

**Brief note**

## PARAMETERS ESTIMATION FOR A PATELLOFEMORAL JOINT OF A HUMAN KNEE USING A VECTOR METHOD

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Position and displacement analysis of a spherical model of a human knee joint using the vector method was presented. Sensitivity analysis and parameter estimation were performed using the evolutionary algorithm method. Computer simulations for the mechanism with estimated parameters proved the effectiveness of the prepared software. The method itself can be useful when solving problems concerning the displacement and loads analysis in the knee joint.

**Key words:** model, knee joint, vector method, estimation, position and displacements.

### 1. Introduction

A simplified model of the human knee joint (Sancisi and Parenti-Castelli, 2010) can be sufficient to study positions of bones (the femur and the tibia) and their relative displacements. However, it can be further completed by adding the patellofemoral joint of the knee that contains: a patella, a patellar tendon and a muscle system.

The models of the patellofemoral joint with two, flexible point contact pairs have been introduced (Caruntu and Hefzy, 2004). However, a set of twelve differential equations has to be solved in order to analyze the model. A different approach for the modeling of the knee joint has been shown in Sancisi and Parenti-Castelli (2008), where the knee model is considered as a parallel platform mechanism. This method will be further explored in this paper.

In comparison with parallel platform mechanisms known from robotics, the considered model of the knee joint is characterized by two nonsymmetrical platforms (the femur and the tibia), connected by one spherical joint (S) and three legs: two passive legs S-S type (the cruciate ligaments) and one active leg S-S-R type (the patellar tendon and the patella with R – revolute joint). The mechanism operates in a limited workspace of one configuration, relatively distant from singular positions. A problem of an inverse position analysis (IPA) for the considered 1-DOF platform mechanism is formulated.

A simplified model (Sancisi and Parenti-Castelli, 2008) of the patellofemoral joint is assumed with the patella connected to the femur by a cylindrical joint (Fig.1a) where angular displacement is analyzed but the linear position is assumed to be constant. An admissible angular displacement of the patella with respect

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to the femur, corresponding to given positions and displacements of the femur with respect to the tibia, is determined by the muscle, simplified and substituted with a single linear actuator. The substitute linear actuator represents the quadriceps muscle of the knee. Therefore, only the extension of the knee can be studied. It is also assumed that the bones are rigid and the ligaments have fixed-length. The knee joint is analyzed in the normal range of flexion (deep flexion is not taken into account).

