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Earth solids and dynamic nonlinear elasticity

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ABSTRACT

The nonlinear response of rock is much different than that of ordinary materials. Especially at ambient conditions, rock responds in an extremely nonlinear fashion, often due to the presence of hysteresis. Nonlinear elastic response in rock is illustrated by dynamic results from resonant bar and pulse-mode experiments.

INTRODUCTION

Our intention is to describe several manifestations of nonlinear behavior in rock. Nonlinear response may manifest itself in a variety of manners, including a nonlinear stress-strain relation, nonlinear attenuation, harmonic generation, resonant peak shift and slow dynamics, all of which are related. We have ample evidence that the responsible mechanism for nonlinear response [to first order] is the presence of compliant features and the influence of fluid. We define compliant features as those features that are the weakest in the rock, e.g., grain-to-grain contacts, low aspect ratio cracks, joints, etc. In addition, there may be other mechanisms responsible as yet unidentified.

In the following, we emphasize the robust nature of observations by illustrating several experimental examples. We do not review the related theoretical framework which can be found in papers by Ostrovsky [1991], Guyer et al. [1995a] and Van Den Abeele et al. [1996, 1997], among others. A more complete review of experimental observations can be found in Johnson and Rasolofosaon [1996]. Finally, we do not present nonlinear parameters derived from these experiments as our purpose in this paper is to illustrate rather than quantify nonlinear response.

EXPERIMENTAL EVIDENCE OF NONLINEAR RESPONSE

1. Resonant Bar Experiments.

1.1 Resonant Bar Apparatus

In resonance experiments, nonlinear elastic behavior is observed by measuring the response of the material in Young's mode resonance. Nonlinear behavior is manifested by resonant peak shift corresponding to a modulus shift as a function of drive level. Independent confirmation of nonlinear behavior is inferred from observation of harmonics. We refer the reader to Johnson et al. [1996] for a complete description of these and other results, as well as for the basic elements of the Young's mode experimental configuration. In short, measurements of acceleration versus frequency are made while sweeping over a frequency interval. Typically, 5-20 frequency sweep experiments are repeated at successively increasing drive levels over the same frequency interval in order to monitor resonant peak change and harmonic generation. Acceleration is the parameter that is actually measured, but strain is of interest and can be directly calculated from the acceleration using the bar length [Johnson et al., 1996].

1.2 Comparison of PVC and Rock Under Young's Mode Resonance

Figure 1 shows a sample sequence of resonance curves for eight different excitation levels in PVC (PolyVinylChloride), a material that is relatively "linear" in comparison with most rocks, but with a similar Q [Q is the inverse of dissipation] to many rocks [Q = 59]. The figure shows detected acceleration versus swept frequency. Both downward and upward resonant sweeps were conducted at each drive level but these are indistinguishable from each other. Within this acceleration/strain range, no harmonics were observed from the time signal.

In Figure 2, a representative result for Young's mode resonant behavior in rock is shown. The material is Lavoux limestone under ambient conditions. The solid lines represent downward frequency sweeps and the dashed lines represent upward frequency sweeps. Note also that the linear resonant response has been expanded vertically in the inset. Contrast this result to that of PVC shown in Figure 1. Two observations are of note. First, the curve bending is dramatic as a function of detected acceleration in the rock. The resonance frequency shift at large drive in rock samples can be as large as 10-15%. Second, the shape of the curve depends on the direction of the sweep, upward or downward in frequency. This second observation is typical of extreme nonlinear oscillators in general [e.g., see Stoker, 1950]. The

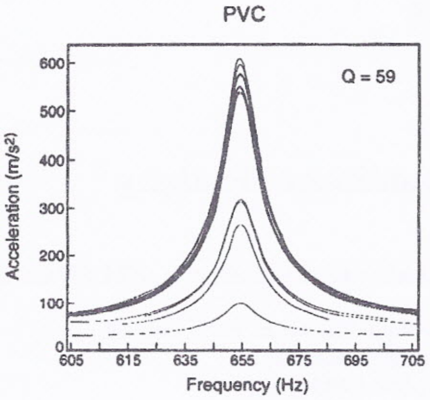

Figure 1. Resonant response of PVC.

