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TWO-DIMENSIONAL ANALYSIS OF FUNCTIONALLY GRADED THERMOELASTIC MICROELONGATED SOLID

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The present research focuses on two-dimensional deformation in a functionally graded thermoelastic micro-elongated medium. It is supposed that the non-homogeneous properties (thermal and mechanical) of FGM are in the x-direction. The normal mode technique is used to acquire the analytic expression for displacement components, stress, micro-elongation and temperature. The cause and effect relationship of non-homogeneity and physical quantities is shown through graphical results.

Keywords: functionally graded, thermoelastic, normal mode analysis, micro-elongation.

1. Introduction

Biot [1] developed the coupled theory of thermoelasticity to study the weakness of the uncoupled theory according to which the elastic deformation does not affect the temperature. To modify the classical coupled and uncoupled theory of thermoelasticity, various researchers developed generalized theories of thermoelasticity. The micro-elongated medium can be categorised into porous media with gaseous pores, or with non-viscous fluid pores in the medium, solid-liquid crystals, composite materials with chopped elastic fibers. In the theory of micro-elongation with classical deformation medium, micro-elongation of the material particles was found to be volumetric. The material points of the deformation medium contract and stretch independently. The nonlinear theory of micro-elastic solids was elaborated by Eringen and Suhubi [2,3]. Furthermore, Eringen [4,5,6] described in its linear theory of micropolar elasticity, the macro deformation and micro rotations of a material particle in solids. Dhaliwal *et al.* [7] reported the impact of a continuous line heat source on thermoelasticity of isotropes. Sharma and Chauhan [8] also studied the impact of thermal as well as mechanical sources in a thermoelastic half-space in a generalized way. Ailawalia and Singla [9] derived the solution and reported a significant effect of laser pulse heating on all quantities in an immersed thermoelastic micro-elongated layer. Furthermore, Ailawalia*et al.* [10] also reported deformation of plain strain in a thermoelastic micro-elongated solid as an effect of laser pulse heating. In addition to this, the effect of variable heat sources on FGMs was also studied by Shaw and Mukhopadhyay [11].Further consequently, thermoelastic interactions were included in the study by Shaw and Mukhopadhyay [12,13] reporting the effect of moving the heat source on an isotropic micro-elongated solid in a homogenous medium. Aliawalia *et al.* [14] also studied the internal heat source at an interface under the G-L theory. Deswal and Kalkal [15] discussed magneto-thermoelastic interactions in an isotropic, microelongated solid which was stressed initially.

Marin *et al.* [16] dealt with problems associated with thetheoryof double porosity structure in a thermoelasticmicropolar body. Said *et al.* [17] derived a solution to the problem related to thermodynamical interaction in a micropolar rotated magneto-elastic medium using the normal mode technique. Khan *et al.* [18] examined a third-grade magnetohydrodynamic fluid with variable thermal conductivity and chemical reaction over an exponentially stretching surface. Bhatti *et al.* [19] presented a theory based on the flow of

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nanoparticles and migratory gyrotactic microorganisms in a non-Newtonian blood-based nanofluid via an artery which was anisotropically tapering.

A report by Reddy and Chin [20] used Lagrangian finite element formulations. Transient thermal stress in a non-homogenous material was discussed by Wang and Mai [21]using the finite element method. This technique was also used to study the electro magneto thermoelastic response in an infinite FGM by Abbas and Zenkour [22]. Aboudi *et al.* [23] observed thermoelastic properties in the functionally graded composite. Radial vibrations under rotation and gravity field in an orthotropic elastic half-space were discussed by Abd-Alla*et al.* [24]. Abbas [25] included one relaxation time in the study of thermoelasticity in a thick-walled FGM. Shankar and Tzeng [26] focused their study on functionally graded beams and found exponential variation with the thickness of the thermoelastic material. Mishra *et al.* [27] studied thermoelastic properties of an annular disk under pressure variations and observed the effect of forced vibrations in a non-homogenous medium. Gunghas *et al.* [28] followed the Green-Naghdi model III to study two-dimensional deformations. Two-dimensional interactions of magnetic as well as thermoelastic properties in a micropo

In the present work, by taking the micro-elongation effect and functionally graded medium, we developed a model for a thermoelastic micro-elongated solid by using the normal mode technique. The thermal and mechanical properties of a functionally graded material vary with an exponential power of the space coordinate. The two-dimensional deformation of a functionally graded thermoelastic micro-elongated solid subjected to mechanical and thermal sources applied along the free surface has been discussed. The analytical expression of displacement components, stress, temperature and micro elongation has been obtained. The numerical calculations are performed using MATLAB software. These numerical results of normal displacements force stress, temperature distribution, and micro-elongation are presented graphically to exhibit the effect of non-homogeneity.

