

Wojciech LIPÍŃSKI*

NUMERICAL ANALYSIS OF CIRCUITS WITH SWITCHED CAPACITORS AND INDUCTANCES

The purpose of the paper is to present numerical calculations for circuits with switched capacitors and inductances. Computer simulations have been carried out for an exemplary circuit using PSpice software. Then the analysis has been performed for the circuit according to the numerical algorithm.

1. CIRCUIT SIMULATION USING PSPICE SOFTWARE

Mathematical models of various systems used in electronics, physics, chemistry, mechanics, economics and biology can often be expressed as ordinary differential equations

$$\frac{d^n}{dt^n} x(t) = F \left[t, x(t), \frac{d}{dt} x(t), \frac{d^2}{dt^2} x(t), \dots, \frac{d^{n-1}}{dt^{n-1}} x(t) \right] \quad (1)$$

The function $x(t)$ is unknown. Using the following substitutions

$$x_0(t) = x(t), \quad x_1(t) = \frac{d}{dt} x(t), \quad x_2(t) = \frac{d^2}{dt^2} x(t), \dots, \quad x_{n-1}(t) = \frac{d^{n-1}}{dt^{n-1}} x(t)$$

we obtain a system of n first-order differential equations with unknowns x_0, x_1, \dots, x_{n-1} .

$$\frac{d}{dt} x_0(t) = x_1(t), \quad \frac{d}{dt} x_1(t) = x_2(t), \dots, \quad \frac{d}{dt} x_{n-1}(t) = F(t, x_0, \dots, x_{n-1}) \quad (2)$$

Parametric equation with time-varying coefficients:

$$a_n(t) \cdot \frac{d^n}{dt^n} x(t) + a_{n-1}(t) \cdot \frac{d^{n-1}}{dt^{n-1}} x(t) + \dots + a_1(t) \cdot \frac{d}{dt} x(t) + a_0(t) \cdot x(t) = f(t)$$

In theory of parametric circuits we adopt coupled fluxes $\psi(t)$ in coils and charges $q(t)$ in capacitors as state variables. Charges $q(t)$ in capacitors and coupled fluxes $\psi(t)$ in coils are continuous functions of time.

$$i(t) = \frac{dq(t)}{dt}, \quad u_L(t) = \frac{d\psi(t)}{dt}$$

* West Pomeranian University of Technology, Szczecin.