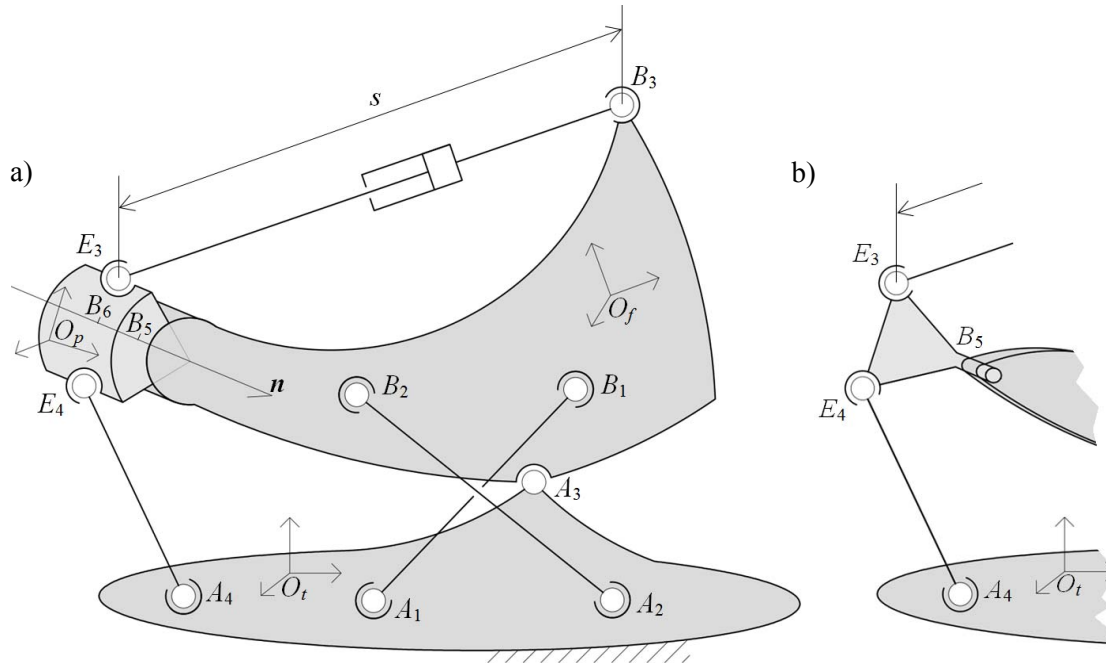


Fig.1. a) The spherical model of the knee joint with the patellofemoral joint and the muscle, where:  $A_1B_1$ ,  $A_2B_2$  – the cruciate ligaments,  $A_3$  – the center of the spherical joint that connects two platforms,  $A_4$  ( $E_4$ ) – the patellar tendon attachment to the tibia (the patella),  $s$  – the length of the substitute muscle,  $E_3$  ( $B_3$ ) – the substitute muscle attachment to the patella (the femur),  $B_5$ ,  $B_6$  – points on the axis of symmetry of the cylindrical joint that connects the patella and the femur with the direction oriented by  $n$ ;  $\{x_t, y_t, z_t\}$  – the tibia reference frame,  $\{x_f, y_f, z_f\}$  – the femur reference frame,  $\{x_p, y_p, z_p\}$  – the patella reference frame; b) the simplified model of the patella with the revolute joint ( $B_5$ ).

## 2. Position analysis of the patellofemoral joint

### 2.1. Research problem

The problem of inverse position analysis (IPA) is formulated as follows. Given a set of successive positions of the femur with respect to the tibia compute corresponding positions and displacements of the patella using the vector method.

The position and displacement analysis of the patellofemoral joint (Fig.1a) using a method of constraint equations leads to a system of nonlinear equations numerically solved in Sancisi and Parenti-Castelli (2010). On the other hand, the vector method yields a closed-form solution. The substitute mechanism for the patella part of the knee joint (Fig.1b) can be described with one tetrahedron presented in Fig.2a. In order to obtain the position of the patella reference frame with respect to the tibia reference frame a unit vector  $e_{54}^o$  describing the patella orientation can be determined using the formula for finding one of three unit vectors, when two unit vectors and two dot products of each these vectors with the unknown unit

vector are known (Ciszkiewicz and Knapczyk, 2014). The unknown unit vector  $\mathbf{e}_{54}^o$  is determined using two known unit vectors  $\mathbf{b}_{56}^o$ ,  $\mathbf{d}_{54}^o$  and three known dot products  $c_1 = \mathbf{b}_{56}^o \cdot \mathbf{d}_{54}^o$ ,  $c_2 = \mathbf{b}_{56}^o \cdot \mathbf{e}_{54}^o$ ,  $c_3 = \mathbf{d}_{54}^o \cdot \mathbf{e}_{54}^o$

$$\mathbf{e}_{54}^o = \left[ (c_2 - c_1 c_3) \mathbf{b}_{56}^o + (c_1 - c_1 c_3) \mathbf{d}_{54}^o \pm \sqrt{D} (\mathbf{b}_{56}^o \times \mathbf{d}_{54}^o) \right] / (1 - c_1^2), \quad (2.1)$$

where  $D = 1 - c_1^2 - c_2^2 - c_3^2 + 2c_1 c_2 c_3$ .

The position vectors  $\mathbf{e}_4$ ,  $\mathbf{b}_5$ ,  $\mathbf{b}_6$  of points  $E_4$ ,  $B_5$  and  $B_6$  in the patella reference frame are given and the distances  $e_{54} = \|\mathbf{e}_4 - \mathbf{b}_5\|$ ,  $e_{64} = \|\mathbf{e}_4 - \mathbf{b}_6\|$  can be computed.