2. Basic equations

Following Shaw and Mukhopadhyay [11], the field equation of motion for a non-homogeneous, isotropic, micro-elongated, thermoelastic solids in the absence of body forces are:

$$\sigma_{kl} = \lambda\delta_{kl}u_{r,r} + \mu(u_{k,l} + u_{l,k}) - \beta_0\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T\delta_{kl} + \lambda_0\delta_{kl}\phi, \tag{2.1}$$

$$m_k = a_0\phi,_k\,, \tag{2.2}$$

$$s - \sigma = \lambda_0 u_{k,k} - \beta_1\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T - \lambda_1\phi, \tag{2.3}$$

$$q_k = \frac{K^* T_{,k}}{T_0}. \tag{2.4}$$

The equation of motion for displacement, micro elongation, and temperature varies with respect to equations defined by Eringen [31], Kiris and Inan [32] as follows:

$$(\lambda + \mu)\, u_{j,ij} + \mu\, u_{i,ij} - \beta_0\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T_{,i} + \lambda_0\phi_{,i} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2.5}$$

$$a_0\phi,_{ii} + \beta_1\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T - \lambda_1\phi - \lambda_0 u_{j,j} = \frac{1}{2}\rho j_0\frac{\partial^2\phi}{\partial t^2}, \tag{2.6}$$

$$K^*T_{,ii}-\rho\,C^*\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}-\beta_0T_0(1+t_0\delta_{1k}\frac{\partial}{\partial t})\frac{\partial}{\partial t}u_{k,k}-\beta_1T_0\frac{\partial\phi}{\partial t}\;=\;0 \tag{2.7}$$

where $\beta_0=\;(3\lambda+2\mu)\alpha_{t_1}$, $\beta_1=\;(3\lambda+2\mu)\alpha_{t_2}$.

For a non-homogenous medium, the parameters $\beta_1,\beta_0,\mu,\lambda,K^*$ and $\lambda_1,\lambda_0,a_0,\rho$ became space dependent. Here $\beta_{10}f(X)$, $\beta_{00}f(X)$, $\mu_0f(X)$, $\lambda_0f(X)$, $K_0^*f(X)$, $\lambda_{10}f(X)$, $\lambda_{00}f(X)$, $a_{00}f(X)$, $\rho_0f(X)$, respectively, replace $\beta_1,\beta_0,\mu,\lambda,K^*,\lambda_1,\lambda_0,a_0,\rho$ with constant values as β_{10}, β_{00}, μ_0, λ_0,K_0^*, λ_{10}, $\lambda_{00},a_{00},\rho_0$ and $f(X)$ is a given non-dimensional space variable $X=(x,y,z)$. It is supposed that properties of materials are dependent on the x coordinate, and notation of $f(X)$ changes into $f(x)$, therefore field Eqs (2.5)-(2.7) become:

$$\begin{aligned}&f(x)\left[(\lambda_0+2\mu_0)\frac{\partial^2u_1}{\partial x^2}+(\lambda_0+\mu_0)\frac{\partial^2u_2}{\partial x\partial y}+\mu_0\frac{\partial^2u_1}{\partial y^2}-\beta_{00}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}+\lambda_{00}\frac{\partial\phi}{\partial x}\right]+\\&+\frac{\partial}{\partial x}f(x)\left[(\lambda_0+2\mu_0)\frac{\partial u_1}{\partial x}+\lambda_0\frac{\partial u_2}{\partial y}-\beta_{00}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_{00}\phi\right]=\rho_0f(x)\frac{\partial^2u_1}{\partial t^2},\end{aligned} \tag{2.8}$$

$$\begin{aligned}&\mu_0f(x)\left[\frac{\partial^2u_1}{\partial x\partial y}+\frac{\partial^2u_2}{\partial x^2}\right]+\mu_0\frac{\partial}{\partial x}f(x)\left[\frac{\partial u_1}{\partial y}+\frac{\partial u_2}{\partial x}\right]+f(x)\left[\lambda_0\frac{\partial^2u_1}{\partial x\partial y}+(\lambda_0+2\mu_0)\frac{\partial^2u_2}{\partial y^2}+\right.\\&\left.-\beta_{00}(1+t_1\delta_{2k}\frac{\partial}{\partial t})\frac{\partial T}{\partial y}+\lambda_{00}\frac{\partial\phi}{\partial y}\right]=\rho_0f(x)\frac{\partial^2u_1}{\partial t^2},\end{aligned} \tag{2.9}$$