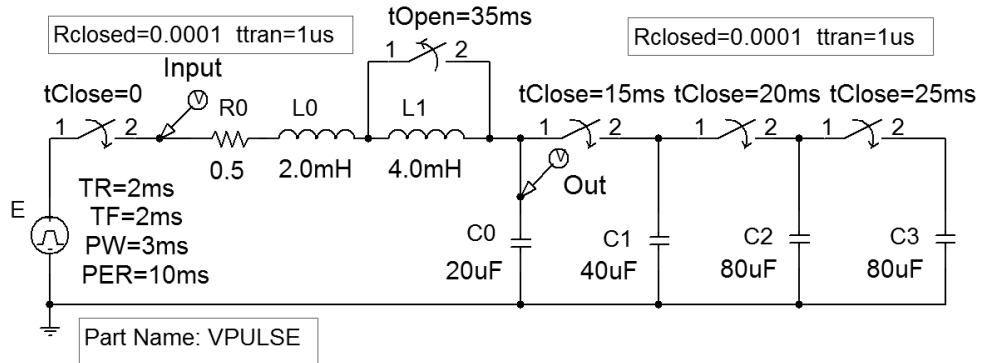


Fig. 1. Circuit with switched capacitors and inductances in PSpice

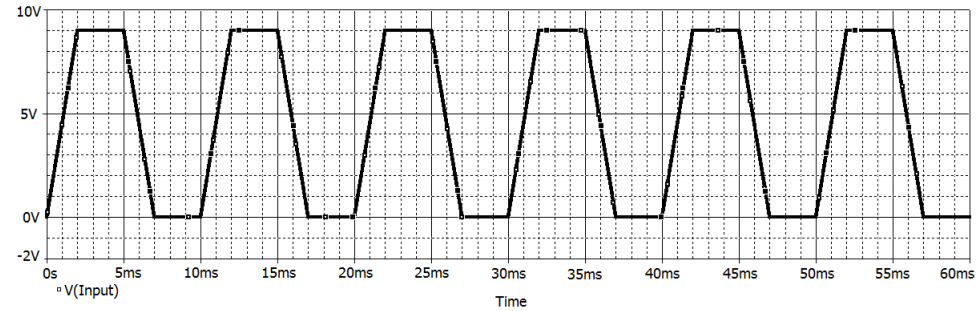


Fig. 2. Input voltage

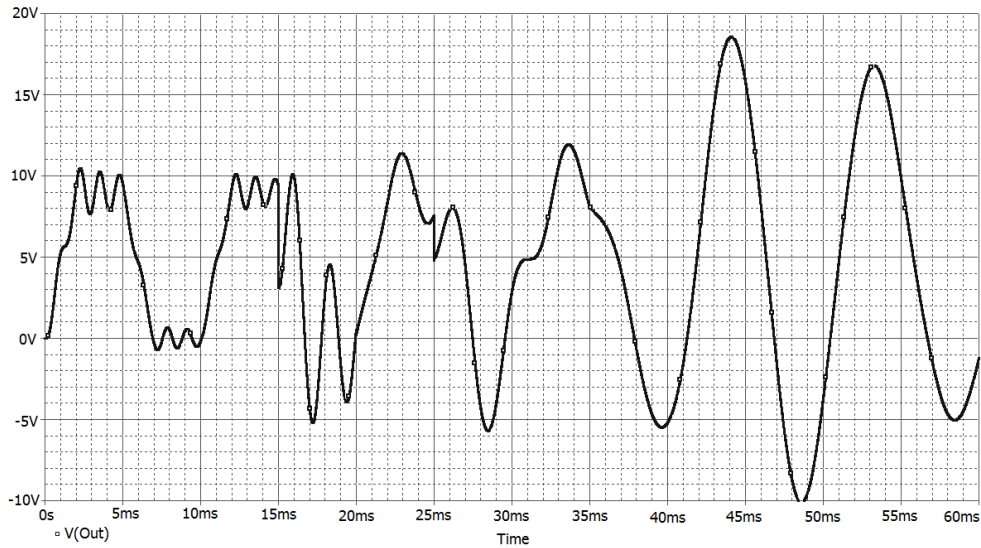
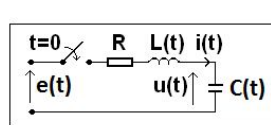


Fig. 3. Voltage across the capacitor C0. Result of computer simulation using PSpice

2. INPUT VOLTAGE AND CHANGES OF L(t) AND C(t)

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The circuit is connected to the voltage $e(t)$ with zero initial conditions.



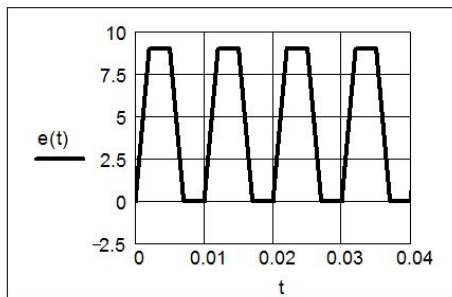
$$(R \ L_0 \ C_0) = (0.5 \ 2.0 \cdot 10^{-3} \ 20 \cdot 10^{-6}) \quad \text{ORIGIN} = 0$$

$$e(t) \equiv \begin{cases} 9 \cdot \frac{t}{0.002} & \text{if } 0 \leq t \leq 0.002 \\ 9 & \text{if } 0.002 < t < 0.005 \\ 9 \cdot \left(\frac{0.007 - t}{0.002} \right) & \text{if } 0.005 \leq t \leq 0.007 \\ 0 & \text{if } 0.007 \leq t \leq 0.01 \\ e(t - 0.01) & \text{otherwise} \end{cases}$$

$$\begin{pmatrix} R_k \\ T_0 \end{pmatrix} \equiv \begin{pmatrix} 2 \cdot \sqrt{\frac{L_0}{C_0}} \\ 2 \cdot \pi \cdot \left[\frac{1}{L_0 \cdot C_0} - \left(\frac{R}{2 \cdot L_0} \right)^2 \right]^{-1} \end{pmatrix}$$