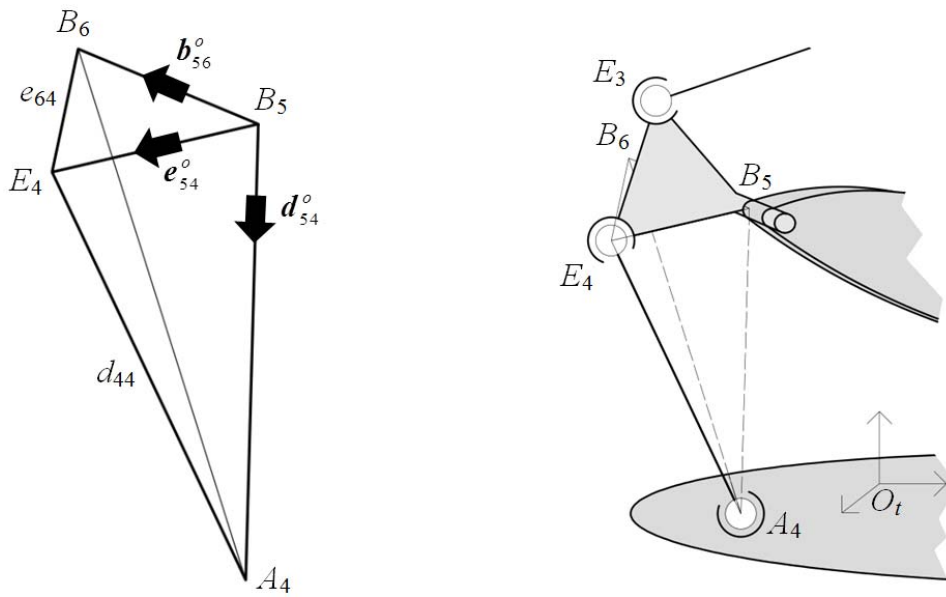


Fig.2. a) The vector tetrahedron (at the patella), b) the patellofemoral joint model.

The unknown components of Eq.(2.1) are computed in the following steps. The procedure starts with the calculation of the vectors  $\mathbf{b}_{56}$ ,  $\mathbf{d}_{54}$  in the tibia reference frame

$$\mathbf{b}_{56} = \mathbf{b}_6 - \mathbf{b}_5; \quad \mathbf{d}_{54} = \mathbf{a}_4 - \mathbf{b}_5; \quad (2.2)$$

where:  $\mathbf{b}_5$  and  $\mathbf{b}_6$  – the position vectors of the points  $B_5$  and  $B_6$  assumed on the femur and described in the tibia reference frame,  $\mathbf{a}_4$  – position vector of the known point  $A_4$  on the tibia and described in the tibia reference frame. In the next step, the unit vectors  $\mathbf{b}_{56}^o$ ,  $\mathbf{d}_{54}^o$ , the distances  $b_{56}$ ,  $d_{54}$  and the dot product  $\mathbf{b}_{56}^o \cdot \mathbf{d}_{54}^o$  are calculated. The dot products  $c_2 = \mathbf{b}_{56}^o \cdot \mathbf{e}_{54}^o$ ,  $c_3 = \mathbf{d}_{54}^o \cdot \mathbf{e}_{54}^o$  can be obtained using a cosine formula

$$\mathbf{b}_{56}^o \cdot \mathbf{e}_{54}^o = \frac{b_{56}^2 + e_{54}^2 - e_{64}^2}{2b_{56}e_{54}}; \quad \mathbf{d}_{54}^o \cdot \mathbf{e}_{54}^o = \frac{d_{54}^2 + e_{54}^2 - d_{44}^2}{2d_{54}e_{54}}. \quad (2.3)$$

Equation (2.1) can now be used to obtain the unit vector  $\mathbf{e}_{54}^o$ . The position vector of the point  $E_4$  can be obtained as follows

$$\mathbf{e}_4 = \mathbf{b}_5 + \mathbf{e}_{54}^o \mathbf{e}_{54}. \quad (2.4)$$

It is worth noting that  $\mathbf{e}_{54}^o$  can assume two different directions depending on the sign before the square root in Eq.(2.1). The proper direction of this unit vector can be chosen after a comparison of the results obtained from the model and the experimental data.