$$\begin{aligned}&a_{00}\left[f(x)\nabla^2\phi+\frac{\partial}{\partial x}f(x)\frac{\partial\phi}{\partial x}\right]+f(x)\beta_{10}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\\&-\lambda_{10}f(x)\phi-f(x)\lambda_{00}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)=\frac{1}{2}\rho_0f(x)j_0\frac{\partial^2\phi}{\partial t^2},\end{aligned} \tag{2.10}$$

$$\begin{aligned}&K_0^*\left[f(x)\nabla^2T+\frac{\partial}{\partial x}f(x)\frac{\partial T}{\partial x}\right]-\rho_0C^*f(x)\left(1+t_0\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}+\\&-\beta_{00}T_0f(x)\left(\frac{\partial}{\partial t}+t_0\delta_{2k}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)-\beta_{10}T_0f(x)\frac{\partial\phi}{\partial t}=0.\end{aligned} \tag{2.11}$$

The constitutive components of microelongational stress tensors are given by:

$$\sigma_{xx}=f(x)\left[(\lambda_0+2\mu_0)\frac{\partial u_1}{\partial x}+\lambda_0\frac{\partial u_2}{\partial y}-\beta_{00}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_{00}\phi\right], \tag{2.12}$$

$$\sigma_{yy}=f(x)\left[\lambda_0\frac{\partial u_1}{\partial x}+(\lambda_0+2\mu_0)\frac{\partial u_2}{\partial y}-\beta_{00}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_{00}\phi\right], \tag{2.13}$$

$$\sigma_{xy}=f(x)\mu_0\left(\frac{\partial u_1}{\partial y}+\frac{\partial u_2}{\partial x}\right). \tag{2.14}$$

3. Exponential variation of non-homogeneity

An assumption is made about $f(x) = e^{-nx}$ where n is a non-dimensional parameter, it can be concluded that materials having mechanical as well as thermal properties alter exponentially along the x-direction.

The commanding Eqs (2.8)-(2.11) and the stress Eqs (2.12)-(2.14) can be modified in the non-dimensional form by establishing the non-dimensional parameters:

$$x' = \frac{\omega^*}{c_1}x,\ y' = \frac{\omega^*}{c_1}y,\ t_0' = \omega^* t_0,\ T' = \frac{T}{T_0},\ t' = \omega^* t,\ u_i' = \frac{\omega^* \rho_0 c_1}{\beta_{00} T_0} u_i,$$

$$\phi' = \frac{\lambda_{00}}{\beta_{00} T_0}\phi,\ \sigma_{ij}' = \frac{\sigma_{ij}}{\beta_{00} T_0},\ \omega^* = \frac{\rho_0 c_1^2 C^*}{K^*},\ c_1^2 = \frac{\lambda_0 + 2\mu_0}{\rho_0}.$$

Using the above mentioned dimensionless variables in Eqs (2.8)-(2.11), after dropping the subscript, we get:

$$\begin{aligned}&\left[\frac{\partial^2 u_1}{\partial x^2} + l_2 \frac{\partial^2 u_2}{\partial x \partial y} + l_3 \frac{\partial^2 u_1}{\partial y^2} - \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} + \frac{\partial \phi}{\partial x}\right] + \\ &-n\left[\frac{\partial u_1}{\partial x} + l_4 \frac{\partial u_2}{\partial y} - \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right)T + \phi\right] = \frac{\partial^2 u_1}{\partial t^2},\end{aligned} \tag{3.1}$$

$$\left[l_3 \frac{\partial^2 u_2}{\partial x^2} + l_2 \frac{\partial^2 u_1}{\partial x \partial y} + \frac{\partial^2 u_2}{\partial y^2} - \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y} + \frac{\partial \phi}{\partial y}\right] - n l_3 \left[\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}\right] = \frac{\partial^2 u_2}{\partial t^2}, \tag{3.2}$$

$$\left[\nabla^2 \phi - n \frac{\partial \phi}{\partial x}\right] + l_5 \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right) T - l_6 \phi - l_7 \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}\right) = l_8 \frac{\partial^2 \phi}{\partial t^2}, \tag{3.3}$$

$$\left[\nabla^2 T - n \frac{\partial T}{\partial x}\right] - l_9 \left(1 + t_0 \delta_{1k} \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} - l_{10} \left(\frac{\partial}{\partial t} + t_0 \delta_{1k} \frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}\right) - l_{11} \frac{\partial \phi}{\partial t} = 0, \tag{3.4}$$

$$\sigma_{xx} = \left[\frac{\partial u_1}{\partial x} + l_4 \frac{\partial u_2}{\partial y} - a_2 T + \phi\right] e^{-nx}, \tag{3.5}$$