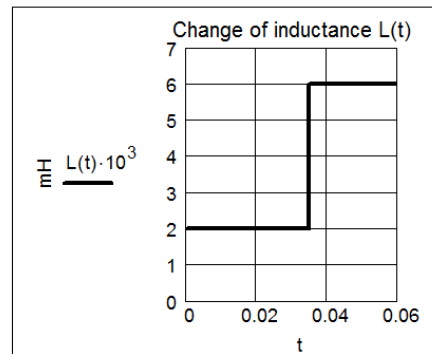
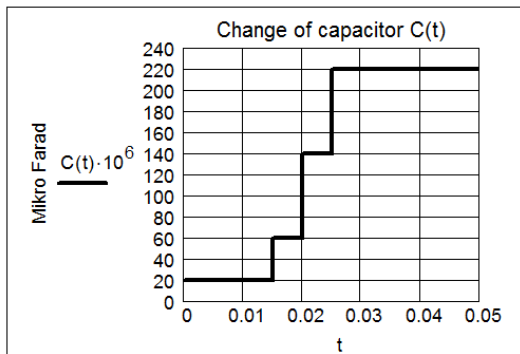
$$\begin{pmatrix} R_k \\ T_0 \end{pmatrix} = \begin{pmatrix} 20 \\ 0.001257 \end{pmatrix} \quad t := 0, 10^{-5} .. 0.06$$

T.0 - The period of oscillation



$$C(t) \equiv \begin{cases} C_0 & \text{if } 0 \leq t \leq 0.015 \\ 3 \cdot C_0 & \text{if } 0.015 < t \leq 0.020 \\ 7 \cdot C_0 & \text{if } 0.020 < t \leq 0.025 \\ 11 \cdot C_0 & \text{otherwise} \end{cases}$$

$$L(t) \equiv \begin{cases} L_0 & \text{if } 0 \leq t \leq 0.035 \\ 3 \cdot L_0 & \text{otherwise} \end{cases}$$



Coupled magnetic flux in the coil $\psi(t)$. Electric charge on the capacitor $q(t)$.

Equations of state for: $x = \psi(t)$, $y = q(t)$.

$i(t)$ - current in the coil, $u(t)$ - voltage on the capacitor.

Fig. 4. Input voltage and changes of C(t) and L(t) obtained using Mathcad

3. RUNGE-KUTTA ALGORITHM IN MATHCAD

Numerical methods for initial value problems are very easily adapted for systems of differential equations of first order. The fourth order Runge-Kutta algorithm is one of the most accurate one-step methods and the most widely used. The RK4 algorithm has local truncation error of order h^5 , so it is more accurate with larger steps than Euler's method. Let an initial value problem be specified as follows. At the start, time is t_0 and y is y_0 .

$$y' = f(t, y), \quad y(t_0) = y_0$$

$$y_{n+1} = y_n + \int_{t_n}^{t_{n+1}} f(t, y) \cdot dt \approx y_n + f(t_n, y_n) \cdot h = y_n + k_1$$