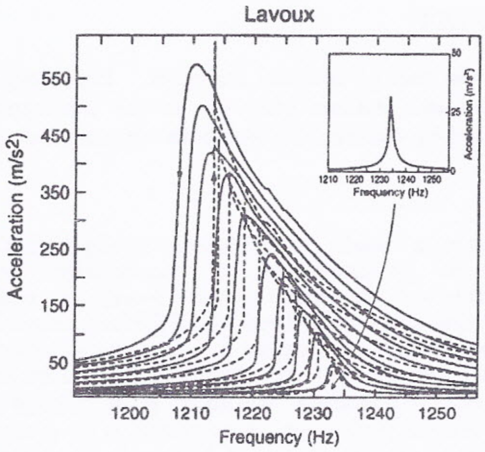

Figure 2. Resonant response of Lavoux Limestone.

corresponding frequency spectrum at peak acceleration illustrates the rich harmonic spectrum associated with large excitation level. Figure 3 illustrates these two manifestations of nonlinear response under resonance conditions for dry Lavoux limestone. The change in resonant frequency $\Delta\omega/\omega_0$ as a function of the detected strain amplitude is shown by the solid line. The harmonic ratios of the second [2×fundamental: 2×f$_1$] and third harmonics [3×fundamental: 3×f$_1$] as a function of detected strain amplitude are shown by the dashed lines. The harmonic ratios are normed to their minimum values, which are identical and appear at the same strain levels. Note that the amplitude of the third harmonic is clearly larger, and that harmonics are observed to appear before a noticeable resonant peak shift is observed.

In general, odd harmonics dominate in amplitude in rock as opposed to all "linear" standards that have been studied. For example, the harmonic ratio (in dB) for the second and third harmonics normed to the fundamental as a function of acceleration response is illustrated for Pyrex glass, in Figure 4. Clearly, the even harmonics dominate in amplitude. The harmonics are primarily electronically generated in this case.

In addition to resonance frequency shift and harmonic generation, resonance experiments in rocks also display intensity dependent attenuation. From theoretical considerations one can argue that the nonlinear attenuation is entirely due to the hysteresis response of the compliant features (see for instance Guyer et al. [1995b], Van Den Abeele et al. [1997]).

It was shown in Ten Cate and Shankland [1996], Johnson et al. [1996], and Zinszner et al. [1997] that Earth solids manifest a high degree of slow dynamics in resonance experiments. When a sample has been excited at large drive level, the materials has softened. The sample will remember this high excitation response. As a result of this memory effect, the "linear" resonance frequency at small excitation levels requires some time to relax to its original value before excitation. Figure 5 illustrates this behavior. It appears that the slow response is due only to the hysteresis in the material.

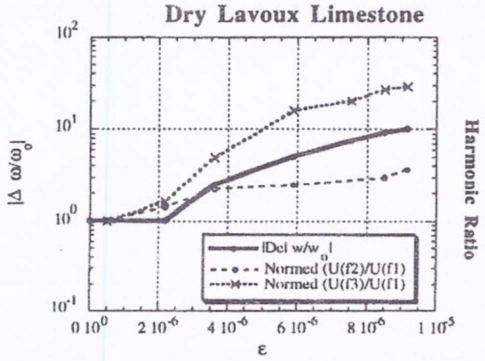

Figure 3. Resonant frequency shift and harmonic ratios in dry Lavoux Limestone.

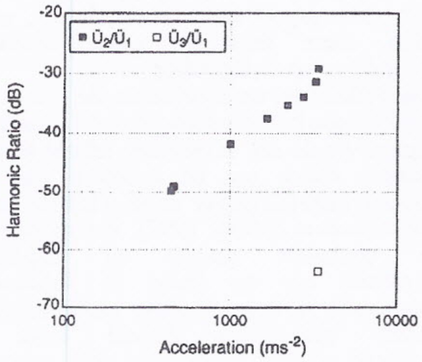

Figure 4. Harmonic generation in Pyrex glass: harmonic ratio of the second [ü$_2$/ü$_1$] and third harmonics [ü$_3$/ü$_1$] in dB versus acceleration, in ms^{-2}.

