APPLICATION OF FLOW GRAPHS IN THE MICROWAVE MEASUREMENTS SYSTEMS

Przemysław Piróg

Central Military Calibration Laboratory, Warsaw

Abstract. Flow graphs are the effective tool for description of the complicated microwave measurement systems and usually are used in the process of creating measurement equation. The algorithm which can find this equation is known as the Mason's gain formula (MGF). This operation needs to perform a lot of symbolic computations so performing of this algorithm without proper software is heavy and is vulnerable to appearing some errors. This paper shows which existing scripts for MATLAB can be use to perform this task.

Keywords: flow graphs, Mason's gain formula (MGF), scattering parameters, microwave measurement

ZASTOSOWANIE GRAFÓW PRZEPŁYWU W POMIARACH MIKROFALOWYCH WIELKOŚCI ELEKTRYCZNYCH

Streszczenie. Grafy przepływu są skutecznym narzędziem umożliwiającym opis skomplikowanych układów pomiarowych w technice mikrofalowej i zwykle są stosowane w procesie tworzenia równania pomiaru. Algorytm umożliwiający znalezienie takiego równania jest znany jako reguła wzmocnienia Masona. Wymaga on znacznej liczby obliczeń symbolicznych toteż jego realizacja bez zastosowania oprogramowania jest uciążliwa oraz podatna na wystąpienie błędów podczas wykonywania przekształceń. W niniejszym artykule wskazano możliwość zastosowania istniejących już skryptów programu MATLAB do realizacji tego zadania.

Słowa kluczowe: grafy przepływu, reguła Masona, macierz rozproszenia S, pomiary mikrofalowe

Introduction

Microwave measurement is the branch of metrology in which some physical phenomena uncommon for other areas are occurred. Voltage and current values can't be used for description of the signals because high frequency signals must be analyzed as the waves.

The reason of it is the fact that in microwave measuring systems the wave is reflected many times between planes (connectors) with different characteristic impedance. Similar situation occurs in optics where rays are reflected between media with a varying index of refraction. In the microwave systems transmitted wave is attenuated and delayed many times. All these operations can change magnitude and phase of the signal. Consequence of it is the fact that measured energy of the wave for each part of measuring system depends on vector sum of all transmitted and reflected voltage waves. One of the difference between optical and microwave systems is that in optical systems usually only one wavelength parameters are analyzed but in the microwave systems parameters for all frequencies of bandwidth have to be found separately.

1. Scattering matrix

The microwave systems can be characterized as n-port network, where n is the sum of numbers of inputs and outputs [6]. For example termination loads or power sensors are characterized as one-port network, attenuator as two-port Fig. 1, splitters or dividers as three-port, directional couplers as three or four-port networks.
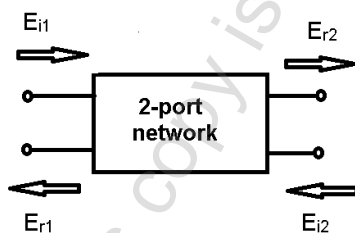

Fig. 1. Sample two-port network diagram, wherein: E_{i1} – *complex value of voltage wave incident to the port 1,* E_{r1} – *complex value of voltage wave reflected from the port 1,* E_{i2} – *complex value of voltage wave incident to the port 2,* E_{r2} – *complex value of voltage wave reflected from the port 2*

Parameters of the n-port network can be characterized by n x n size S-matrix (1). The network can be created from splitters, terminations, attenuators, power sensors and other components and can be described as one n-port network. Number of components which form the network has no influence on the size of substitute S-matrix.

Scattering matrix called S-matrix is very common form used for describing microwave networks. The S_{xy} elements reflect the physical characteristic of the whole system. These coefficients are complex numbers, magnitude represents attenuation or amplification, phase represents signal delay.

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & \cdots & S_{1n} \\ S_{21} & S_{22} & & & S_{2n} \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ S_{n1} & S_{2n} & \cdots & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_n \end{bmatrix} \tag{1}$$

wherein:

$$a_j = \frac{E_{ij}}{\sqrt{Z_o}} \tag{2}$$

$$b_j = \frac{E_{rj}}{\sqrt{Z_o}} \tag{3}$$

Z_o – characteristic impedance of system (typically 50 Ω),

$|a_j|^2 = P_{ij}$ – power incident to the port j,

$|b_j|^2 = P_{rj}$ – power reflected from the port j.

For the microwave networks S-matrix coefficients are usually different for different frequencies and need to be measured for every specified frequency because there is no global rule to describe relationship between values for separate frequencies.

