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IMPROVEMENT OF REGULARITY OF URBAN PUBLIC TRANSPORT LINES BY MEANS OF INTERVALS SYNCHRONIZATION

Summary. This article describes a way of synchronization of communication lines in urban public transport. In the literature, no comprehensive methods have been presented to ensure the regularity of running public transport vehicles, except specific cases of this problem, which have little practical application. It demonstrates how this problem is difficult.

In the article, the problem was presented more broadly – running of vehicles in different intervals, in lots of common fragments of routes, and running periods was considered. The objective function for this problem was defined, and then the algorithms to solve it were discussed. In the next part of the work, a model was verified by making synchronization of the timetables of selected lines in Ostrowiec Świętokrzyski. Three lines from the twelve were included in the analysis. The routes of these lines created seven communication bundles (i.e. the common fragments of the routes) for which synchronization was required. The results of synchronization (obtained by an author software) were new departure times of the lines from their start stops. Finally, they were confronted with the existing timetables, which confirmed the usefulness of the proposed method.

1. INTRODUCTION

In bus urban transport, there are extensive networks including even several dozen communication lines. These lines are run along such routes as the layout of the streets of a given city allows. This causes overlapping of the communication lines running in different directions on some fragments of the routes, i.e., creation of so-called communication bundles. Very often the side effect of this phenomenon is the bus ride "one after another", causing duplication of communication trips.

Fig. 1 shows an example of the bundle created by two lines: *L1* and *L2*. The lines *L1* and *L2* begin to run from the start stops $S_{L1,0}$ and $S_{L2,0}$, respectively. Then they run on their routes and meet at the B_1 stop. From the B_1 stop to the B_4 stop, the lines run along the common route, creating the communication bundle. After passing the B_4 stop, the lines split in the various directions.

While ensuring regular running of one line is a simple task, it is much more difficult to coordinate the trips of the several different lines in the bundles so that the time intervals between the trips running in the same direction are constant. For example, in Fig. 1, passengers who start their journey at the B_1 stop (the first bundle stop) can travel to the B_4 stop either by the line *L1* or *L2*. If these lines depart from the start stops every x minutes, then – in order to keep regular departures on the common segments of the routes – their trips in the bundle should be arranged alternately every $x/2$ minutes.

The aforementioned example describes the simplest case of the trips departure coordination. In the real conditions, the communication lines run in a lot of bundles. Ensuring regular running of the lines in the bundles is not easy, among others owing to the different running intervals and the different arrival times from the start stops of the lines to the first stops of the bundles. This task can be realized only by intervals synchronization of the timetables.

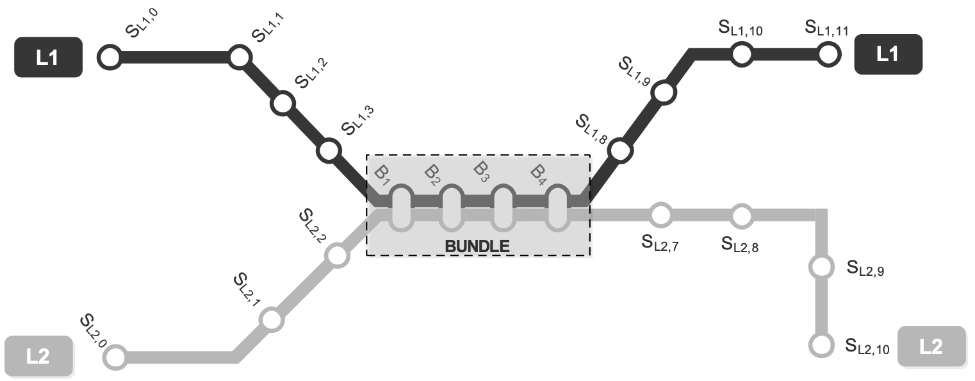

Fig. 1. The example of the bundle of two communication lines: $S_{L1,n}$ – n-th stop of the line $L1$; $S_{L2,n}$ – n-th stop of the $L2$ line; B_i – i-th stop of the communication bundle

2. RELATED LITERATURE

The concept of synchronization has a lot of meanings in transport, but it always refers to minimizing the waiting time for a means of transport. Minimizing the waiting time for the means of transport reduces the duration of the total travel. In the literature, the aspect of shortening the duration of the travel is related to the line-planning problem, consisting in a choice of the lines and their frequencies such that a given travel demand can be satisfied [3, 4, 35, 36]. The line-planning problem does not take into account the relationships between the trips on the common route fragments for the several lines.

Formally, the literature is divided into transfers synchronization and intervals synchronization. In the classic form, taken in [2, 10 - 12, 20, 40], the purpose of transfers synchronization is to minimize the total waiting times for all passengers in all transfer nodes, which are the sum of individual waiting times within given operation hours. These models refer to the quadratic assignment problem (QAP) and mixed integer programming (MIP), and heuristic algorithms are the most frequently used to solve them [8, 19].