$$\sigma_{yy} = \left[l_4 \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} - a_2 T + \phi\right] e^{-nx}, \tag{3.6}$$

$$\sigma_{xy} = l_3 \left[\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right] e^{-nx} \tag{3.7}$$

where

$$l_2 = \frac{\lambda_0 + \mu_0}{\rho_0 c_1^2},\ l_3 = \frac{\mu_0}{\rho_0 c_1^2},\ l_4 = \frac{\lambda_0}{\rho_0 c_1^2},\ l_5 = \frac{\lambda_{00}\beta_{10} c_1^2}{\beta_{00}\omega^{*2}},\ l_6 = \frac{\lambda_{10} c_1^2}{\omega^{*2}},$$

$$l_7 = \frac{\lambda_{00}^2}{\omega^{*2}\rho_0},\ l_8 = \frac{1}{2}\rho_0 j_0 c_1^2,\ l_9 = \frac{c^* c_1^2}{K_0^* \omega^*},\ l_{10} = \frac{\beta_{00} T_0}{\rho_0 K_0^* \omega^{*2}},\ l_{11} = \frac{\beta_{10}\beta_{00} T_0 c_1^2}{\lambda_{00} K_0^* \omega^*}.$$

4. Solution of the problem

To find the solution of physical variables in the above equations, normal mode analysis is used as follows:

$$(u_i, T, \phi, \sigma_{ij})(x, y, t) = (u_i^*, T^*, \phi^*, \sigma_{ij}^*)(x) e^{\omega t + \iota b y} \tag{4.1}$$

where ω is the complex frequency, b defines the wave number in the y-direction, and $u_i^*, T^*, \phi^*, \sigma_{ij}^*$ are the amplitudes of the field quantities.
Using Eq.(6.1) in Eqs (3.1)-(3.7), we get

$$(D^2 - nD - a_1)u_1^* + \iota b(l_2 D - n l_4)u_2^* - a_2(D - n)T^* + (D - n)\phi^* = 0\ , \tag{4.2}$$

$$\iota b(l_2 D - n l_3)u_1^* + (l_3 D^2 - n l_3 D - a_3)u_2^* - a_2 \iota b T^* + \iota b \phi^* = 0, \tag{4.3}$$

$$-l_7 D u_1^* - l_7 \iota b u_2^* + l_5 a_2 T^* + (D^2 - nD - a_4)\phi^* = 0, \tag{4.4}$$

$$-l_{10} a_5 D u_1^* - \iota b l_{10} a_5 u_2^* + (D^2 - nD - a_6)T^* - l_{11}\omega\phi^* = 0, \tag{4.5}$$

$$\sigma_{xx} = D u_1^* + l_4 \iota b u_2^* - a_2 T^* + \phi^*, \tag{4.6}$$

$$\sigma_{yy} = l_4 D u_1^* + \iota b u_2^* - a_2 T^* + \phi^*, \tag{4.7}$$

$$\sigma_{xy} = l_3\left(\iota b u_1^* + D u_2^*\right) \tag{4.8}$$

where

$$D = \frac{d}{dx},\ a_1 = l_3 b^2 + \omega^2,\ a_2 = \left(1 + t_1 \delta_{2k}\omega\right),\ a_3 = b^2 + \omega^2,$$

$$a_4 = b^2 + l_6 + l_8\omega^2,\ a_5 = \omega\left(1 + t_1\delta_{2k}\omega\right),\ a_6 = b^2 + l_9\left(1 + t_0\delta_{1k}\omega\right).$$

In order to find the differential equation for $u_1^*(x)$ eliminating $u_2^*(x)$, $T^*(x)$, $\phi^*(x)$ from Eqs (4.2)-(4.5), we get

$$(D^8 + nF_1 D^7 + F_2 D^6 + nF_3 D^5 + F_4 D^4 + nF_5 D^3 + F_6 D^2 + nF_7 D + F_8)u_1^* = 0 \tag{4.9}$$

where

$$F_1 = \frac{-(r_2 r_{12} + r_1 r_{12} - r_{16} l_3 + 2 l_3 r_{15})}{l_3 l_{11} \omega},$$

$$F_2 = \frac{-(r_3 r_{12} - n^2 r_2 r_{12} + r_1 r_{21} - r_{17} l_3 - 2n^2 l_3 r_{16} - r_{15} r_4)}{l_3 l_{11} \omega},$$

$$F_3 = \frac{(r_3 r_{12} + r_2 r_{21} - r_1 r_{22} - r_{18} l_3 + 2 l_3 r_{17} - r_{16} r_4 + r_{15} r_5)}{l_3 l_{11} \omega},$$