All these reasons cause that calculations are complicated and it can be difficult to predict behavior of the expanded systems when all S-matrixes of components are available. The tool which can improve process of analysis is flow graph.

2. Flow graphs

Flow graph is the graphical method for presenting system of equations [3]. It is widely used in the theory of control. The transmittance in control systems describes dependencies similarly like S-matrix for microwave system, feedback has similar function to repeated reflection of voltage wave. Fig. 2 shows sample conversion from system of equation (4) to flow graph.
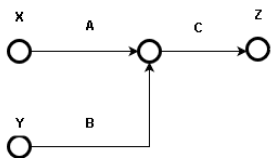
$$Z = (X \cdot A + Y \cdot B) \cdot C \tag{4}$$

Fig. 2. Conversion from equation to flow graph

Fig. 3 presents flow graph of 2-port network (green) connected to the power source (blue) and termination load (red).
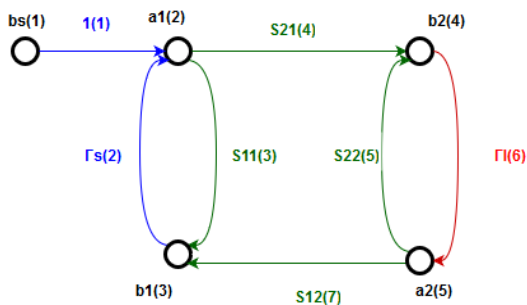

Fig. 3. Flow graph of 2–potrt network connected to power source and load, wherein: $\Gamma_s(2)$ –equivalent source match, branch 2, $\Gamma_l(6)$ –reflection coefficient of load, branch 6, $b_s(1)$ –internally generated wave in power source, node 1

Using of flow graphs brings very important advantage during process of designing microwave system. Connecting of two microwave components is equal to connecting their flow graphs, output node of the first flow graph becomes input node of the second.

The representation of microwave system as flow graph gives ability to apply algorithm which is known in control theory as the Mason's gain formula MGF [4].

3. Mason's gain formula (MGF)

MGF is the algorithm which provides the way to find the gain between any specified nodes of flow graph [1]. This gain is expressed by equations:

$$G = \frac{\sum_{k=1}^{N} G_k \cdot \Delta_k}{\Delta} \qquad (5)$$

$$\Delta = 1 - \sum L_i + \sum L_i L_j - \sum L_i L_j L_k + \cdots + (-1)^m \sum \cdots + \cdots \qquad (6)$$

wherein:

Δ – determinant of the flow graph,

G – total gain between input (IN) and output node (ON),

N – number of possible forward paths between IN and ON,

G_k – gain of k-th forward path between IN and ON,

L_i – loop gain of each closed loop in the graph,

$L_i L_j$ – product of the loop gains of any two non-touching loops,

$L_i L_j L_k$ – product of the loop gains of any three pairwise two non-touching loops,

Δ_k – the cofactor value of Δ for the k-th forward path, with the loops touching the k-th forward path removed.

Definitions:

path – a continuous set of branches traversed in the direction that they indicate;

forward path – a path from an input node to an output node in which no node is touched more than once;

loop – a path that originates and ends on the same node in which no node is touched more than once;

path gain – the product of the gains of all the branches in the path;

loop gain – the product of the gains of all the branches in the loop.

4. No-automatic realization of MGF

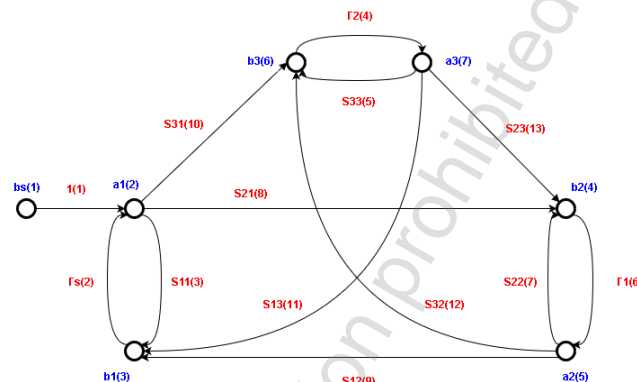Fig. 4 presents sample flow graph of power splitter connected to power source and two power sensors.