1.3 Experiments at Pressure

Numerous experiments have been conducted under confining pressure by Zinszner et al. [1997] in order to explore the effects of external pressure on the nonlinear response of rock. The experi-

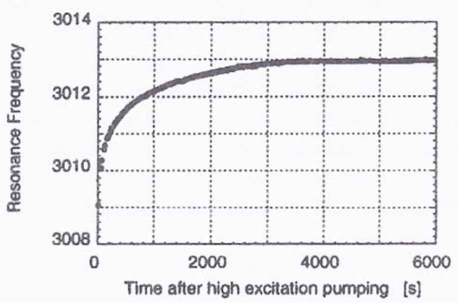

Figure 5. Relaxation of the "linear" resonance frequency in Berea Sandstone after being driven at a high excitation level for 10 minutes.

mental configuration is identical to the one at used at ambient conditions, except that the sample is placed inside a pressure vessel and jacketed with an impermeable material. Confining pressure is applied by helium gas.

Figure 6 illustrates results from dry Fontainebleau sandstone at five different confining pressures listed at the bottom of the figure. Figure 6a displays the normalized ratio of excitation intensity [measured at the source] to acceleration [measured at the detector] versus dynamic strain [in nanostrain] for these five pressure levels. The excitation-intensity-to-acceleration ratio, measured in mA/ms^{-2}, has been normalized to the initial values in each case. Excitation level must be increased for equivalent strain as the sample responds more and more nonlinearly (see Zinszner et al. [1997] for details). Figure 6b illustrates the relative change in resonant frequency $|\Delta\omega/\omega_o|$ plotted against dynamic strain amplitude for the same sample. Figure 6c illustrates the ratio of the third harmonic level to the fundamental [$\ddot{u}_3/\ddot{u}_1$] in dB versus dynamic strain. It is clear that the nonlinear response decreases as a function of confining pressure. This is due to the closure of compliant features in the material as a function of confining pressure. By approximately 10 MPa confining pressure, resonant frequency shift and third harmonic generation are undetectable with this experimental configuration.

2. Dynamic studies of pulsed waves.

In this set of experiments we illustrate the creation of harmonics along the propagation path for pulse-mode waves. Experiment 1 shows results of a single frequency drive for a wave propagating in a sandstone bar. Experiment 2 shows results for waves propagating from a broad band input in a similar bar. For a complete description of these experiments see Meegan et al. [1993], Ten Cate et al. [1996] and Johnson and McCall [1994].

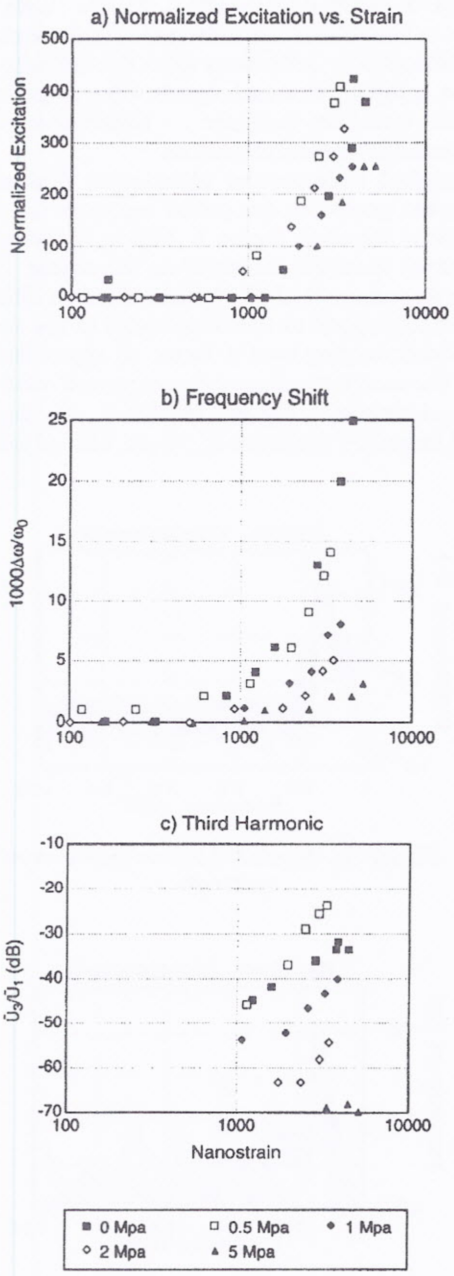

Figure 6. Nonlinear response of a sandstone at various applied confining pressures.

