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FORWARD KINEMATICS ALGORITHM FOR ANTHROPOMORPHIC MANIPULATORS

Abstract. The paper presents the problem of forward kinematics of an anthropomorphic manipulators. The proposed forward kinematics algorithm based on equations of classical mechanics. The sample simulations have been done for four degrees of freedom manipulator mounted on the Martian rover. The correctness of the proposed algorithm has been verified with the help of results obtained using Denavit-Hartenberg notation. The shown derivations are basis to consider of dynamics problem. The presented algorithm can be also used for other devices having only rotational joints.
Keywords: algorithm, anthropomorphic manipulator, forward kinematics, DH notation.

ALGORYTM ROZWIĄZYWANIA KINEMATYKI PROSTEJ MANIPULATORÓW ANTROPOMORFICZNYCH

Streszczenie. W pracy przedstawiono zagadnienie kinematyki prostej manipulatorów antropomorficznych. Zaproponowany algorytm rozwiązywania kinematyki prostej sformułowano, bazując na metodzie wykorzystującej równania mechaniki klasycznej. Przedstawiono przykładowe wyniki symulacyjne dla manipulatora o czterech stopniach swobody zamontowanego na łaziku marsjańskim. Działanie zaproponowanego algorytmu sprawdzono, porównując uzyskane wyniki z rezultatami otrzymanymi z wykorzystaniem notacji Denavita-Hartenberga. Otrzymane parametry ruchu manipulatora stanowią podstawę do rozważań zagadnień dynamicznych. Zaprezentowany algorytm może być wykorzystywany także do innych urządzeń posiadających tylko przeguby obrotowe.
Słowa kluczowe: algorytm, manipulator antropomorficzny, kinematyka prosta, notacja DH.

Introduction

Motion analysis of manipulators without taking into account theirs physical features and forces acting on them, is the first stage of full robots movement description. Together with the technological development, the issue of analysis of kinematic systems is continuously improving. Currently, the appropriate conventions of calculation [10] are used to describe of this problem. In the case of manipulators, the Denavit-Hartenberg (DH) notation or its modified versions [2-5, 11] are mostly applied.

The paper presents alternative approach which based on the equations of classical mechanics. This approach was used inter alia to study of rotary cranes [1, 8] and it has been described, in more detail, in books [12, 13]. The advantage of the proposed approach is the possibility of directly connection of kinematics analysis with dynamics analysis of examined objects.

The paper focuses on solution of forward kinematics of anthropomorphic (equipped only with rotary joints) manipulator and working out of the algorithm to determination of coordinates and orientation of gripping device of manipulator. The proposed algorithm can be used for anthropomorphic manipulator with any number of robot arms. Using the presented algorithm, the sample numerical calculations have been carried out for real manipulator [7] mounted on the Martian rover. Moreover the obtained results were confronted with the numerical results obtained on the basis of DH method [2].

The algorithm for determination the forward kinematics of the anthropomorphic manipulators

In the Figure 1 the kinematics scheme of any anthropomorphic manipulator is presented. In the computational model, it has been assumed that: the members of manipulator have infinite rigidity, the axes of local Cartesian coordinate systems are parallel regarding to each other. On this base it can be stated that each relative motion of the system is the translational motion [8, 12]. The motion of the tip of manipulator is the relative motion, therefore it is necessary to determine coordinates of each characteristic point (O_1, O_2, …, O_n) and make their proper summation.