The RK4 method for this problem is given by the following equations:

$$y_{n+1} = y_n + (k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4), \quad t_{n+1} = t_n + h$$

$$k_1 = f(t_n, y_n) \cdot h$$

$$k_2 = f(t_n + 0.5 \cdot h, y_n + 0.5 \cdot k_1) \cdot h \quad (3)$$

$$k_3 = f(t_n + 0.5 \cdot h, y_n + 0.5 \cdot k_2) \cdot h$$

$$k_4 = f(t_n + h, y_n + k_3) \cdot h$$

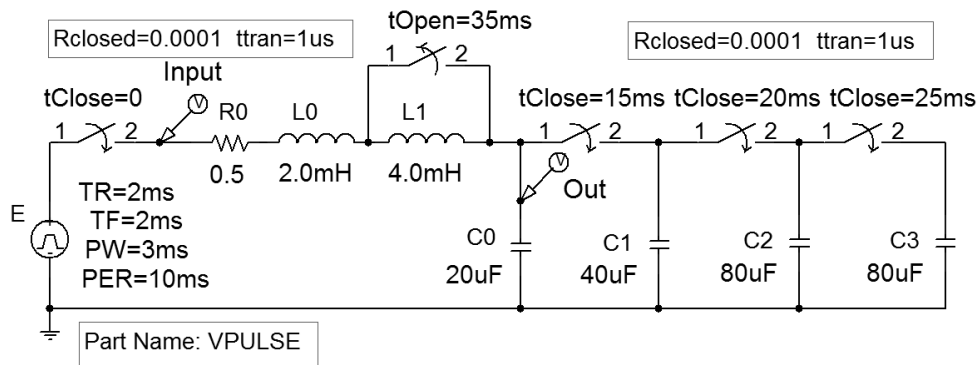


Fig. 1a. Circuit with switched capacitors and inductances

$$\begin{bmatrix} \frac{d}{dt} \psi(t) \\ \frac{d}{dt} q(t) \end{bmatrix} = \begin{bmatrix} a_1(t) & a_2(t) \\ a_3(t) & a_4(t) \end{bmatrix} \cdot \begin{bmatrix} \psi(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} e(t) \\ 0 \end{bmatrix} \quad (4)$$

Coupled magnetic flux in the coil is $\psi(t)$. Electric charge on the capacitor is $q(t)$.
 Equations of state for: $x = \psi(t)$, $y = q(t)$.
 $i(t)$ - current flowing through the coil, $u(t)$ - voltage across the capacitor.

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$$\frac{d}{dt} \begin{pmatrix} \psi(t) \\ q(t) \end{pmatrix} = \begin{pmatrix} -\frac{R}{L(t)} & -\frac{1}{C(t)} \\ \frac{1}{L(t)} & 0 \end{pmatrix} \cdot \begin{pmatrix} \psi(t) \\ q(t) \end{pmatrix} + \begin{pmatrix} e(t) \\ 0 \end{pmatrix}$$

time discretization,
 h - time step $h=\Delta t$,
 N - number of time steps

$n := 0..N$

$N := 60000$

$\Delta t := \frac{0.06}{N}$

$t_n := n \cdot \Delta t$

$h := \Delta t$

$h = 1 \times 10^{-6}$

$\frac{d}{dt} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} F(t, x, y) \\ G(t, x, y) \end{pmatrix}$ This procedure RK depends on h and N, it declares functions F(t,x,y) and G(t,x,y)

$F(t, x, y) = -\frac{R}{L(t)} \cdot x - \frac{1}{C(t)} \cdot y + e(t)$

$G(t, x, y) = \frac{1}{L(t)} \cdot x$

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RK(h, N) :=
  x0 ← 0
  y0 ← 0
  t ← 0
  for i ∈ 0.. 2
    A0,i ← 0
  for n ∈ 0.. N
    k1 ← F(t, xn, yn)·h
    K1 ← G(t, xn, yn)·h
    k2 ← F(t + 0.5·h, xn + 0.5·k1, yn + 0.5·K1)·h
    K2 ← G(t + 0.5·h, xn + 0.5·k1, yn + 0.5·K1)·h
    k3 ← F(t + 0.5·h, xn + 0.5·k2, yn + 0.5·K2)·h
    K3 ← G(t + 0.5·h, xn + 0.5·k2, yn + 0.5·K2)·h
    k4 ← F(t + h, xn + k3, yn + K3)·h
    K4 ← G(t + h, xn + k3, yn + K3)·h
    t ← t + h
    xn+1 ← 1/6 · (k1 + 2·k2 + 2·k3 + k4) + xn
    yn+1 ← 1/6 · (K1 + 2·K2 + 2·K3 + K4) + yn
    An+1,0 ← t
    An+1,1 ← xn+1
    An+1,2 ← yn+1
    
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A

A fourth-order Runge-Kutta method is used. It returns the matrix A in which the zero column contains the time points at which the solution is valued. First and second column contain the solution.

