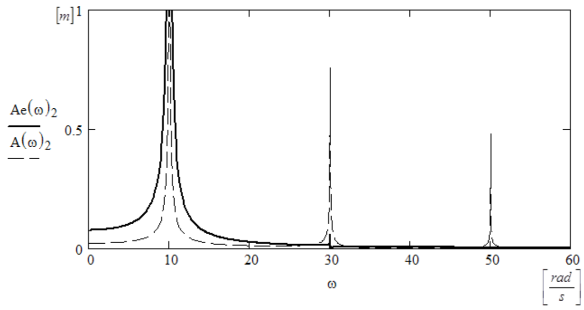

Fig. 22. Diagram of A_2 amplitude and Ae_2 displacement of system with electric elements (Fig. 14) at $\omega = \omega_5 = 50$ rad/s

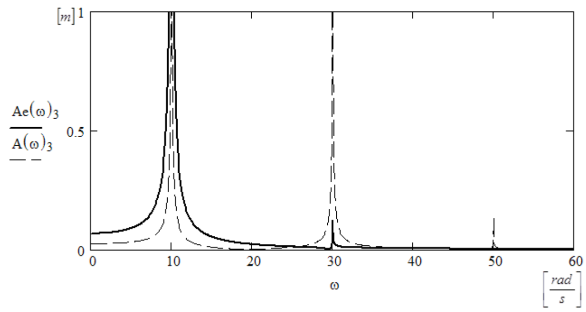

Fig. 23. Diagram of A_3 amplitude and Ae_3 displacement of system with electric elements (Fig. 14) at $\omega = \omega_5 = 50$ rad/s

to modify systems irrespective of the number of the degree of freedom possessed by the systems in question.

An important issue brought up in this paper is physical realisability of active subsystems as well as the analysis of mutual relations between the primary system and the subsystem reducing the vibration.

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