Fig. 4. Flow graph of power splitter connected to power source and two power sensors, wherein: Sxy – complex coefficients of power splitter S-matrix, $\Gamma_{1(2)}$ – complex reflection coefficient of power sensor 1(2), Red color – name and number of branch, Blue color – name and number of node

The practical calculation of the gain between input node $b_s(1)$ and output node $b_2(4)$:

Step 1 (find all the forward paths between $b_s(1)$ and $b_2(4)$)

- Path 1: $1(1)S21(8)$
- Path 2: $1(1)S31(10)\Gamma2(4)S23(13)$

Step 2 (find all closed loops)

- $\Gamma s(2)S11(3)$
- $S21(8)\Gamma1(6)S12(9)\Gamma s(2)$
- $S21(8)\Gamma1(6)S32(12)\Gamma2(4)S13(11)\Gamma s(2)$
- $S31(10)\Gamma2(4)S13(11)\Gamma s(2)$
- $S31(10)\Gamma2(4)S23(13)\Gamma1(6)S12(9)\Gamma s(2)$
- $\Gamma1(6)S22(7)$
- $\Gamma1(6)S32(12)\Gamma2(4)S23(13)$
- $\Gamma2(4)S33(5)$

Step 3 (find product of any two non-touching loops)

- $\Gamma s(2)S11(3) \cdot \Gamma1(6)S22(7)$
- $\Gamma s(2)S11(3) \cdot \Gamma1(6)S32(12)\Gamma2(4)S23(13)$
- $\Gamma s(2)S11(3) \cdot \Gamma2(4)S33(5)$
- $S21(8)\Gamma1(6)S12(9)\Gamma s(2) \cdot \Gamma2(4)S33(5)$
- $S31(10)\Gamma2(4)S13(11)\Gamma s(2) \cdot \Gamma1(6)S22(7)$
- $\Gamma1(6)S22(7) \cdot \Gamma2(4)S33(5)$

Step 4 (find product of any three non-touching loops)

- $\Gamma s(2)S11(3) \cdot \Gamma1(6)S22(7) \cdot \Gamma2(4)S33(5)$

The microwave system which was presented at Fig. 4 is relatively simple (it's typical system for calculating power sensor cal factor) however when all path gains and loop gains are substituted to equation (5) and (6) calculation becomes very complicated. This is the reason why some publications [5,6] show simplified method for creating measuring equation and uncertainty estimation. Taking all these aspects into consideration it's obvious that full analysis of microwave system in all frequency bandwidth without adequate software is unreasonable. To solve this problem is possible to use MATLAB.

5. Automatic realization of MGF

The MATLAB software offers script which provides realization the Mason's gain formula algorithm [9]. This script named *mason.m* is very easy for use. It needs only net-file with defined microwave system and m-file with information which gain should be find. If MATLAB has Symbolic Math Toolbox it's possible to simplify result and use equations for further calculation. The syntax of net-file is:

[Coefficient #][Start Node #][Stop Node #][Coefficient Name]

Every line of code contains start node, stop node and coefficient name for every branch of flow graph.

Figure 5 shows net-file which describes the microwave system presented at Fig. 4 in the form acceptable for *mason.m* script:

```
 1    1    2    1
 2    3    2    Gs
 3    2    3    S11
 4    6    7    G2
 5    7    6    S33
 6    4    5    G1
 7    5    4    S22
 8    2    4    S21
 9    5    3    S12
10    2    6    S31
11    7    3    S13
12    5    6    S32
13    7    4    S23
```

Fig. 5. MATLAB net-file for Fig. 4 flow graph

The script which calculates symbolically gain between input node $b_s(1)$ and output node $b_2(4)$ is presented at Fig. 6. The start and stop node during the gain calculation process can be changed very easily by setting integer numbers in the formula:

```
[G_Numerator,G_Denominator] = mason('splitter.net',1,4);
G_Numerator=sym(G_Numerator);
b2_Denominator=sym(G_Denominator);
G=G_Numerator/G_Denominator
```