In the special cases, the minimum waiting time for the transfer may be too short in view of the time required by the passengers. Therefore, some models take into account shifts of the departures of the communication lines so as to realize the largest possible number of the transfers [7, 38].

In another approach, the problem of transfers synchronization is based on maximization of the number of simultaneous arrivals of public transport vehicles [5, 6, 24]. Sometimes, in these models, the avoidance of the so-called ride the buses “one after another” on the common fragments of the communication lines is considered [17, 18].

Nowadays, it is necessary to carry out synchronization of the communication lines of various branches of transport, e.g., buses and trains. In this case, the optimization criterion is on the one hand the maximum of capacity of the railway line and on the other the minimum waiting time for transfer [9, 21, 39]. It is worth mentioning that in the case of rail transport, the issue of periodic event scheduling problem is also considered (e.g. [23, 27, 37]), for which minimizing the waiting time for the transfer can be the rate of the quality of the solution.

Regardless of the model, transfers synchronization does not guarantee regular running of the public transport vehicles on the common fragments of the routes.

Intervals synchronization occurs when it is possible to get from the given node of the communication network to another node in the same network with using the several alternative communication lines. In this approach, it is required to equalize the time intervals between every two consecutive trips running in the same direction [32].

The first and the simplest case of intervals synchronization was presented in the study by Adamski [1] and concerned running two or more lines only in one common segment of route and in one running period. In the real conditions (in which generally synchronization in lots of communication bundles and periods is required), the usefulness of such a model is rather insignificant.

Synchronization of the lines in the multiple bundles but still in one running period can be done by the model presented in Kwaśnicka, H. and Molecki, B. [22]. However, all the lines must run in the same intervals. To solve this type of problem, genetic algorithms were proposed. The practice shows that urban transport networks with constant and the same frequencies of running of all the lines represent a small percentage of all communication systems. They are usually the tram networks or groups of the selected bus lines and they operate only at certain times of the day.

The execution of the complete synchronization, i.e. taking into account the lines running at various frequencies in lots of bundles and running periods, is possible with a model presented in Ibarra-Rojas and Muñoz [16]. However, these lines must run with the same intervals. The main doubts in this model are an evaluation method of line synchronization degree, no information about ranges in which the departure times may change, and no information about a way of a selection parameter showing the importance of synchronization on the given stop in comparison to the other.

Sometimes the regularity of bus running is considered as a measure of evaluation of public transport quality [13, 15, 28, 29, 34]. This evaluation is based on so-called indicators of regularity, including the indicator of service failure due to the lack of regularity[1], standard deviation of intervals, percentage regularity deviation mean[2]. These indicators provide general advice on how the timetables with regular departures of public transport vehicles should be constructed.

Finally, the literature does not give any "universal" model to solve the intervals synchronization problem, that is, such a model that would take into account lines running in the same intervals (not necessarily constant at a given time), in lots of communication bundles and running periods.

3. A MODEL OF THE INTERVALS SYNCHRONIZATION PROBLEM

An analysis of literature and own observations allowed the authors to draw two important conclusions. First of all, there are no literature studies dealing with the intervals synchronization problem in a comprehensive manner. Second, ensuring the regular departures on the common segments of the routes of many transport lines has not been satisfactorily implemented in small, medium, and large cities.

In this article, the following assumptions were made:

– all sizes are of deterministic type;
– the set of communication lines is given;
– the set of running periods is given, among which peak hours and pre-peak hours are specified;
– the earliest possible departure times from the start stops, the travel times between the stops in each running period, and the intervals of communication lines in each running period are known; and
– the travel times between the stops and the intervals of communication lines can prove changeable in particular running periods.

The aim of the intervals synchronization problem is to determine such departure times of communication lines from their start stops, which guarantee that public transport vehicles will run in the bundles regularly. Regularity of running has a significant effect on the perceived quality of public transport by passengers, mainly through a better use of available transport capacity [25, 31].

In urban public transport, regularity of running occurs when the interval between each pair of adjacent trips in the given bundle and the running period is constant. The interval $\tilde{h}_{o_h}^{w_r}$ for the given bundle w_r and the running period o_h is expressed by the following:

[1] The indicator of service failure owing to the lack of regularity means the probability of the lack of space in an arriving vehicle at the stop as a result of overfilling [34].

[2] The percentage regularity deviation mean (PRDM) is defined such that if its value is 0% regularity is perfect, whereas a value of 100% implies bunched arrivals [15].

$$\tilde{h}_{o_h}^{w_r} = \frac{\bar{\rho}_{o_h}}{n_{o_h}^{w_r}} \tag{1}$$

where $\bar{\rho}_{o_h}$ – length of running period o_h; $\bar{\rho}_{o_h} = \rho_{o_h}^{konc} - 1 - \rho_{o_h}^{pocz}$, $\rho_{o_h}^{pocz}$, $\rho_{o_h}^{konc}$ – start/end of running period o_h (it is assumed that in each running period departures from the moment $\rho_{o_h}^{pocz}$ to the moment $\rho_{o_h}^{konc} - 1$ are possible; in the moment $\rho_{o_h}^{konc}$ the departure is not possible), and $n_{o_h}^{w_r}$ – number of trips in bundle w_r in running period o_h.