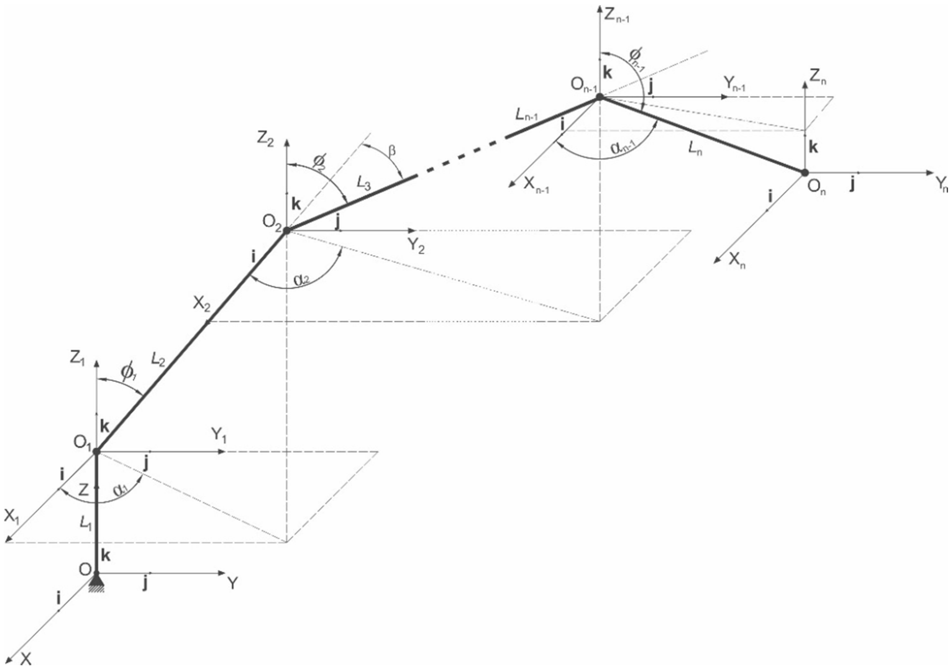

Fig. 1. Kinematics scheme of anthropomorphic manipulator for any number of robot arms

The position of points in Cartesian coordinate system can be written using vector:

$$\mathbf{l}_m = X_m\mathbf{i} + Y_m\mathbf{j} + Z_m\mathbf{k}, \tag{1}$$

and generalized coordinates in curvilinear coordinate system:

$$q^1, q^2, q^3, \tag{2}$$

where m is the characteristic point number of the anthropomorphic manipulator.

By differentiation of the equation with time:

$$\mathbf{l}_m = \mathbf{l}_m\left(q^1, q^2, q^3\right), \tag{3}$$

relation between velocities in Cartesian coordinate system and curvilinear coordinate system is obtained:

$$\mathbf{v}_m = \frac{\partial \mathbf{l}_m}{\partial q^i}\dot{q}^i. \tag{4}$$

Denote [8]:

$$A_{ki} = \frac{\partial b_k}{\partial q^i}, \tag{5}$$

it is possible to get the relation:

$$\dot{b}_k = A_{ki}\,\dot{q}^i, \tag{6}$$

$$\dot{q}^i = B^{ik}\,\dot{b}_k, \tag{7}$$

where:

B – inverse matrix to **A,** $\dot{q}^i$ – components of the velocity vector.

If **A** is nonsingular matrix, equation (7) can be solved relative to generalized velocities [12]. The velocities have different titers, because the generalized coordinates have also different titers. It is therefore necessary to project velocities to directions of generalized coordinates using unit vectors [6, 8, 12]:

$$\mathbf{e}_j = \frac{1}{\left|\frac{\partial \mathbf{l}_m}{\partial q^j}\right|}\frac{\partial \mathbf{l}_m}{\partial q^j}, \qquad \left|\frac{\partial \mathbf{l}_m}{\partial q^j}\right| = \sqrt{\sum_{i=1}^{3}\left(\frac{\partial b_i}{\partial q^j}\right)^2}. \tag{8}$$

Physical components of velocity in curvilinear coordinate system can be defined as functions of coordinate velocity vector X, Y and Z in the Cartesian coordinate system.

Analogical to velocities, physical components of accelerations can be determined by multiplication velocity vector by unit vector [8, 12]:

$$\tilde{\tilde{a}}_j = \left\{\frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}^j}\left(\frac{1}{2}v^2\right)\right] - \frac{\partial}{\partial q^j}\left(\frac{1}{2}v^2\right)\right\}\cdot\frac{1}{\left|\frac{\partial \mathbf{l}_m}{\partial q^j}\right|}, \tag{9}$$

where:

$$v^2 = g_{lr}\dot{q}^l\dot{q}^r, \tag{10}$$

$$g_{lr} = \delta_{ik}\frac{\partial x_i}{\partial q^l}\frac{\partial x_k}{\partial q^r}, \tag{11}$$

δ_{ik} - Kronecker delta.

In analogical way, it is possible to determine relative motion of arbitrary point for anthropomorphic manipulators.