The position analysis of the patella part model for the knee joint using the vector method provides the solution in the form of position vectors of the three points located on the patella and described in the tibia reference frame  $\{x_t, y_t, z_t\}$ . For the known coordinates of these vectors in the femur reference frame a rotation matrix  $\mathbf{R}_{pf}(\alpha^p, \beta^p, \gamma^p)$  and a translation vector  $(\mathbf{p}_{pf}^f = [p_x^p, p_y^p, p_z^p]^T)$  of the patella reference frame with respect to the femur reference frame can be solved using Horn's method (Horn, 1987). A sequence of rotations for the rotation matrix is assumed as per Sancisi and Parenti-Castelli (2008). In a medical terminology the following description is used:  $\alpha^p$  – flexion(+) and extension(-),  $\beta^p$  – abduction(-) and adduction(+),  $\gamma^p$  – external(-) and internal(+) rotation. This form of the results will be used in the parameters estimation. However, the results are also presented by parameters of the screw displacements (Fenton and Shi, 1990).

## 2.2. Data preparation

The input data set for the position analysis contains:  $\mathbf{b}_5^f$  ( $\mathbf{b}_5^p$ ),  $\mathbf{b}_6^f$  ( $\mathbf{b}_6^p$ ) – the position vectors of the points  $B_5$  and  $B_6$  on the cylindrical joint axis in the femur (the patella) reference frame,  $\mathbf{a}_4$  – the position vector of the point  $A_4$  – the center of the patellar tendon attachment to the tibia in the tibia reference frame,  $\mathbf{e}_4$  – the position vector of the point  $E_4$  – the center of the patellar tendon attachment to the patella in the patella reference frame,  $d_{44}$  – the length of the patellar tendon. The assumed 19 parameters are listed below (the coordinates of position vectors and  $d_{44}$  are expressed in [mm])

$$\mathbf{a}_4 = \begin{bmatrix} 20.0 \\ -30.0 \\ 5.0 \end{bmatrix}, \quad \mathbf{b}_5^f = \begin{bmatrix} 4.8 \\ 10.5 \\ 0.0 \end{bmatrix}, \quad \mathbf{b}_5^p = \begin{bmatrix} -43.0 \\ 6.5 \\ -2.6 \end{bmatrix},$$

$$\mathbf{b}_6^f = \begin{bmatrix} 4.5 \\ 10.2 \\ -2.7 \end{bmatrix}, \quad \mathbf{b}_6^p = \begin{bmatrix} -42.5 \\ 6.1 \\ 0.0 \end{bmatrix}, \quad \mathbf{e}_4^p = \begin{bmatrix} 0.0 \\ -21.0 \\ 0.0 \end{bmatrix}, \quad d_{44} = 48.7.$$

The parameter set and the experimental data required for the parameters estimation are assumed as per Sancisi and Parenti-Castelli (2008). The input data set also contains a set of the rotation matrices  $\mathbf{R}_{ft}(\alpha, \beta, \gamma)$ , the translation vectors  $\mathbf{p}_{ft} = [p_x, p_y, p_z]^T$  from the femur reference frame  $\{x_f, y_f, z_f\}$  to the tibia reference frame  $\{x_t, y_t, z_t\}$  obtained from the spherical model (Ciszkievicz and Knapczyk, 2014). The position vectors of the points  $B_5$  and  $B_6$  in the tibia reference frame can be computed as follows

$$\mathbf{b}_5 = \mathbf{p}_{ft} + \mathbf{R}_{ft} \mathbf{b}_5^f; \quad \mathbf{b}_6 = \mathbf{p}_{ft} + \mathbf{R}_{ft} \mathbf{b}_6^f. \quad (2.5)$$

### 2.3. Parameters estimation

The parameters estimation is performed so that displacements of the simplified patellofemoral joint can reproduce the displacements of the actual joint more accurately. In this case 19 parameters are estimated.

In order to compare the results obtained from the patellofemoral joint model with the experimental data six relative indicators are calculated as follows (over the full flexion range of the knee)

$$\Delta x = \frac{1}{n z_x} \sum_{i=1}^n |x_s(\alpha_i) - x_r(\alpha_i)| \tag{2.6}$$

where:  $\Delta x$  – indicator of  $x \in \{\alpha^p, \beta^p, \gamma^p, p_x^p, p_y^p, p_z^p\}$ ;  $x_s(\alpha_i)$  – the value of  $x$  obtained using the spherical model at  $\alpha_i$  ( $\alpha$  is the flexion angle of the spherical model of the knee or the knee when the experimental data is considered); here:  $\alpha_i \in \langle 6^\circ; 108^\circ \rangle$ ,  $\Delta\alpha_i = 6^\circ$ ;  $x_r(\alpha_i)$  – the measured value of  $x$  at  $\alpha_i$ ,  $z_x$  – the range of  $x$  obtained from the experimental data,  $n$  – the number of the flexion angle set elements (here:  $n = 18$ ).