The interval defined by the equation (1) is called an ideal interval. Basing on the ideal interval, it is easy to see that synchronization of the trips in the bundle will occur when deviations of intervals between the successive trips will be the smallest. In addition, the "evenly spaced" trips from the beginning and end of the running period are desirable [30]. Finally, the mathematical model for the intervals synchronization problem for urban transport lines in the single bundle w_r and one running period o_h has the following form:

$$F_{o_h}^{w_r} = \sum_{i=1}^{n_{o_h}^{w_r}-1} \left(\left[t\left(k_{i+1;o_h}^{w_r}\right) - t\left(k_{i;o_h}^{w_r}\right)\right] - \tilde{h}_{o_h}^{w_r}\right)^2 + \left[t\left(k_{1;o_h}^{w_r}\right) - \rho_{o_h}^{pocz}\right]^2 + \left[\rho_{o_h}^{konc} - 1 - t\left(k_{x;o_h}^{w_r}\right)\right]^2 \to min \tag{2}$$

where $t\left(k_{i;o_h}^{w_r}\right)$ – departure time of i-th trip in the bundle w_r and the running period o_h, and $t\left(k_{x;o_h}^{w_r}\right)$ – departure time of the last trip in the bundle w_r and the running period o_h.

Alternatively, in the equation (2), instead of the quadratic function, the absolute value of the deviation from the ideal interval can be used. However, the function (2) is more sensitive to extreme values [30].

The aforementioned objective function can be written for each bundle and each running period, but obtaining the solution minimizing all of them is usually not possible. This difficulty may be avoided by the weighted sum method. However, it requires to introduce a notion of a bundle importance $\lambda_{o_h}^{w_r}$ in the bundle w_r and the running period o_h, i.e., a parameter that shows the significance of the given bundle in the network in the set of all the bundles and all the running periods. One possible way of determining the parameter is given by the formula (3). Regardless of how the parameter is constructed, the bundle importance must meet conditions specified by the formulae (4) – (5).

$$\lambda_{o_h}^{w_r} = \frac{n_{o_h}^{w_r}}{\sum_{h=1}^{p}\sum_{r=1}^{q} n_{o_h}^{w_r}} \tag{3}$$

$$\lambda_{o_h}^{w_r} = [0,1] \tag{4}$$

$$\sum_{h=1}^{p}\sum_{r=1}^{q} \lambda_{o_h}^{w_r} = 1 \tag{5}$$

where p – number of running periods and q – number of bundles.

Using the previous information, it can be said that the weighted sum method is multiplying each of the partial objective functions (2) by the corresponding bundle importance (3) and then summing of the obtained results as in the equation (6).

$$F = \sum_{h=1}^{p}\sum_{r=1}^{q}\left(\lambda_{o_h}^{w_r} \cdot F_{o_h}^{w_r}\right) \to min \tag{6}$$

The formula (6) describes the intervals synchronization problem in a lot of communication bundles and running periods.

4. THE SOLUTION OF THE INTERVALS SYNCHRONIZATION PROBLEM

The solution of the intervals synchronization problem is the set of such departure times from the start stops for which the objective function (6) assumes the smallest value. A correction of the departure time of trips from the start stops at least one line influences the change of departure times in the bundles created by this line, which makes that is possible to compare the degree of synchronization of the trips in the bundles depending on the departure from the start stops. However, the departure times of the trips may not shift totally free. Knowing the earliest possible departure times of each line l_m, the number of possible departure moments (i.e. values of which the departure time of the line from it start stop can be shifted) for each of them should be so constructed in order to:

– not affect the number of the trips run on the start stop in the earliest period for this line:

$$C'_{l_m} = \rho^{konc}_{o_\Delta} - t(l_m; \alpha; o_\Delta) \tag{7}$$

where Δ – the earliest running period of the line l_m; $\Delta = \min_t o_t$ such that $K^{l_m}_{o_\Delta} > 0$, $K^{l_m}_{o_\Delta}$ – number of the trips run by the line l_m in the period o_Δ, $t(l_m; \alpha; o_\Delta)$ – the departure time of the last trip (α) of the line l_m in the period o_Δ from the start stop.

– arrival of the latest trip at the final stop was provided before the end of the latest running period for this line:

$$C''_{l_m} = \rho^{konc}_{o_\Omega} - t'(l_m; \beta; o_\Omega) \tag{8}$$

where Ω – the latest running period of the line l_m; $\Omega = \max_t o_t$ such that $K^{l_m}_{o_\Omega} > 0$, $t'(l_m; \beta; o_\Omega)$ – the arrival time of the last trip (β) of the line l_m in the period o_Ω at the final stop.