$$F_4 = \frac{-(r_3 r_{21} - n^2 r_2 r_{22} + r_1 r_{23} + r_{19} l_3 - 2n^2 l_3 r_{18} + r_{17} r_4 + n^2 r_{16} r_5)}{l_3 l_{11} \omega},$$

$$F_5 = \frac{(r_3 r_{22} + r_2 r_{23} - r_1 r_{24} - r_{20} l_3 + 2 l_3 r_{19} - r_{18} r_4 - r_{17} r_5)}{l_3 l_{11} \omega},$$

$$F_6 = \frac{-(r_3 r_{23} - n^2 r_2 r_{24} + r_1 r_{25} - 2n^2 l_3 r_{20} + r_{19} r_4 + n^2 r_{18} r_5)}{l_3 l_{11} \omega},$$

$$F_7 = \frac{(r_3 r_{24} + r_2 r_{25} - r_{20} r_4 - r_{19} r_5)}{l_3 l_{11} \omega}, \; F_8 = \frac{(r_3 r_{25} - n^2 r_5 r_{20})}{l_3 l_{11} \omega},$$

$$r_1 = (1 - l_2), \; r_2 = (1 - l_2 + l_3), \; r_3 = (a_1 + n^2 l_3), \; r_4 = (l_2 b^2 - a_3 + n^2 l_3),$$

$$r_5 = (a_3 - b^2 l_4), \; r_6 = l_{10} a_5, \; r_7 = (l_{10} a_5 a_4 - l_7 l_{11} \omega), \; r_8 = n^2 - a_6 - a_4,$$

$$r_9 = a_6 + a_4, \; r_{10} = l_5 l_{11} \omega a_2 + a_4 a_6, \; r_{11} = l_2 l_{11} \omega - l_{10} a_5, \; r_{12} = l_3 l_{11} \omega,$$

$$r_{13} = b^2 l_{10} a_5 - a_3 l_{11} \omega, \; r_{14} = a_2 l_{11} \omega + a_6, \; r_{15} = l_2 l_{11} \omega, \; r_{16} = l_3 l_{11} \omega + 2 l_2 l_{11} \omega,$$

$$r_{17} = a_2 a_5 l_{10} l_{11} \omega - l_7 l_{11} \omega - 2n^2 l_3 l_{11} \omega - \left(n^2 - a_6 - a_4\right) l_2 l_{11} \omega,$$

$$\begin{aligned} r_{18} = {} & a_2 a_5 l_{10} l_{11}{}^2 l_3 \omega^2 - a_2 a_5 l_{10} l_{11} \omega - a_6 a_5 l_{10} - l_3 l_7 l_{11}{}^2 \omega^2 + \\ & - 2n^2 l_3{}^2 l_{11}{}^2 \omega^2 - \left(n^2 - a_6 - a_5\right) l_2 l_3 l_{11}{}^2 \omega^2 - a_6 l_2 l_{11} \omega, \end{aligned}$$

$$\begin{aligned} r_{19} = {} & - a_2 a_5 a_4 l_{10} l_{11} \omega + a_2 l_7 l_{11}{}^2 \omega^2 + a_6 l_7 l_{11} \omega + n^2 l_3 l_{11} a_6 \omega + n^2 l_3 l_{11} a_4 \omega + \\ & - l_5 l_{11}{}^2 l_2 a_2 \omega^2 - a_4 a_6 l_2 l_{11} \omega + a_2 a_5 l_5 l_{10} l_{11} \omega, \end{aligned}$$

$$r_{20} = r_{10} r_{12}, \; r_{21} = b^2 r_6 - r_{13} - 2n^2 r_{12} - r_8 r_{12}, \; r_{22} = -2b^2 r_6 + 2 r_{13} + r_{12} r_8 - r_{12} r_9,$$

$$r_{23} = b^2 r_7 + b^2 n^2 r_6 - b^2 r_6 r_{14} - r_8 r_{13} + n^2 r_{12} r_9 - r_{10} r_{12},$$

$$r_{24} = -b^2 r_7 + b^2 r_6 r_{14} - r_{19} r_{13} + r_{10} r_{12},\; r_{25} = -b^2 r_7 r_{14} - r_{10} r_{13},$$

$$r_{26} = -r_1 r_2 - r_{15} l_3,\;\; r_{27} = r_{12}\left(r_2 + r_1\right) + l_3\left(r_{16} + 2r_{15}\right),$$

$$r_{28} = r_{12}\left(r_3 - n^2 r_2\right) + r_1 r_{21} + l_3\left(r_{17} - 2n^2 r_{16}\right) - r_{15} r_4,$$