Analyzing absolute motion it is necessary to remember, that motion of particular point relative to fixed coordinate system is translational motion. Displacement of gripper tip of the manipulator can be defined by the equation:

$$\mathbf{l}_n^* = \mathbf{l}_1 + \mathbf{l}_2 + \ldots + \mathbf{l}_n \ , \tag{12}$$

and velocity as well as acceleration vectors of gripper tip of the manipulator have the following form:

$$\mathbf{v}_n^* = \mathbf{v}_1 + \mathbf{v}_2 + \ldots + \mathbf{v}_n \ , \tag{13}$$

$$\mathbf{a}_n^* = \mathbf{a}_1 + \mathbf{a}_2 + \ldots + \mathbf{a}_n \ , \tag{14}$$

where n is a number of characteristic points of the anthropomorphic manipulator.

Taking into account above derivations, an algorithm (Fig. 2) which allows to determine the motion parameters of the gripping device of any anthropomorphic manipulator has been formulated.

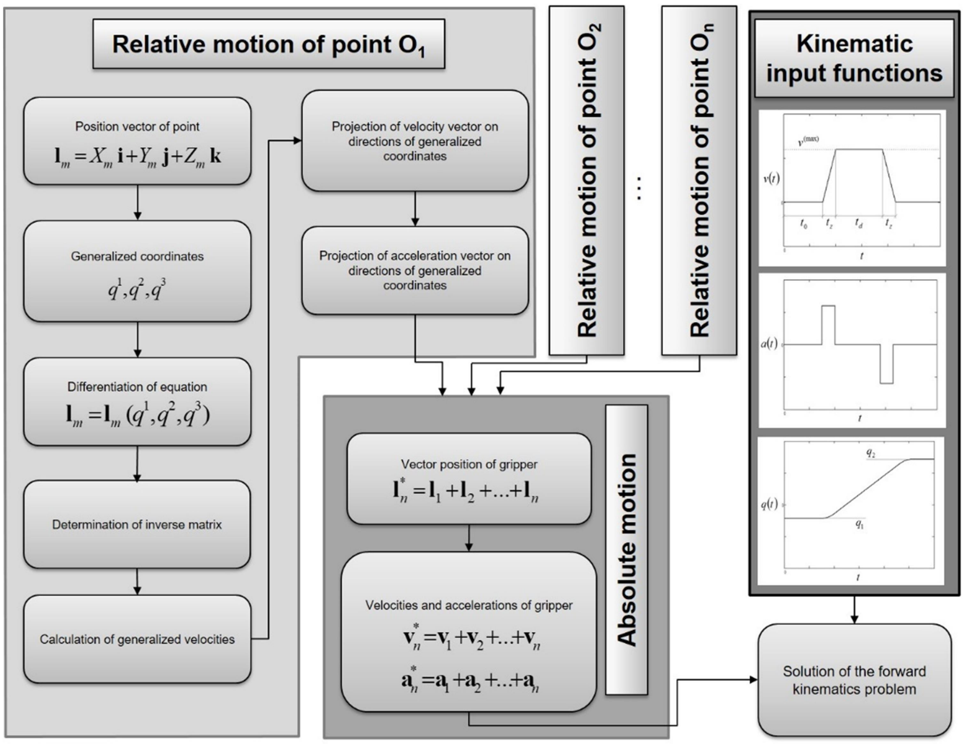

Fig. 2. Solution of forward kinematics problem - block diagram

Sample numerical results

On the basis of proposed algorithm, the sample numerical calculations have been done for the manipulator mounted on the Martian rover [7] (Fig. 3).