Taking into account both conditions, the number of the possible moments of the departure times of the line l_m results from the formula:

$$C_{l_m} = \min\left(C'_{l_m}; C''_{l_m}\right) \tag{9}$$

The effect of adding the further departure moments to the earliest departure times of the trips (the departure times from the start stops) specific to the communication line is a set of possible solutions for the intervals synchronization problem. Finding the solutions with the smallest value of the objective function is possible by accurate or approximate (heuristic) methods. The accurate methods always return the optimal solution, but approximate ones give only suboptimal solution, that is “close enough” to the best one. The main reason for the use of the approximate methods is the shorter duration of calculation, which is usually achieved at the expense of optimal solutions.

The simplest method in implementation is a brute force method. It is based on generating all feasible solutions, calculating for each of them values of the objective function, and selecting the solution with the smallest value of the objective function [33]. Generating acceptable solutions in this case means generating all possible departure times for all lines from their start stops.

The brute force method always returns the optimal solution. Nevertheless, the use of the brute force method in practice is limited owing to the duration of calculation. Even for small networks, the time needed to get the best solution may be too long to use this method in real life, but it can be successfully used to synchronize groups of the lines in the several bundles.

Most heuristic methods are effectively devoid of the time-consuming aspect of calculations. One of them – and at the same time the simplest one – is a random search algorithm. Its idea is to random the solutions from the set of the feasible solutions, then to calculate the value of the objective function and finally to select the solution with the smallest value of the objective function. The solution obtained in this way is considered the best solution for the optimization problem.

The number of iterations made by the random search method can be specified in two ways. These methods can be over when the satisfactory solution was obtained and then the operation time and the number of iterations made by the algorithm are unknown, or they can be over after making a certain number of the iterations and then they return the solution with some accuracy [26, 41]. In the second case, increasing the accuracy of the calculations can be done by increasing the number of randomizations.

The second heuristic algorithm is a beam search algorithm. This is a modification of the greedy algorithm. The principle of the greedy algorithm operation is to find the solutions step-by-step. At every stage, the best local solution is chosen, hoping to receive the best global solution. The modification made in the beam search algorithm is based on the number of the best local solutions (δ) selected after each stage of the algorithm [14]. In the beam search algorithm, it is greater than 1.

In the case of the intervals synchronization problem, the first step is to generate all possible departure times for the first communication line from the start stop and to calculate the value of the objective function. Among the obtained solutions, δ the best solutions is remembered – i.e. the solutions with the smallest value of the objective function. In the n-th step ($n > 1$), the departure times from the δ remembered solutions are read and then all possible departure times of the n-th line are generated and the value of the objective function is calculated. Later δ solutions (in the last step only one) are selected and saved with the smallest value of the objective function.

5. SYNCHRONIZATION OF SELECTED LINES IN OSTROWIEC ŚWIĘTOKRZYSKI

For practical considerations referring to the intervals synchronization problem, the communication network in Ostrowiec Świętokrzyski will be used. It consists of 12 communication lines serviced completely by Miejskie Przedsiębiorstwo Komunikacji. These lines are numbered from 0 to 11. A fragment of this network, which consists of lines 1, 3 and 4, was considered. The lines number 1 and 4 are pendulum lines, and the line 3 is a circular line[3]. Their routes are schematically illustrated in Fig. 2.