$$r_{29} = r_1 r_{22} + l_3\left(r_{18} - 2r_{17}\right) - r_3 r_{12} - r_2 r_{21} + r_4 r_{16} - r_5 r_{15},$$

$$r_{30} = r_1 r_{22} + l_3\left(r_{19} - 2n^2 r_{18}\right) - r_3 r_{21} - n^2 r_2 r_{22} + r_4 r_{17} + n^2 r_{16} r_5,$$

$$r_{31} = r_1 r_{24} + r_{20} l_3 - r_3 r_{22} - r_2 r_{23} - 2l_3 r_{19} + r_4 r_{18} - r_5 r_{17},$$

$$r_{32} = r_1 r_{25} - r_3 r_{23} - n^2 r_2 r_{24} - 2n^2 l_3 r_{20} + r_4 r_{19} + n^2 r_{18} r_5,$$

$$r_{33} = r_{20} r_4 - r_3 r_{24} - r_2 r_{25} + r_5 r_{19},\; r_{34} = n^2 r_5 r_{20} - r_3 r_{25}.$$

The general solution of Eq.(4.9) which is bounded as $x \to \infty$ is given by:

$$u_1^*(x) = \sum_{i=1}^{4} A_i(m,\omega) e^{-k_i x}\ , \tag{4.10}$$

$$u_2^*(x) = \sum_{i=1}^{4} H_{1i} A_i(m,\omega) e^{-k_i x}, \tag{4.11}$$

$$T^*(x) = \sum_{i=1}^{4} H_{2i} A_i(m,\omega) e^{-k_i x}, \tag{4.12}$$

$$\phi^*(x) = \sum_{i=1}^{4} H_{3i} A_i(m,\omega) e^{-k_i x} \tag{4.13}$$

where k_i's $\left(i = 1,2,3,4\right)$ denotes roots of Eq. (4.9) and the parameters $A_i(m,\omega)$ $\left(i = 1,2,3,4\right)$ depend upon ω and m

$$H_{1i} = \frac{-\left[\iota b r_1 k_i^2 + n r_2 k_i - r_3\right]}{l_3 k_i^3 + 2n l_3 k_i^2 + r_4 - n r_5}, \tag{4.14}$$

$$H_{2i} = \frac{-\left[\left(-\iota b r_{11} k_i - n r_{12}\iota b\right) + H_{1i}\left(r_{12} k_i^2 + n r_{12} + r_{13}\right)\right]}{\iota b k_i^2 + \iota b n k_i - \iota b r_{14}}, \tag{4.15}$$

$$H_{3i} = \frac{-l_7 k_i + l_7 \iota b H_{1i} - l_5 a_2 H_{2i}}{k_i + n k_i - a_4} \tag{4.16}$$

In view of solution (4.10)-(4.13), stress components (4.6)-(4.8) become:

$$\sigma_{xx}{}^{*}=\sum_{i=1}^{4}U_iA_i(m,\omega)e^{-k_ix-nx}, \tag{4.17}$$

$$\sigma_{yy}{}^{*}=\sum_{i=1}^{4}V_iA_i(m,\omega)e^{-k_ix-nx}, \tag{4.18}$$

$$\sigma_{xy}{}^{*}=\sum_{i=1}^{4}W_iA_i(m,\omega)e^{-k_ix-nx} \tag{4.19}$$

where

$$\begin{aligned} U_i &= (-k_i+l_4\iota bH_{1i}-a_2H_{2i}+H_{3i}), \\ V_i &= (-k_il_4+\iota bH_{1i}-a_2H_{2i}+H_{3i}), \\ W_i &= l_3(\iota b-k_iH_{1i})\,. \end{aligned} \tag{4.20}$$

5. Boundary conditions

To calculate the values of $A_i\left(i=1,2,3,4\right)$, we use the mechanical and thermal boundary condition at the free surface $x=0$ as:

5.1. Mechanical boundary conditions

a)

$$\sigma_{xx} = -\,Fe^{\omega t+\iota by}, \tag{5.1}$$

b)

$$\sigma_{xy} = 0, \tag{5.2}$$

c)

$$\frac{\partial T}{\partial x}= 0, \tag{5.3}$$

d)

$$\phi=0, \tag{5.4}$$

The above boundary conditions in the form of a non-homogeneous matrix showing non dimensional expression of stress as well as temperature as mentioned above in Eqs (5.1)-(5.4) are shown as follows

$$\begin{bmatrix} U_1 & U_2 & U_3 & U_4 \\ W_1 & W_2 & W_3 & W_4 \\ S_1 & S_2 & S_3 & S_4 \\ Y_1 & Y_2 & Y_3 & Y \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}=\begin{bmatrix} -F \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{5.5}$$