Fig. 5. The fourth order Runge-Kutta algorithm in Mathcad

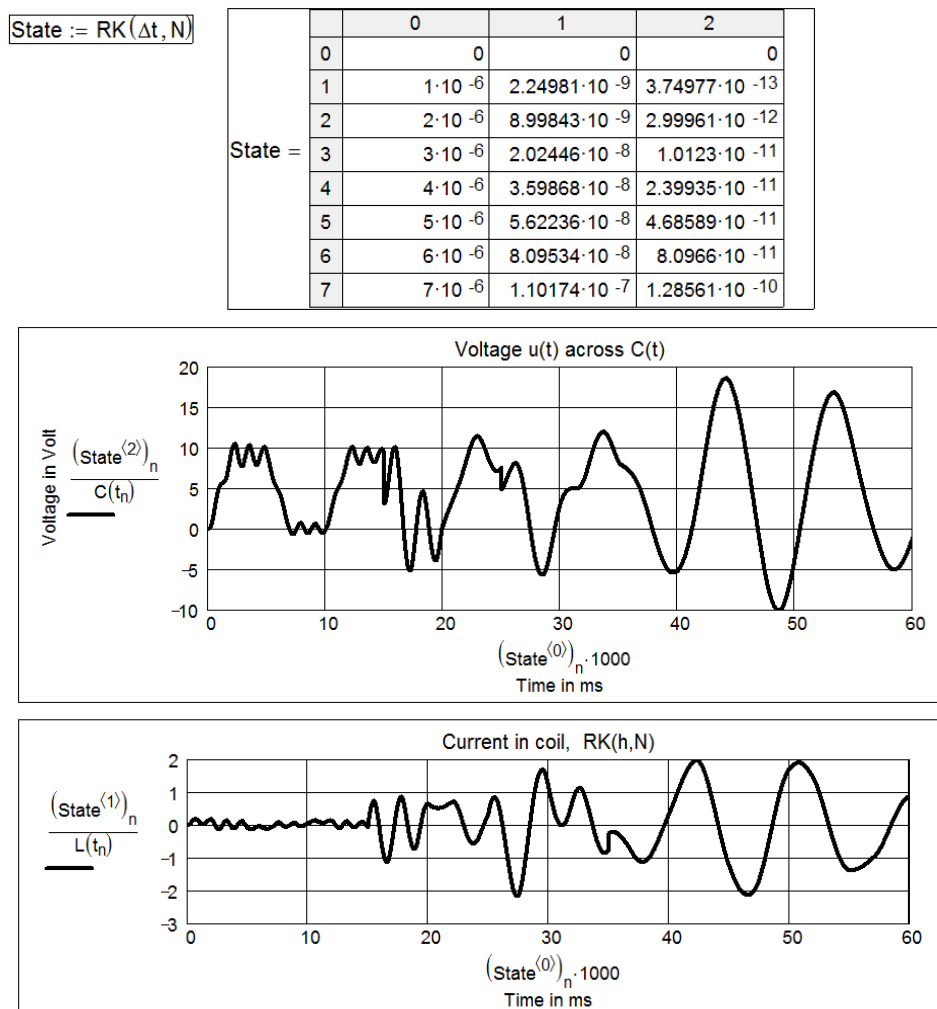


Fig. 6. Voltage across the capacitor. Current in the coil. Calculations using Mathcad

The results given in Fig. 6 have been compared with those shown in Fig. 3. A consistency of results is found.

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