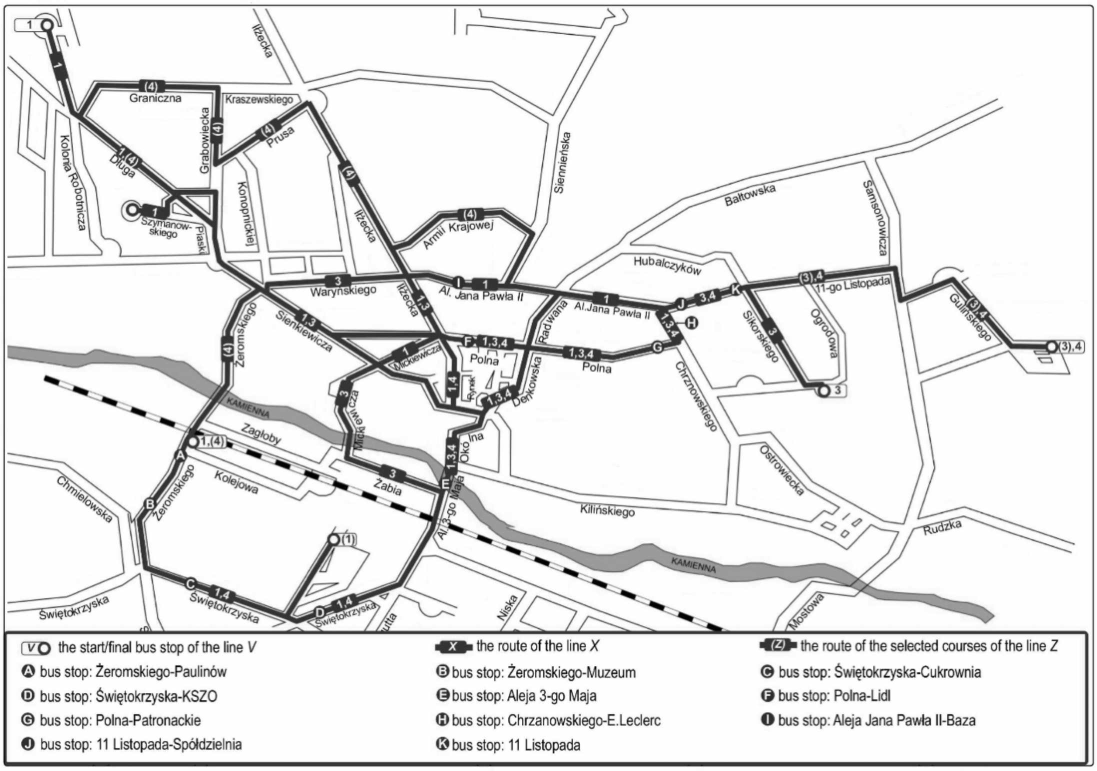

Fig. 2. Routes of the communication lines 1, 3 and 4 – own elaboration based on [42]

The total number of the trips run on each line during the working day is known, as well as the earliest possible departures from the start stops, which are a timetable for the bus lines number 1, 3 and 4 valid from June 24th. It is worth noticing that the number of the trips run at different times of the day (running periods) changes according to a passenger traffic. In this case, six such periods can be specified: 04:20 ÷ 06:30 – traffic pre-peak period; 06:30 ÷ 09:00 – morning traffic peak; 09:00 ÷ 12:30 – period between morning and afternoon traffic peak; 12:30 ÷ 17:00 – afternoon traffic peak; 17:00 ÷ 20:30 – period between afternoon and evening traffic peak; and 20:30 ÷ 23:00 – evening traffic peak.

The earliest possible departure times of the lines from the start stops are presented in Table 1.

The Fig. 2 shows that these lines run on the several common parts of the route, creating the communication bundles. It is assumed, that the bundles consisting of at least 4 stops and at least 10 trips in the day will be considered. In the aforementioned diagram, twelve of them can be pointed. They differ from each other in the first and last stop of the bundle, the travel time and the number of trips. Table 2 shows details of the bundles created by the lines number 1, 3 and 4.

6. RESULTS OF SYNCHRONIZATION AND CONCLUSIONS

The brute force method and beam search method were used to synchronize the timetables. Both methods were implemented in the *Lazarus* programming environment. Before doing the calculation, the

[3] The pendulum line runs in two direction: there and back; the circular line runs in one direction: there or back.

number of the possible departures was set on 20, 16, and 10 for lines 1, 3 and 4, respectively. It means that in case of brute force method, 3200 feasible solutions were generated, and in case of beam search method (with parameter $\delta = 3$), 98.

Table 1

The timetable of buses of the lines 1, 3 and 4 from their start stops
(the earliest possible departure times of the trips) [42]

Line	Start stop	Departure times of trips
1	Kolonia Robotnicza	05:11, 06:10, 06:50, 07:10, 07:41, 08:50[a], 09:35, 10:20[a], 11:20, 12:10[a], 13:05, 13:53, 14:30, 15:10, 15:52[b], 16:10, 17:05, 17:52, 18:50[a], 20:10, 21:05[c]
	Świętokrzyska	05:26, 09:37, 11:20, 13:00, 19:30
	Żeromskiego	06:05, 06:55, 07:40, 08:00, 08:30, 10:30, 12:15, 13:36, 14:15, 15:00, 15:20, 16:10, 17:05, 18:05, 20:21
	Kolejowa	22:15[b]
3	Ogrodowa	05:45, 06:35, 07:25, 08:20, 09:10, 10:10, 11:00, 12:00, 13:10, 14:10, 15:00, 16:00, 17:00, 18:00, 19:00, 20:00, 21:15[d]
	Gulińskiego	22:10
4	Gulińskiego	06:15[e], 06:52, 07:52, 14:20[e], 15:08, 22:20
	Żeromskiego	05:01, 06:26, 07:21, 08:21, 13:01, 21:01

[a] – trip to Świętokrzyska street
[b] – trip to Jana Pawła II street
[c] – trip to Kolejowa street
[d] – trip to Gulińskiego street
[e] – trip through Prusa street to Jana Pawła II street

Table 2

The bundles created by the lines 1, 3 and 4
(symbols from Ⓐ to Ⓚ point location of the stops in Fig. 1) – own elaboration