The objective function can be written as follows

$$h = (w_1\Delta\alpha^p + w_2\Delta\beta^p + w_3\Delta\gamma^p + w_4\Delta p_x^p + w_5\Delta p_y^p + w_6\Delta p_z^p) + r \tag{2.7}$$

where:  $w_i$  – the weight factor of the respective indicator (here:  $w_i = 1 [i = 1..6]$ ),  $r$  – the penalty function (assumes a large value if for a dataset there is no real solution or the range of the flexion angle is unsatisfactory).

The estimation was implemented using the evolutionary algorithm (Fig.3). Note that a group of specimens is called a population and an individual specimen is a solution (a set of geometrical parameters).

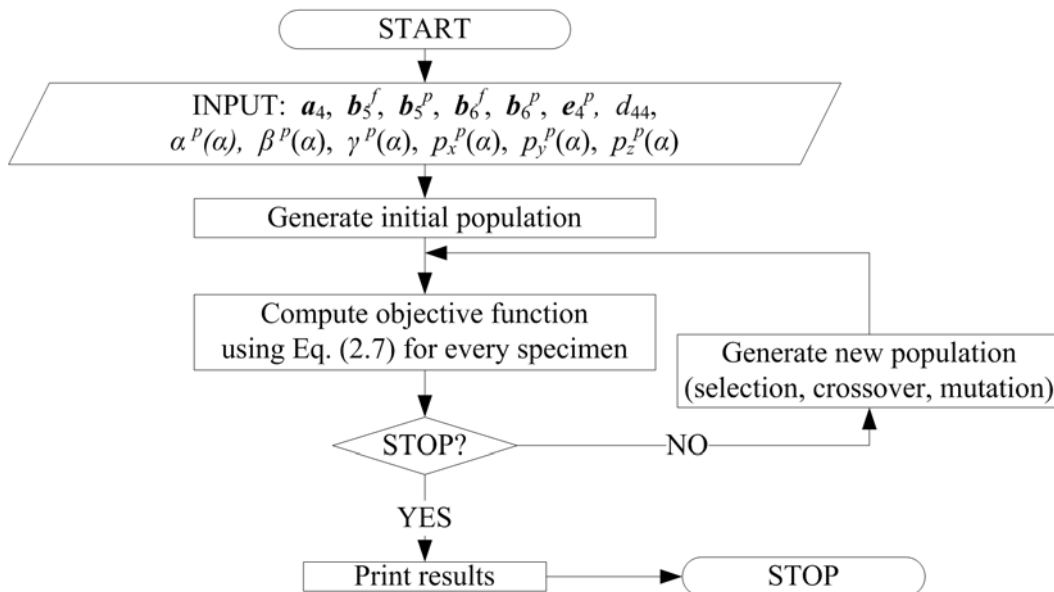


Fig.3. Flow chart – parameters estimation procedure.

**3. Results**

Summarized results of the parameters estimation of the patella part model with 19 parameters are presented below (the coordinates of the position vectors and  $d_{44}$  are expressed in [mm])

$$\mathbf{a}_4 = \begin{bmatrix} 20.1 \\ -29.7 \\ 10.0 \end{bmatrix}, \quad \mathbf{b}_5^f = \begin{bmatrix} 7.6 \\ 16.8 \\ 0.0 \end{bmatrix}, \quad \mathbf{b}_5^p = \begin{bmatrix} -45.4 \\ 8.9 \\ -0.9 \end{bmatrix},$$

$$\mathbf{b}_6^f = \begin{bmatrix} 7.6 \\ 16.9 \\ -1.1 \end{bmatrix}, \quad \mathbf{b}_6^p = \begin{bmatrix} -45.1 \\ 8.6 \\ 0.0 \end{bmatrix}, \quad \mathbf{e}_4^p = \begin{bmatrix} 0.0 \\ -31.7 \\ 0.0 \end{bmatrix}, \quad d_{44} = 38.9.$$

Figure 4a illustrates successive positions of the patellofemoral joint mechanism when added to the spherical model of the knee joint (Caruntu and Hefzy 2004). Table 1 presents a comparison of the results obtained using the vector method and the experimental data (Sancisi and Parenti-Castelli, 2008). The considered mechanism cannot fully reproduce complex displacements of the patellofemoral joint. However, the results obtained can be improved by estimating two sets of parameters (for  $\alpha \in \langle 6^\circ; 54^\circ \rangle$  and  $\alpha \in \langle 60^\circ; 108^\circ \rangle$ ).