Fig. 6. The code to calculate gain G between node no. 1 and 4

The script automatically finds all paths and both single loops and non-touching loops. The result is as below:
```
----- Paths -----
P1 : 1 8
P2 : 1 10 4 13
- Order 1 Loops -
L11 : 3 2
L12 : 8 6 9 2
L13 : 8 6 12 4 11 2
L14 : 10 4 11 2
L15 : 10 4 13 6 9 2
L16 : 6 7
L17 : 6 12 4 13
L18 : 4 5

- Order 2 Loops -
L21 : 3 2 6 7
L22 : 3 2 6 12 4 13
L23 : 3 2 4 5
L24 : 8 6 9 2 4 5
L25 : 10 4 11 2 6 7
L26 : 6 7 4 5
- Order 3 Loops -
L31 : 3 2 6 7 4 5
```

```
G = (S21*(G2*S33 - 1)-G2*S23*S31)/(G1*S22+G2*S33+
Gs*S11- G1*G2*S22*S33 + G1*G2*S23*S32 - G1*Gs*S11*S22
+G1*Gs*S12*S21-G2*Gs*S11*S33+G2*Gs*S13*S31+
G1*G2*Gs*S11*S22*S33-G1*G2*Gs*S11*S23*S32-
1*G2*Gs*S12*S21*S33+G1*G2*Gs*S12*S23*S31+G1*G2*Gs
*S13*S21*S32 - G1*G2*Gs*S13*S22*S31 - 1)
```

Fig. 7. Gain computed by the mason.m script

The gain equation (Fig. 7) allows to model the microwave system. All the coefficients are complex because of the scattering matrix properties. Fig. 7 measuring equation can be use for estimation of uncertainty by the Monte Carlo method.

It's possible to modify magnitude, phase and probability distribution of each coefficient, what can be useful for assessing how specific components have influence on expanded uncertainty of the measurement.

6. MGF for power splitter ratio measurements

Power splitter is typical microwave element used for power ratio measurements. For power splitter ratio measurements the value of the ratio R of two output voltage waves Fig. 4, $b_2(4)$ and $b_3(6)$ is key. The mason.m script can be used for this purpose.

$$R = \frac{b3(6)}{b2(4)} = \frac{bs(1) \cdot G16}{bs(1) \cdot G14} \qquad (7)$$

wherein:
$G16$ - gain between node $b_s(1)$ and $b_3(6)$
$G12$ - gain between node $b_s(1)$ and $b_2(4)$

```
[G14_Numerator,G14_Denominator] = mason('splitter.net',1,4);
G14_Numerator=sym(G14_Numerator);
G14_Denominator=sym(G14_Denominator);
b2='bs'*G14_Numerator/G14_Denominator;
[G16_Numerator,G16_Denominator] = mason('splitter.net',1,6);
G16_Numerator=sym(G16_Numerator);
G16_Denominator=sym(G16_Denominator);
b3='bs'*G16_Numerator/G16_Denominator;
R=b3/b2
```

Fig. 8. Script for calculating power ratio of power splitter

Fig. 8 shows how fast and how easy the gain between nodes and symbolic computations on partial results can be done. The result of code above is presented at Fig. 9

```
R =(S31*(G1*S22 - 1) - G1*S21*S32)/(S21*(G2*S33 - 1) - G2*S23*S31)
```

Fig. 9. Automatically calculated power ratio

7. Summary

The microwave measurement in some aspects needs specific approach in the process of creation measurement equation and uncertainty estimation [2]. Even relatively simple measuring system in many cases has complicated measuring equation.

For the people who are novice in microwave technique some clues can be useful:
- elements of scattering matrix (reflection coefficient, transmission coefficient) are complex, be sure when in equations occurs complex value or the magnitude of the value;
- the probability distribution of reflection coefficient isn't normal distribution [2];
- never average measurements in magnitude - phase form, use only real - imaginary form. The result is completely different for these two methods. The reason becomes obvious while

considering the example, why average value of 160° and -160°is 0° but it should be value 180° for this case;

- when derivatives are calculated during uncertainty estimation process in some cases the ones don't exist in specified ranges. Software can often find the result but it has no physical sense. For example, reflection coefficient $\Gamma = 0,001$, standard uncertainty $u(\Gamma) = 0,006$. For converting Γ to return loss RL and estimating standard uncertainty $u(RL)$ equations below are frequently used:

$$RL(\Gamma) = 20 \cdot \log_{10}(\Gamma) \qquad (8)$$

$$RL(\Gamma) = -20 \cdot \log_{10}(0,001) = 60[dB] \qquad (9)$$

$$\frac{\partial RL(\Gamma)}{\partial \Gamma} = \frac{-20}{\Gamma \cdot \ln(10)} \qquad (10)$$

$$u(RL) = \sqrt{\left(\frac{\partial RL(\Gamma)}{\partial \Gamma}\right)^2 \cdot (u(\Gamma))^2}\,[dB] \qquad (11)$$

$$u(RL) = \sqrt{\left(\frac{-20}{0,001 \cdot \ln(10)}\right)^2 \cdot (0,006)^2} = 52,11[dB] \qquad (12)$$