Bundle	Lines in bundle	First stop of bundle (symbol and common name)	Last stop of bundle (symbol and common name)	Number of stops in bundle	Number of trips in bundle
1	3,4	Ⓔ Aleja 3-go Maja	Ⓙ 11 Listopada - Spółdzielnia	6	25
2	3,4	Ⓚ 11 Listopada	Ⓕ Polna - Lidl	6	24
3	1,3,4	Ⓗ Chrzanowskiego - E.Leclerc	Ⓕ Polna - Lidl	4	46
4	1,3,4	Ⓔ Aleja 3-go Maja	Ⓖ Polna - Patronackie	5	46
5	1,3	Ⓘ Aleja Jana Pawła II - Baza	Ⓕ Polna - Lidl	6	22
6	1,4	Ⓐ Żeromskiego - Paulinów	Ⓖ Polna - Patronackie	12	21
7	1,4	Ⓓ Świętokrzyska - KSZO	Ⓖ Polna - Patronackie	7	27
8	1,4	Ⓒ Świętokrzyska - Cukrownia	Ⓖ Polna - Patronackie	9	22
9	1,4	Ⓗ Chrzanowskiego - E.Leclerc	Ⓑ Żeromskiego - Muzeum	14	21
10	1,4	Ⓗ Chrzanowskiego - E.Leclerc	Ⓓ Świętokrzyska - KSZO	10	27
11	1,4	Ⓗ Chrzanowskiego - E.Leclerc	Ⓒ Świętokrzyska - Cukrownia	12	22
12	1,4	Ⓗ Chrzanowskiego - E.Leclerc	Ⓐ Żeromskiego - Paulinów	15	17

Exactly one optimal and approximate solution was found by means of the brute force method and beam search method. The value of the objective function for the optimal solution was 3441 and for the suboptimal was 3602. In comparison, the value of the objective function for all departure times equal 0 (i.e. for the current timetable) was 3776, whereas for the worst solution – 4202.

As mentioned previously, the solutions of the intervals synchronization problem are such departure times of the lines 1, 3 and 4 from their start stops, that were generated as a result of shifting the earliest possible departure times (Table 1) for certain values expressed in minutes and obtained in the optimization process (Table 3). In the case of the optimal solution, 18, 2, and 9 minutes respectively should add to the earliest possible departure times for the lines 1, 3, 4, and 8, 1, 6 minutes for the approximate solution obtained by the beam search method. The optimal solution is the best layout of the trips in the bundles (the most satisfactory) in the given conditions in the aspect of the intervals synchronization. The layouts of the trips in the bundles resulting from the suboptimal solution are the layouts with “partial” synchronization.

Table 3

The values of the departure moments for optimal and suboptimal solution for the intervals synchronization problem – own elaboration

Line	**Departure moments [min]**	
	Optimal solution	**Suboptimal solution**
1	18	8
3	2	1
4	9	6

A detailed comparison of the synchronization results with the existing timetable, including such rates as the number of the trips in the bundle (l), the average interval ($\bar{h}$), the standard deviation of intervals (σ), and the interval coefficient of variation[4] (V) is shown in Table 4. It should be noted that depending on the analysed case (the current timetable, the timetable synchronized by the brute force method, the timetable synchronized by the beam search method), there may be different number of the trips in selected bundles and the running periods, but the total number of the trips in the bundles during the day does not change.

Table 4

Comparison between the current timetable, synchronized by brute force method and beam search method – own elaboration