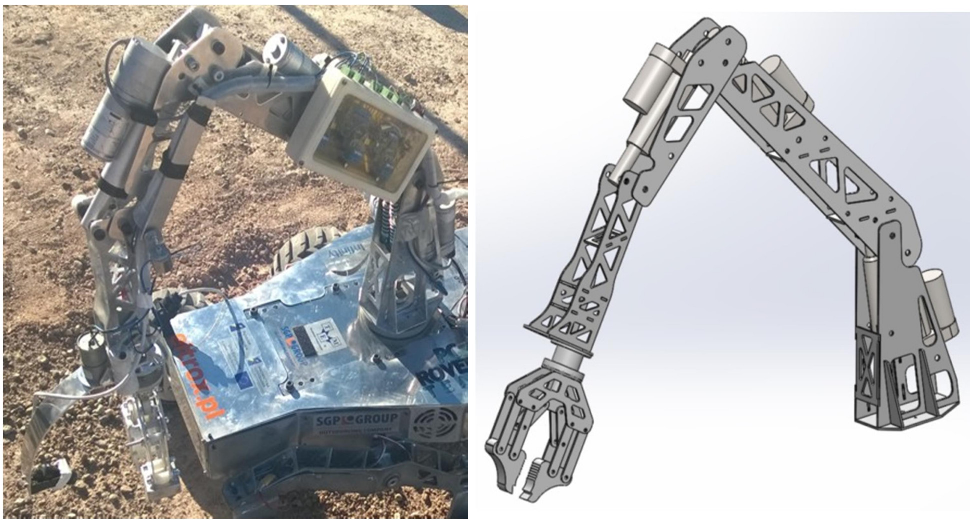

Fig. 3. Manipulator mounted on the Martian rover: real object (on the left), geometrical model (on the right) [7]

The considered system has four degrees of freedom (fig. 4) (the movement of gripper is not taken into account) and assumed that:

- the upper arms of the manipulator rotate together with the base column:

$$\alpha_1 = \alpha_2 = \ldots = \alpha_n = \alpha(t) + \pi \,, \qquad (26)$$

- the arms of the manipulator have constant length: L_1=57mm, L_2=300mm, L_3=280mm, L_4=150mm.

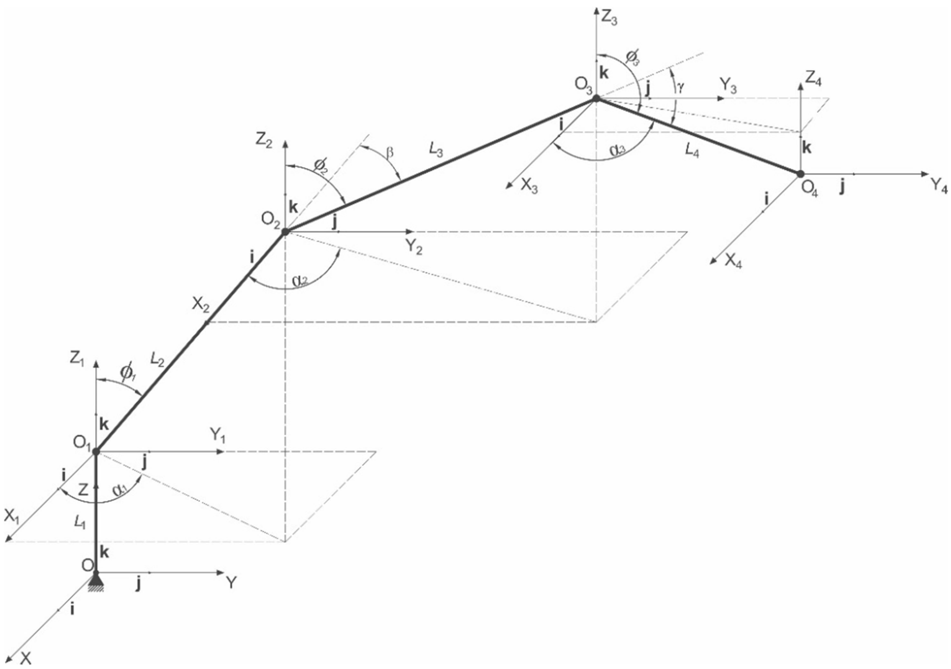

Fig. 4. Kinematic scheme of analyzed manipulator

The components of position, velocities and accelerations for tip of the manipulator have the following form (based on the presented algorithm):

- position components:

$$l_X^* = [L_2 \sin(\phi_1) + L_3 \sin(\phi_2) + L_4 \sin(\phi_3)]\cos(\alpha), \qquad (15)$$

$$l_Y^* = [L_2 \sin(\phi_1) + L_3 \sin(\phi_2) + L_4 \sin(\phi_3)]\sin(\alpha), \qquad (16)$$

$$l_Z^* = L_1 + L_2 \cos(\phi_1) + L_3 \cos(\phi_2) + L_4 \cos(\phi_3), \qquad (17)$$