2.1 Pulse-Mode Apparatus and Experimental Procedure

A drive transducer composed of a piezoelectric crystal with a backload was bonded to a 2-m long rod of Berea sandstone. The source transducer was amplified and driven with the desired drive at source displacements from 10^{-9} - 10^{-6} m.

In the case of the *monofrequency* measurements, a single frequency pulsed wave train was used. Individual frequencies of 8 to 24-kHz were used for these experiments, but frequencies of up to 10^6-Hz have been studied in a variety of pulse-mode experiments as well [e.g., Johnson and Shankland, 1989]. Detectors were either embedded in the sample or placed on the sample surface. Care was taken to assure that the measured signals were not contaminated by reflections from the opposite end of the sample. Detected signals were output to a 16 bit waveform analyzer. Experiments were conducted at ambient conditions.

A typical experimental observation illustrating harmonic generation for pulsed waves in the rock sample is shown in Figure 7. Figure 7a shows the frequency spectrum measured at the source for a drive frequency (f) of 13.75-kHz. The five different curves correspond to five amplitudes of the source transducer varying over a factor of approximately 50. The source displacement spectrum is relatively monochromatic. Figure 7b shows the displacement frequency spectrum at 58-cm, also at increasing drive level. For detected displacements as small as 2×10^{-8} m at the fundamental frequency, the composition of the displacement frequency spectrum at 58-cm is rich in harmonics which are not present at the source. These higher harmonic displacement fields have amplitudes that are a sensitive function of the drive amplitude. Figure 7c illustrates the relative growth of the second harmonic amplitude with distance (see Meegan et al. [1993] for details).

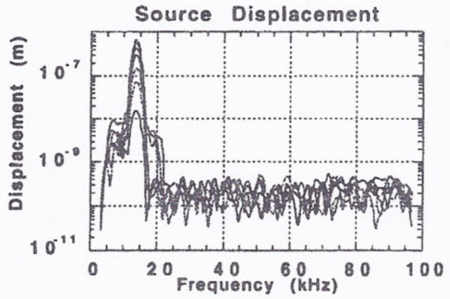

Figure 7a. Measured source displacement spectrum.

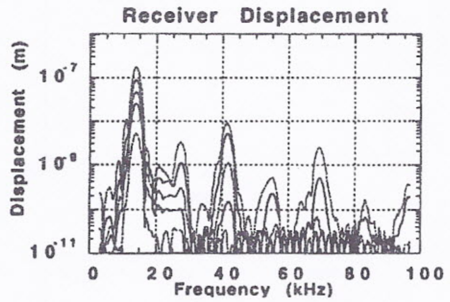

Figure 7b. Measured receiver displacement spectrum at 58 cm from source.

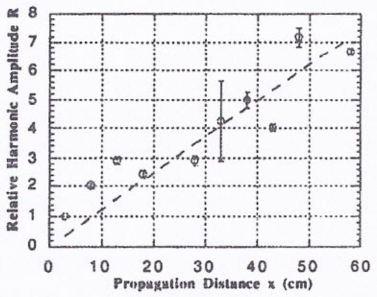

Figure 7c. Growth of second harmonic amplitude as a function of distance.

The results in Figure 7b show spectral growth at harmonics higher than 2×f. Note also that, as in the resonance experiments in rock, odd harmonics tend to dominate in amplitude. This suggests that higher order terms [i.e. quartic anharmonicity] and/or hysteresis in the stress strain relationship are necessary to provide a complete description of nonlinear elastic behavior in rock. Van den Abeele [1996] has included higher order terms in the perturbative solution and Nazarov et al. [1987], Guyer et al. [1995a], and Van Den Abeele et al. [1997], among others, have offered models that contain discontinuities in the pressure-density relation.