Bundle	**Period***	**Current timetable**				**Timetable obtained by brute force method**				**Timetable obtained by beam search method**			
		l	$\bar{h}$	σ	V	l	$\bar{h}$	σ	V	l	$\bar{h}$	σ	V
1	O_1	4	34,67	14,46	42%	3	41,00	8,33	20%	3	41,00	9,93	24%
	O_2	5	26,25	11,53	44%	6	25,20	15,77	63%	6	25,60	14,12	55%
	O_3	4	56,67	5,59	10%	4	56,67	5,59	10%	4	56,67	5,59	10%
	O_4	5	48,00	14,85	31%	5	46,25	17,87	39%	5	46,75	17,00	36%
	O_5	4	60,00	6,71	11%	4	60,00	6,71	11%	4	60,00	6,71	11%
	O_6	3	42,50	13,21	31%	3	39,00	17,08	44%	3	40,00	15,97	40%
2	O_1	2	32,00	22,98	72%	2	39,00	18,03	46%	2	37,00	19,45	53%
	O_2	5	26,25	5,16	20%	5	26,25	6,37	24%	5	26,25	5,35	20%
	O_3	4	56,67	5,59	10%	4	56,67	5,59	10%	4	56,67	5,59	10%
	O_4	6	34,00	20,80	61%	6	34,00	17,64	52%	6	34,00	18,42	54%
	O_5	4	60,00	6,71	11%	4	60,00	6,71	11%	4	60,00	6,71	11%
	O_6	3	33,50	23,29	70%	3	37,00	19,33	52%	3	36,00	20,46	57%
3	O_1	5	28,50	7,80	27%	4	27,33	19,67	72%	5	29,75	12,63	42%
	O_2	9	14,25	8,74	61%	10	12,89	6,04	47%	9	13,38	8,10	61%
	O_3	8	24,86	10,42	42%	8	24,29	18,27	75%	8	24,29	14,09	58%
	O_4	12	21,82	13,64	63%	12	21,82	11,23	51%	12	21,82	11,08	51%
	O_5	7	30,00	18,06	60%	7	30,00	6,53	22%	7	30,00	12,17	41%
	O_6	5	29,00	18,85	65%	5	26,75	7,24	27%	5	28,50	13,05	46%
4	O_1	6	20,80	9,28	45%	4	27,33	5,77	21%	5	23,75	6,32	27%
	O_2	9	13,13	8,60	65%	11	14,60	5,64	39%	10	14,67	6,45	44%
	O_3	8	24,43	21,98	90%	7	28,33	7,59	27%	7	28,33	15,16	54%
	O_4	11	19,70	15,68	80%	12	21,36	13,15	62%	12	21,45	16,41	76%
	O_5	8	28,00	20,61	74%	7	30,00	17,42	58%	7	30,33	22,34	74%

[4] Interval coefficient of variation – a measure of dispersion (expressed as a percentage), defined as the ratio of the standard deviation to the mean.

	O_6	4	28,33	14,59	51%	5	28,50	16,93	59%	5	29,50	16,23	55%
5	O_1	4	41,00	14,52	35%	3	40,00	13,29	33%	3	35,50	11,21	32%
	O_2	3	25,50	20,24	79%	4	30,33	9,28	31%	4	30,33	9,28	31%
	O_3	5	50,00	9,15	18%	4	50,00	6,43	13%	4	50,00	6,43	13%
	O_4	5	46,25	10,44	23%	6	48,00	8,45	18%	6	48,00	8,45	18%
	O_5	4	61,67	14,41	23%	3	52,50	14,72	28%	3	52,50	14,72	28%
	O_6	1	–	–	–	2	55,00	13,79	25%	2	55,00	13,79	25%
6	O_1	3	40,00	19,75	49%	2	73,00	6,01	8%	3	40,00	21,37	53%
	O_2	6	19,00	7,35	39%	7	23,00	9,51	41%	6	19,00	6,63	35%
	O_3	2	105,00	0,35	0%	1	–	–	–	2	105,00	0,35	0%
	O_4	6	37,80	11,36	30%	7	39,17	8,86	23%	6	38,20	11,10	29%
	O_5	3	98,00	38,70	39%	2	60,00	31,47	52%	3	98,00	38,70	39%
	O_6	1	–	–	–	2	31,00	30,76	99%	1	–	–	–
7	O_1	4	26,67	11,25	42%	3	36,50	7,51	21%	3	33,00	11,55	35%
	O_2	6	19,00	7,35	39%	7	23,00	9,51	41%	7	21,83	6,93	32%
	O_3	4	54,00	5,18	10%	3	51,50	15,50	30%	4	54,00	5,18	10%
	O_4	7	32,33	15,34	47%	8	33,57	13,88	41%	7	32,33	15,92	49%
	O_5	4	65,33	14,95	23%	3	70,50	8,60	12%	3	70,50	8,60	12%
	O_6	2	74,00	0,35	0%	3	57,00	22,06	39%	3	57,00	16,63	29%
8	O_1	3	40,00	19,75	49%	2	73,00	6,01	8%	2	66,00	1,06	2%
	O_2	6	19,00	7,35	39%	7	23,00	9,51	41%	7	21,83	6,93	32%
	O_3	2	105,00	0,35	0%	1	–	–	–	2	105,00	0,35	0%
	O_4	6	37,80	11,36	30%	7	39,17	8,86	23%	6	38,20	11,10	29%
	O_5	3	98,00	38,70	39%	2	60,00	31,47	52%	2	60,00	31,47	52%
	O_6	2	74,00	0,35	0%	3	57,00	22,06	39%	3	57,00	16,63	29%
9	O_1	2	50,00	10,25	21%	1	–	–	–	2	48,00	11,67	24%
	O_2	6	18,20	10,68	59%	7	18,17	3,95	22%	6	18,20	9,46	52%
	O_3	2	105,00	0,35	0%	2	105,00	0,35	0%	2	105,00	0,35	0%
	O_4	7	30,83	17,41	56%	7	30,83	18,90	61%	7	30,83	17,53	57%
	O_5	2	47,00	40,66	87%	2	47,00	40,66	87%	2	47,00	40,66	87%
	O_6	2	116,00	29,34	25%	2	107,00	22,98	21%	2	114,00	27,93	25%
10	O_1	3	37,00	11,69	32%	2	24,00	28,64	119%	3	36,00	11,34	32%
	O_2	6	18,20	10,68	59%	7	18,17	3,95	22%	6	18,20	9,46	52%
	O_3	4	50,00	6,43	13%	4	50,00	6,43	13%	4	50,00	6,43	13%
	O_4	8	34,29	16,88	49%	8	34,29	18,23	53%	8	34,29	16,99	50%
	O_5	3	52,50	14,72	28%	3	52,50	14,72	28%	3	52,50	14,72	28%
	O_6	3	58,00	7,23	12%	3	53,50	3,36	6%	3	57,00	6,21	11%
11	O_1	2	50,00	10,25	21%	1	–	–	–	2	48,00	11,67	24%
	O_2	6	18,20	10,68	59%	7	18,17	3,95	22%	6	18,20	9,46	52%
	O_3	2	105,00	0,35	0%	2	105,00	0,35	0%	2	105,00	0,35	0%
	O_4	7	30,83	17,41	56%	7	30,83	18,90	61%	7	30,83	17,53	57%
	O_5	2	47,00	40,66	87%	2	47,00	40,66	87%	2	47,00	40,66	87%
	O_6	3	58,00	7,23	12%	3	53,50	3,36	6%	3	57,00	6,21	11%
12	O_1	1	–	–	–	1	–	–	–	1	–	–	–
	O_2	6	18,20	10,68	59%	6	18,20	6,72	37%	6	18,20	9,46	52%
	O_3	2	105,00	0,35	0%	2	105,00	0,35	0%	2	105,00	0,35	0%
	O_4	5	46,25	10,44	23%	5	46,25	10,44	23%	5	46,25	10,44	23%
	O_5	2	47,00	40,66	87%	2	47,00	40,66	87%	2	47,00	40,66	87%
	O_6	1	–	–	–	1	–	–	–	1	–	–	–