- velocity components:

$$\begin{aligned} v_X^* = {} & v_{L_2} \cos(\alpha)\sin(\phi_1) - v_\alpha \sin(\alpha) + v_{\phi_1} \cos(\alpha)\cos(\phi_1) + \\ & + v_{L_3} \cos(\alpha)\sin(\phi_2) - v_\alpha \sin(\alpha) + v_{\phi_2} \cos(\alpha)\cos(\phi_2) + \\ & + v_{L_4} \cos(\alpha)\sin(\phi_1) - v_\alpha \sin(\alpha) + v_{\phi_3} \cos(\alpha)\cos(\phi_3), \end{aligned} \qquad (18)$$

$$
\begin{aligned}
v_Y^* &= v_{L_2} \sin(\alpha)\sin(\phi_1) - v_\alpha \cos(\alpha) + v_{\phi_1} \sin(\alpha)\cos(\phi_1) + \\
&+ v_{L_3} \sin(\alpha)\sin(\phi_2) - v_\alpha \cos(\alpha) + v_{\phi_2} \sin(\alpha)\cos(\phi_2) + \\
&+ v_{L_4} \sin(\alpha)\sin(\phi_1) - v_\alpha \cos(\alpha) + v_{\phi_3} \sin(\alpha)\cos(\phi_3),
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
v_Z^* &= v_{L_2} \cdot \cos(\phi_1) - v_{\phi_1} \cdot \sin(\phi_1) + v_{L_3} \cdot \cos(\phi_2) + \\
&- v_{\phi_2} \cdot \sin(\phi_2) + v_{L_4} \cdot \cos(\phi_4) - v_{\phi_3} \cdot \sin(\phi_3),
\end{aligned}
\tag{20}
$$

- acceleration components:

$$
\begin{aligned}
a_X^* &= a_{L_2} \cos(\alpha_1)\sin(\phi_1) - a_{\alpha_1} \sin(\alpha_1) + a_{\phi_1} \cos(\alpha_1)\cos(\phi_1) + \\
&+ a_{L_3} \cos(\alpha_2)\sin(\phi_2) + a_{\alpha_2} \sin(\alpha_2) + a_{\phi_2} \cos(\alpha_2)\cos(\phi_2) + \\
&+ a_{L_4} \cos(\alpha_3)\sin(\phi_3) - a_{\alpha_3} \sin(\alpha_3) + a_{\phi_3} \cos(\alpha_3)\cos(\phi_3),
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
a_Y^* &= a_{L_2} \sin(\phi_1)\sin(\alpha_1) + a_{\alpha_1} \cos(\alpha_1) + a_{\phi_1} \cos(\phi_1)\sin(\alpha_1) + \\
&+ a_{L_3} \sin(\phi_2)\sin(\alpha_2) + a_{\alpha_2} \cos(\alpha_2) + a_{\phi_2} \cos(\phi_2)\sin(\alpha_2) + \\
&+ a_{L_4} \sin(\phi_3)\sin(\alpha_3) + a_{\alpha_3} \cos(\alpha_3) + a_{\phi_3} \cos(\phi_3)\sin(\alpha_3),
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
a_Z^* &= a_{L_2} \cos(\phi_1) - a_{\phi_1} \sin(\phi_1) + a_{L_3} \cos(\phi_2) - a_{\phi_2} \sin(\phi_2) + \\
&+ a_{L_4} \cos(\phi_3) - a_{\phi_3} \sin(\phi_3),
\end{aligned}
\tag{23}
$$

where:

$$\phi_2 = \beta + \phi_1, \tag{24}$$

$$\phi_3 = \gamma + \phi_2. \tag{25}$$

The motion of arms is controlled by the linear induction motors, but the plane motion is turned into the rotary motion. Therefore, in the presented numerical calculations, the kinematic input functions (Fig. 5) concern the angular velocities.