The result $RL = 60$ dB with standard uncertainty $u(RL) = 52,11$ dB is completely useless. This result is the consequence of fact that logarithmic function and its derivative don't exist for negative values $(0,001 - 0,006 < 0)$. For many other branches of metrology fact of discontinuity of the derivative of the function does not have practical importance for uncertainty estimation;

- in some cases reflection coefficient can be converted directly into its standard uncertainty according to equations [3]:

$$u(\Gamma) = \frac{|\Gamma|}{\sqrt{2}} \quad \text{(for uniform ring assumption)} \qquad (13)$$

$$u(\Gamma) = \frac{|\Gamma|}{2} \quad \text{(for uniform disk assumption)} \qquad (14)$$

The methods, software and clues indicated in this paper can improve process of measuring in daily practice and let avoid some basic errors which can be very difficult to detect. The ones can be used in high-precision calibration as well as routine low-precision calibration.

Table 1. The result of averaging in magnitude degree form and real imaginary form

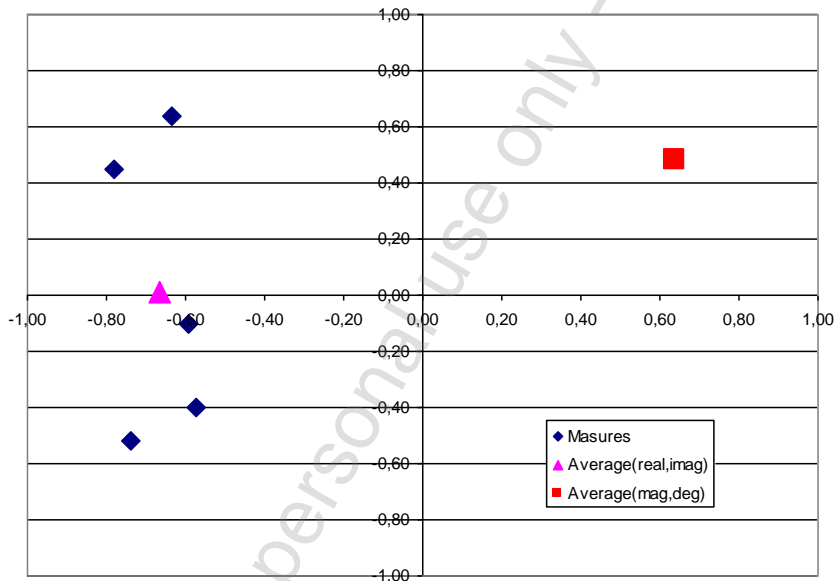| Measurement no.	r (magnitude)	fi (degrees)	X (real part)	Y (imaginary part)
1	0,9	135	-0,64	0,64
2	0,9	-145	-0,74	-0,52
3	0,6	190	-0,59	-0,10
4	0,9	150	-0,78	0,45
5	0,7	-145	-0,57	-0,40
Mean:	0,8	37	-0,66	0,01
X_converted=	**0,64**	X_direct=	**-0,66**	
Y_converted=	**0,48**	Y_direct=	**0,01**	

Fig. 10. Completely different result of two averaging methods

References

[1] Beillahi S. M., Siddique U., Tahar S.: Towards the Application of Formal Methods in Process Engineering, Department of Electrical and Computer Engineering, Concordia University, 2014.
[2] Hall B. D.: Notes on complex measurement uncertainty, Measurements Standards Laboratory of New Zealand, 2010.
[3] Lorens C. S.: Theory and application of flow graphs, Technical Report 317, MIT, 1956.
[4] Mason S. J.: Feedback theory – further properties of signal flow graphs, Technical Report 303, MIT, 1955.
[5] Fundamentals of RF and Microwave Power Measurements, Agilent, Application Note 64-1C, 2001.
[6] Guidelines on the Evaluation of Vector Network Analysers (VNA), EURAMET cg-12, 2011.
[7] S-parameter design, Agilent, Application Note AN154, 2006.
[8] https://www.draw.io/ - flow chart maker [20.04.2016]
[9] http://www.mathworks.com/matlabcentral/fileexchange/22-mason-m, Walton R. TRLabs, Calgary, Alberta, Canada, [25.03.2016]

M. Sc. Przemysław Piróg
e-mail: przemekpirog@gmail.com

The author has graduated metrology specialization in Military University of Technology. He works as senior engineer in Central Military Calibration Laboratory in Warsaw. In his work he deals with the issues in the field of microwave measurements, automatization of measurements and software engineering.
His scientific interests include modeling of microwave systems and their uncertainty budgets.