We obtain the values of $A_i\ (i=1,2,3,4)$ from the solution of the system (5.5) as

$$A_i=\frac{\Delta_i}{\Delta}$$

where

$$\Delta=U_1L_1-U_2L_2+U_3L_3-U_4L_4,$$

$$\Delta_i=(-1)^i FL_i,\ (i=1,2,3,4),$$

$$L_1=W_2(S_3Y_4-Y_3S_4)-W_3(S_2Y_4-Y_2S_4)+W_4(S_2Y_3-Y_2S_3),$$

$$L_2=W_1(S_3Y_4-Y_3S_4)-W_3(S_1Y_4-Y_1S_4)+W_4(S_1Y_3-Y_1S_3), \tag{5.6}$$

$$L_3=W_1(S_2Y_4-Y_2S_4)-W_2(S_1Y_4-Y_1S_4)+W_4(S_1Y_2-Y_1S_2),$$

$$L_4=W_1(S_2Y_3-Y_2S_3)-W_2(S_1Y_3-Y_1S_3)+W_3(S_1Y_2-Y_1S_2)$$

where

$$S_1=-H_{21}K_1,\ S_2=-H_{22}K_2,\ S_3=-H_{23}K_3,\ S_4=-H_{24}K_4,$$

$$Y_1=-H_{31},\ Y_2=-H_{32},\ Y_3=-H_{33},\ Y_4=-H_{34}.$$

5.2. Thermal boundary conditions

To determine the constants $A_i\ (i=1,2,3,4)$ the boundary conditions at the free surface $x=0$ are

a)

$$\sigma_{xx}=0, \tag{5.7}$$

b)

$$\sigma_{xy}=0, \tag{5.8}$$

c)

$$\frac{\partial T}{\partial x}=\mathrm{P}e^{\omega t+\iota by}, \tag{5.9}$$

d)

$$\phi=0. \tag{5.10}$$

The above boundary conditions in the form of a non-homogeneous matrix showing non dimensional expression of stress as well as temperature as mentioned above in Eqs (5.7)-(5.10), are shown as follows:

$$\begin{bmatrix} U_1 & U_2 & U_3 & U_4 \\ W_1 & W_2 & W_3 & W_4 \\ M_1 & M_2 & M_3 & M_4 \\ Y_1 & Y_2 & Y_3 & Y \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ P \\ 0 \end{bmatrix}. \tag{5.11}$$

We obtain the values of $A_i\ (i=1,2,3,4)$ from the solution of the system (5.11) as

$$A_i = \frac{\Delta_i^*}{\Delta^*}$$

where

$$\Delta^* = U_1 L_1^* - U_2 L_2^* + U_3 L_3^* - U_4 L_4^*,$$

$$\Delta_i^* = P L_i^*, \ (i = 1,2,3,4)$$

$$L_1^* = W_2(M_3 Y_4 - Y_3 M_4) - W_3(M_2 Y_4 - Y_2 M_4) + W_4(M_2 Y_3 - Y_2 M_3),$$

$$L_2^* = W_1(M_3 Y_4 - Y_3 M_4) - W_3(M_1 Y_4 - Y_1 M_4) + W_4(M_1 Y_3 - Y_1 M_3), \tag{5.12}$$

$$L_3^* = W_1(M_2 Y_4 - Y_2 M_4) - W_2(M_1 Y_4 - Y_1 M_4) + W_4(M_1 Y_2 - Y_1 M_2),$$

$$L_4^* = W_1(M_2 Y_3 - Y_2 M_3) - W_2(M_1 Y_3 - Y_1 M_3) + W_3(M_1 Y_2 - Y_1 M_2),$$

$$S_1 = H_{21}, \ S_2 = H_{22}, \ S_3 = H_{23}, \ S_4 = H_{24}.$$

6. Numerical results

To compute the results numerically, constant values of an aluminum epoxy-like material are taken as in Shaw and Mukhopadhyay [12,13]

$$\lambda = 7.59 \times 10^{10}\,Nm^{-2}, \quad \mu = 1.89 \times 10^{10}\,Nm^{-2}, \quad a_0 = 0.61 \times 10^{-10}\,N,$$

$$\rho = 2.19 \times 10^{3}\,Kgm^{-3}, \quad \beta_1 = \beta_0 = 0.05 \times 10^{5}\,Nm^{-2}K^{-1}, \quad C^* = 966\,JKg^{-1}K^{-1},$$