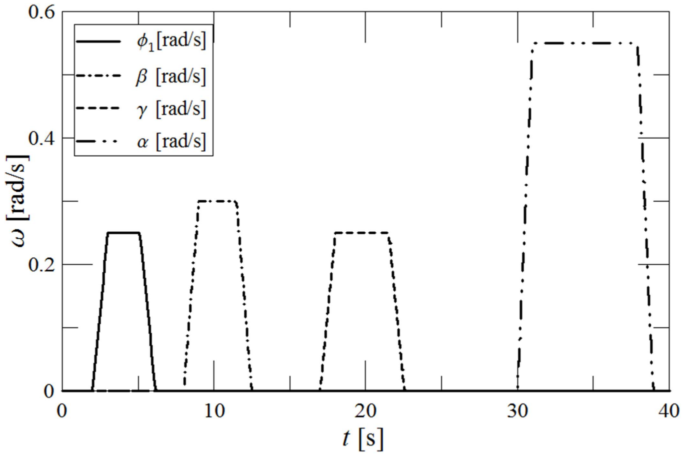

Fig. 5. Kinematic input functions (control signals)

The gripper trajectory (Fig. 6), velocity and acceleration components (Fig. 7, 8) have been determined with the help of formulae (15-23), kinematic input functions and the following initial positions: $\alpha=0^\circ$, $\phi_1=15^\circ$, $\beta=0^\circ$, $\gamma=0^\circ$.

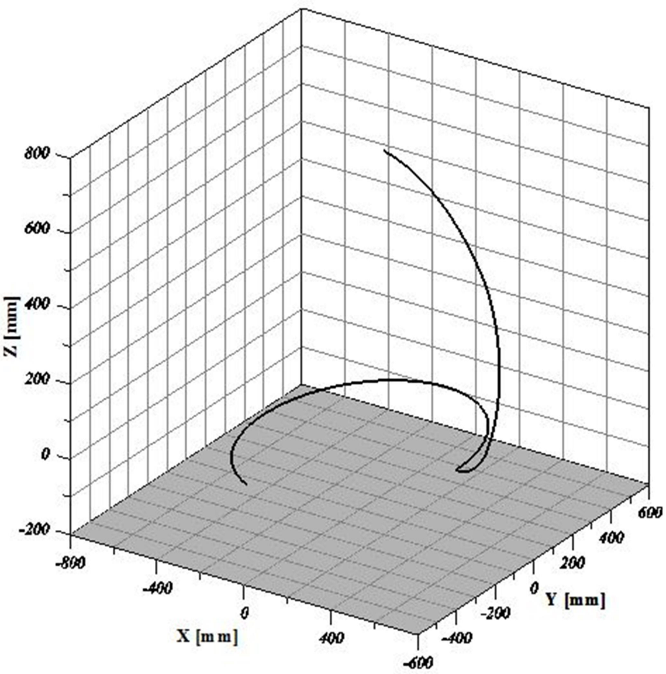

Fig. 6. Motion trajectory of the tip of the analyzed manipulator in the global coordinate system

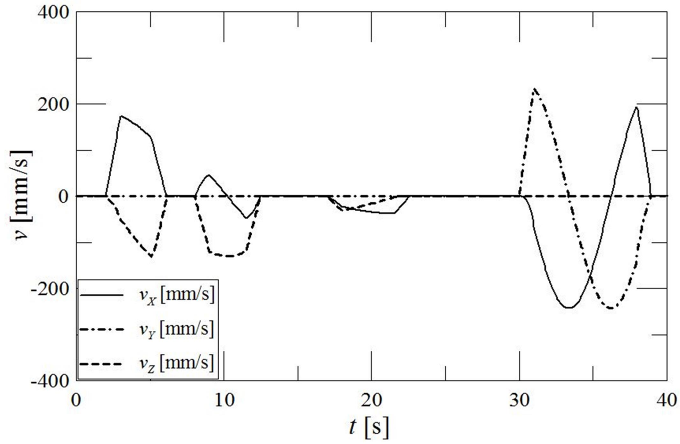

Fig. 7. Velocity of the tip of the analyzed manipulator in the global coordinate system

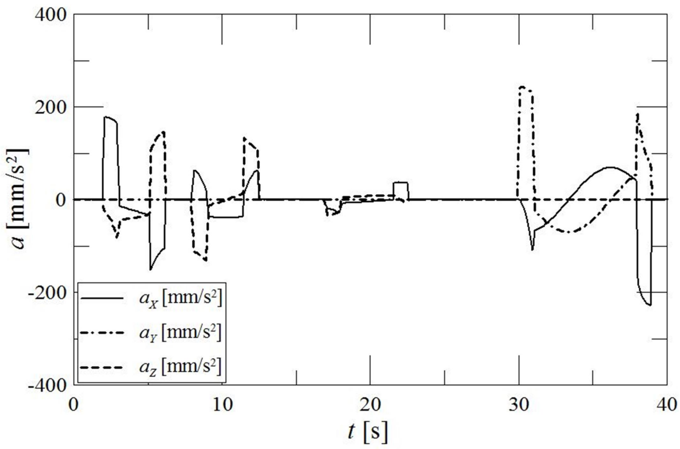

Fig. 8. Acceleration of the tip of the analyzed manipulator in the global coordinate system