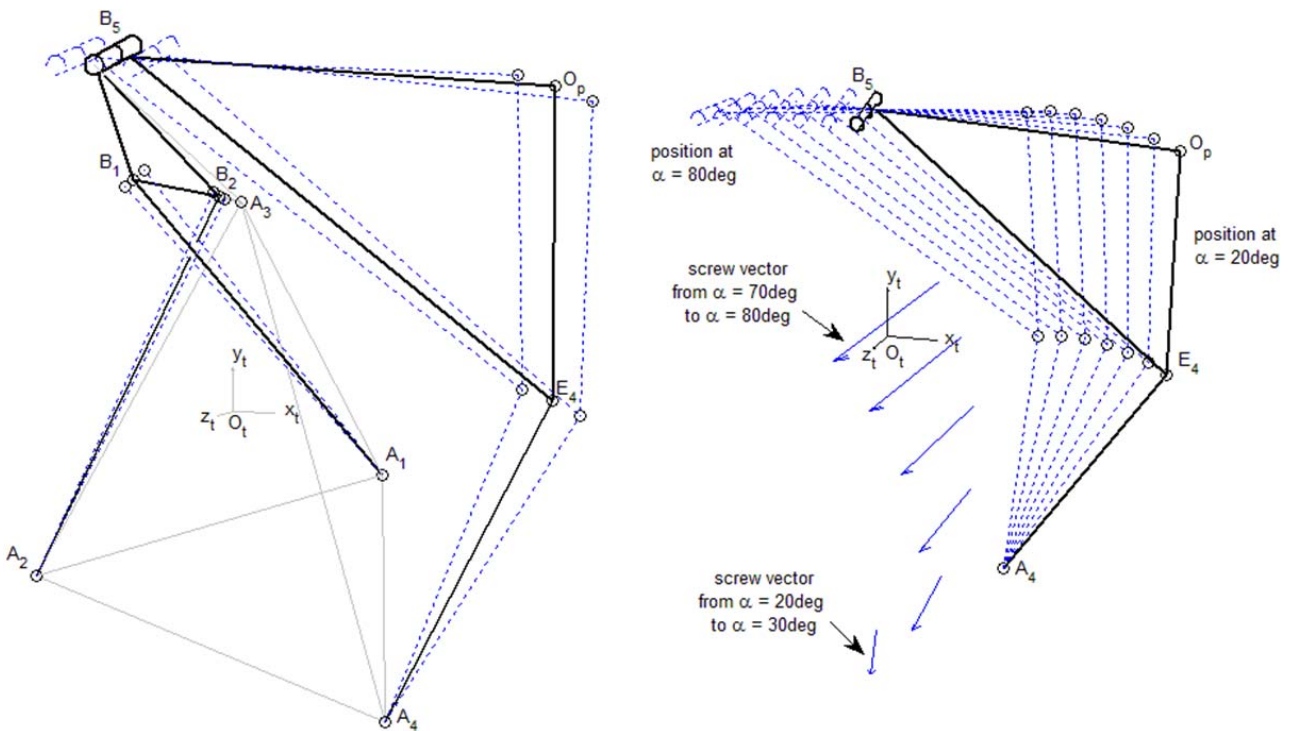


Fig.4.a) The spherical model of the knee with the patella model in three positions, where:  $A_1B_1, A_2B_2$  – the cruciate ligaments,  $A_3$  – the center of the spherical joint that connects two platforms,  $B_5$  – the revolute joint,  $A_4 (E_4)$  – the patellar tendon attachment to the tibia (the patella),  $\{x_t, y_t, z_t\}$  – tibia reference frame,  $O_p$  – the origin of the patella reference frame, b) the patellofemoral joint model in successive positions with the set of instantaneous screw vectors of the patella with respect to the tibia (length of the screw vector is proportional to the angular displacement along the axis), where:  $\alpha$  is the flexion angle of the spherical model of the knee joint.