* O_1: 04:20 – 06:29, O_2: 06:30 – 08:59, O_3: 09:00 – 12:29, O_4: 12:30 – 16:59, O_5: 17:00 – 20:29, O_6: 20:30 – 22:59.

In order to better illustrate the difference between the current timetable and synchronized, the Fig. 3 shows the layout of the trips depending on time in three selected bundles and running periods.

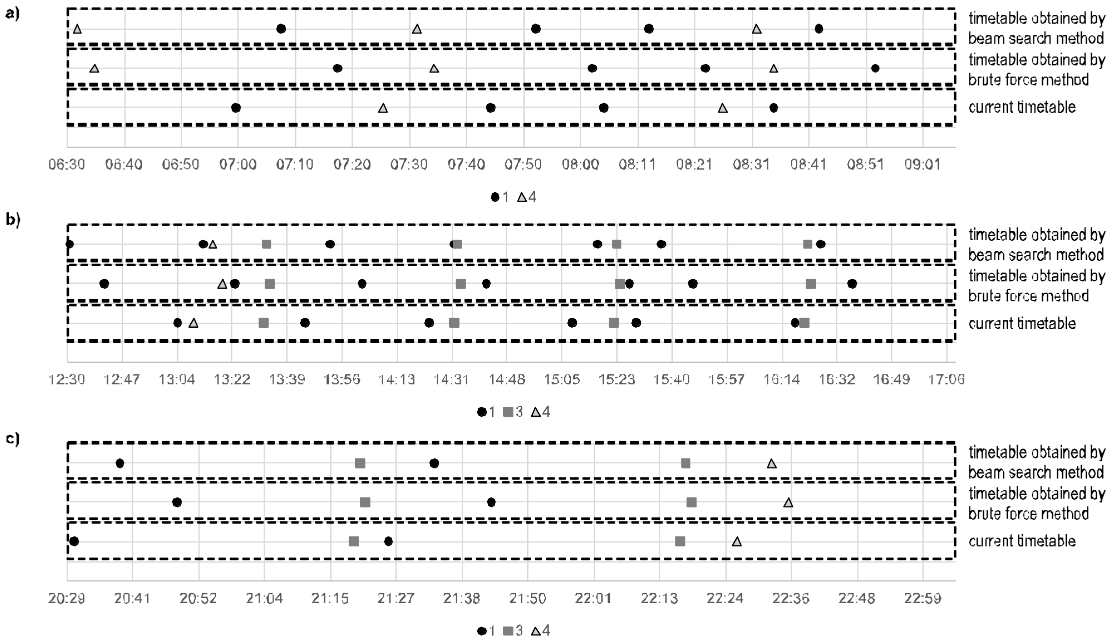

Fig. 3. The departures of the trips from the first stop of the bundle 7 in the period 06:30 – 08:59 (a), the bundle 4 in the period 12:30 – 16:59 (b), the bundle 3 in the period 20:29 – 22:59 (c) – own elaboration

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