Verification of the proposed algorithm

To check correctness of the proposed algorithm the forward kinematics problem using DH parameters [2, 3, 5, 14] for presented case has been solved. In Table 1 the DH kinematics parameters for the considered anthropomorphic manipulator (Fig. 4) are presented. The following denotations: θ_i – joint angle rotation, a_i – length of element, α_i – torsion of element, d_i – offset of joint [4] have been introduced.

Table 1. DH parameters for the analyzed manipulator

Member number	θ_i	d_i	a_i	α_i
1	α	l_1	0	90°
2	φ_1	0	l_2	0
3	β	0	l_3	0
4	γ	l_4	0	– 90°

The determined transformation matrices between individual coordinate systems in the matrix product form create main transformation matrix of the system. On account of the fact that position of the gripping device of manipulator can be described with coordinates of point O_n in effector coordinate system, it can be obtained [9,11]:

$$\begin{bmatrix} \mathrm{l}_X \\ l_Y \\ l_Z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\alpha)[L_2\cos(\phi_1)+L_3\cos(\phi_1+\beta)+L_4\cos(\phi_1+\beta+\gamma)] \\ \sin(\alpha)[L_2\cos(\phi_1)+L_3\cos(\phi_1+\beta)+L_4\cos(\phi_1+\beta+\gamma)] \\ L_1+L_2\sin(\phi_1)+L_3\sin(\phi_1+\beta)+L_4\sin(\phi_1+\beta+\gamma) \\ 1 \end{bmatrix} \tag{28}$$

Finally the coordinates of the gripper have the following form:

$$l_X = \cos(\alpha)[L_2\cos(\phi_1)+L_3\cos(\phi_1+\beta)+L_4\cos(\phi_1+\beta+\gamma)] \tag{29}$$

$$l_Y = \sin(\alpha)[L_2\cos(\phi_1)+L_3\cos(\phi_1+\beta)+L_4\cos(\phi_1+\beta+\gamma)] \tag{30}$$

$$l_Z = L_1+L_2\sin(\phi_1)+L_3\sin(\phi_1+\beta)+L_4\sin(\phi_1+\beta+\gamma) \tag{31}$$

The position components of the tip of manipulator for the selected time moments get on the basis of the elaborated algorithm and DH parameters are compared in Table 2.

Table 2. Comparison of coordinates obtained using the proposed algorithm and DH parameters

Time [s]	Coordinates determined using the proposed algorithm			Coordinates determined using DH parameters		
	X	Y	Z	X	Y	Z
2	111,3	0	772,4	111,3	0	772,4
4	365,6	0	681,9	365,6	0	681,9
8	584,9	0	484,1	584,9	0	484,1
12	592,0	0	67,7	592,0	0	67,7
18	574,2	0	38,5	574,2	0	38,5
30	441,6	0	-20,3	441,6	0	-20,3
34	-153,2	414,1	-20,3	-153,2	414,1	-20,3
40	-151,0	-414,9	-20,3	-151,0	-414,9	-20,3

Based on the showed results (Table 2), the compatibility of the formulated algorithm can be confirmed.

Summary

The paper presents the forward kinematics algorithm for anthropomorphic manipulators with any number of robot arms. Motion parameters have been determined using classical mechanics equations. The proposed method, in contrast to the Denavit-Hartenberg (DH) notation, allows to determine except for position, the velocity and acceleration of the tip of manipulator. These parameters are essential to solution of dynamics problem.

Applying the presented algorithm, the computer calculations for the manipulator mounted on the Martian rover, have been conducted. Position, velocity and acceleration of the gripping device of manipulator have been determined. Moreover the correctness of the algorithm have been verified with the help of DH parameters. The presented algorithm can be used for any anthropomorphic manipulators as well as other devices having only rotational joints. The presented derivations will be the basis for the solution of manipulator dynamics.

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