Figures 8a and 8b show the results for a similar experiment to the above but using a *broad-band* source. Figure 8a shows the frequency spectrum measured at the source for a drive frequency of 7 - 32 kHz [the source was a Blackman window] with peak amplitude at approximately 20 kHz. The spectral shape was heavily influenced by the electrical-mechanical transfer function of the source itself. The seven different curves correspond to progressively increasing drive amplitudes varying over a factor of approximately 100. The source displacement spectrum contains only a small fraction of harmonic and intermodulation effects and only at large drive levels. Figure 8b shows the resulting displacement frequency spectrum at 1-m. For detected displacements at the fundamental frequency as small as 3×10^{-8} m, the composition of the displacement frequency spectrum at 1-m is extremely rich in frequencies not present at the source. As drive amplitude is increased the spectrum becomes progressively richer due to the nonlinear elastic wave interaction in the material.

The broadening in the spectrum is due to the fact that a source composed of two frequencies gives rise to waves propagating at harmonics of both source frequencies in addition to waves at the sum and difference frequencies between the two source frequencies. Additionally, each individual frequency will interact with all other frequencies present creating waves at their respective sum and difference frequencies [Note that this experiment is the analogue to that conducted by Pestorious and Blackstock [1973] in gas].

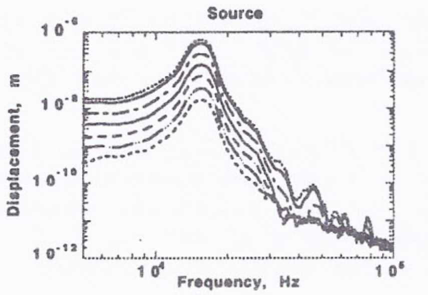

Figure 8a. Measured source spectrum using broad band input.

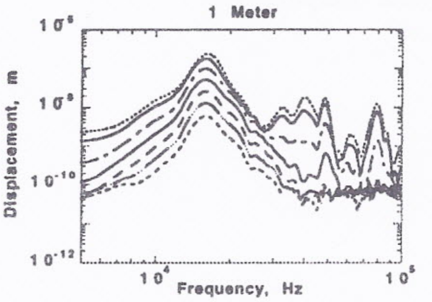

Figure 8b. Detected spectrum at 1-m.

DISCUSSION

1. Nonlinear Elasticity in Geoscience.

The results from numerous static and dynamic laboratory experiments, some illustrated in this paper, indicate that nonlinear elastic response is a pervasive characteristic of rock over orders of magnitude in frequency and strain. In Geoscience, the dynamic nonlinear response of rock has been largely ignored in the past, especially at dynamic strain levels illustrated in this paper. Clearly, even at very low drive levels, and confining pressures less than 10-MPa, nonlinear are significant.

2. Mechanism of nonlinear elasticity.

The primary mechanisms that produce nonlinear response in rock are due to the low-aspect ratio compliant features (cracks, grain-to-grain contacts, etc.) and the presence of fluids [see. e.g., Gist [1994], Johnson et al. [1996] and Zinszner et al. [1997]]. Anharmonicity in the individual minerals of the solid matrix produces very small contributions. Sharma and Tutuncu [1994] and Tutuncu et al. [1995] have suggested that contact adhesion hysteresis is the fundamental cause of volumetric hysteretic behavior. Contact adhesion may well be primarily responsible for dynamic hysteretic behavior, and we are in the process of further studies exploring this effect.

CONCLUSIONS

We have illustrated the extreme nonlinear response of rock from several laboratory experiments. The ramifications of nonlinear response in rock may ultimately affect many areas of research in Geoscience including seismology and strong motion analysis, where the spectral distortion of seismic waves during propagation must be considered [e.g., Johnson and McCall [1994]; Beresnev and Wen [1997]]. Other areas of research include rock mechanics and materials science where the nonlinear response of a material may be used for characterization purposes. In addition, characterization of material property change by monitoring nonlinear response may be of value. For instance, these changes include variations in water saturation for porous media, change in response to variations in stress, change induced by fatigue damage, etc.

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