Table 1. Simulation results: *MaxDiff* – maximum; *MeanDiff* – mean difference between the results obtained from the model (a – for  $\alpha \in \langle 6^\circ; 108^\circ \rangle$ ; b – for  $\alpha \in \langle 6^\circ; 54^\circ \rangle$ ,  $\alpha \in \langle 60^\circ; 108^\circ \rangle$ ) and the experimental data presented in Sancisi and Parenti-Castelli (2010).

		$\alpha^p$ [deg]	$\beta^p$ [deg]	$\gamma^p$ [deg]	$p_x^p$ [mm]	$p_y^p$ [mm]	$p_z^p$ [mm]
a	<i>MaxDiff</i>	4.00	3.71	4.94	1.81	6.20	1.28
	<i>MeanDiff</i>	1.24	1.84	1.70	0.92	1.03	0.39
	<i>Range</i>	84.42	5.61	28.57	50.19	30.35	2.00
b	<i>MaxDiff</i>	2.95	4.71	4.28	1.60	5.17	0.79
	<i>MeanDiff</i>	1.11	1.50	1.59	0.59	1.07	0.25
	<i>Range</i>	81.37	8.36	22.96	48.34	29.73	3.20

#### 4. Conclusion

Numerical simulations proved effectiveness of the prepared software and the parameters estimation using the vector method. Estimation of two parameters sets for  $\alpha \in \langle 6^\circ; 54^\circ \rangle$  and  $\alpha \in \langle 60^\circ; 108^\circ \rangle$  instead of one parameter set for  $\alpha \in \langle 6^\circ; 108^\circ \rangle$  can improve the results obtained from the model. It seems expedient to consider the linear displacement along the axis of the patellofemoral cylindrical joint as it is allowed in the actual joint.

The vector method can be advantageous in research concerning the patellofemoral joint – allowed ranges of the displacements and possible collisions between ligaments and bones can be investigated. It is also worth mentioning that the vector method provides a closed-form solution for the position of the patellofemoral joint. The method can be used to solve direct position analysis (DPA) of the considered mechanism, using numerical methods.

It is possible to use the results to solve static and elasto-static problems for the knee joint model. The model with assumed coordinates of muscle attachments could be used to study the relation between the muscle force and the resulting screw displacement. Then, medical conditions such as patellofemoral dislocation could be analyzed.

#### Nomenclature

$\mathbf{p}_{pf}$  – translation vector of the origin  $O_p$  of the patella reference frame with respect to the femur reference frame,

$$\mathbf{p}_{pf} = [p_x^p \ p_y^p \ p_z^p]^T$$

$\alpha$  – flexion(+)/extension(-) angle of the femur with respect to the tibia

$\alpha^p$  – flexion(+) and extension(-) angle of the patella with respect to the femur

$\beta^p$  – abduction(-)/adduction(+) angle of the patella with respect to the femur

$\gamma^p$  – external(-)/internal(+) angle of the patella with respect to the femur

#### References

Caruntu D.I. and Hefzy M.S. (2004): *3-D anatomically based dynamic modeling of the human knee to include tibio-femoral and patello-femoral joint.* – Trans. ASME, Journal of Biomechanical Engineering, vol.126, No.1, pp.44-53.

- Ciszkiewicz A. and Knapczyk J. (2014): *Parameters estimation for the spherical model of the human knee joint using vector method.* – International Journal of Applied Mechanics and Engineering, vol.19, No.3.
- Fenton R.G. and Shi X. (1990): *Comparison of methods for determining screw parameters of finite rigid body motion from initial and final position data.* – Trans. ASME, Journal of Mechanical Design, vol.112, pp.472-479.
- Horn B. (1987): *Closed-form solution of absolute orientation using quaternions.* – Journal of the Optical Society of America A, vol.4, pp.629-642.
- Sancisi N. and Parenti-Castelli V. (2010): *A 1-Dof parallel spherical wrist for the modelling of the knee passive motion.* – Mechanism and Machine Theory, vol.45, No.4, pp.658-665.
- Sancisi N. and Parenti-Castelli V. (2008): *A new approach for the dynamic modelling of the human knee.* – Ph.D., University of Bologna.

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