$$T_0 = 293K, \quad j_0 = 0.196 \times 10^{-4}\,m^2, \quad \lambda_0 = \lambda_1 = 0.37 \times 10^{10}\,Nm^{-2},$$

$$t_0 = 0.02, \quad t_1 = 0.003, \quad K^* = 252\,Jm^{-1}s^{-1}K^{-1}.$$

Assuming $\omega = \omega_0 + \iota\xi$, $\omega_0 = -0.3$, $\xi = 0.2$, *and* $b = 0.3$, the value of non-dimensional time $t = 0.1 \ (0 \le x \le 20.0)$ surface value $y = 1.0$, are derived in numerical results and the values for microelongation, normal displacement, temperature as well as forced stress are reflected in Figs 1 to 4. These numerical values are in accordance with the Lord-Shulman (L-S) and Green-Lindsay (G-L) theories of thermoelasticity.

7. Discussions

Figures 1 to 4 show that values (in the range of $n = 0$ and $n = 1$) for microelongation, normal displacement, forced stress and temperature are very close according to the L-S and G-L theories. An exponential increase of these quantities is also observed with respect to 'n'. The variations of all the above mentioned quantities are similar in nature but the magnitude of normal displacement is greater as compared to microelongation and temperature distribution. In contrast to the variation in normal displacement, microelongation and temperature distribution, the values of normal force stress increase sharply for a homogeneous medium i.e. for $n = 0$. Figure 2 illustrates changes in magnitude of normal forced stress with n and it is observed that the magnitude of normal forced stress was quite smaller in a non-homogeneous medium (i.e. $n = 1$; 3).

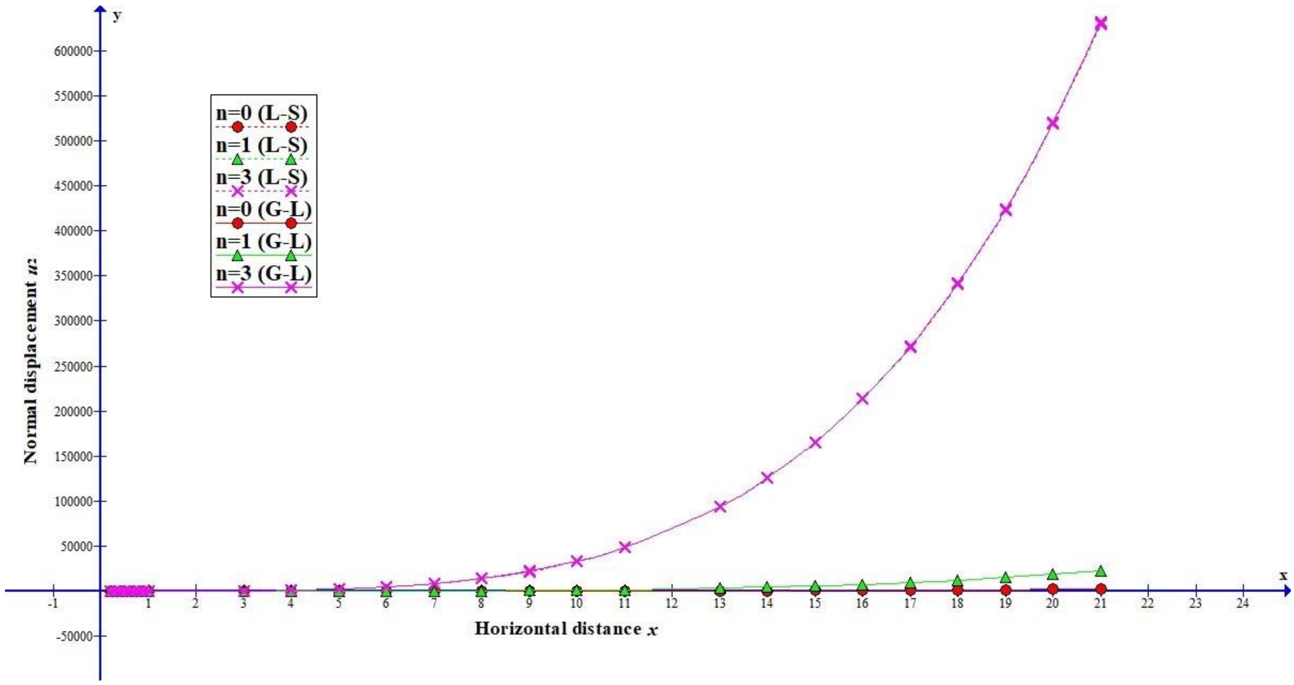

Fig.1. Variation of normal displacement u_2 with distance x.

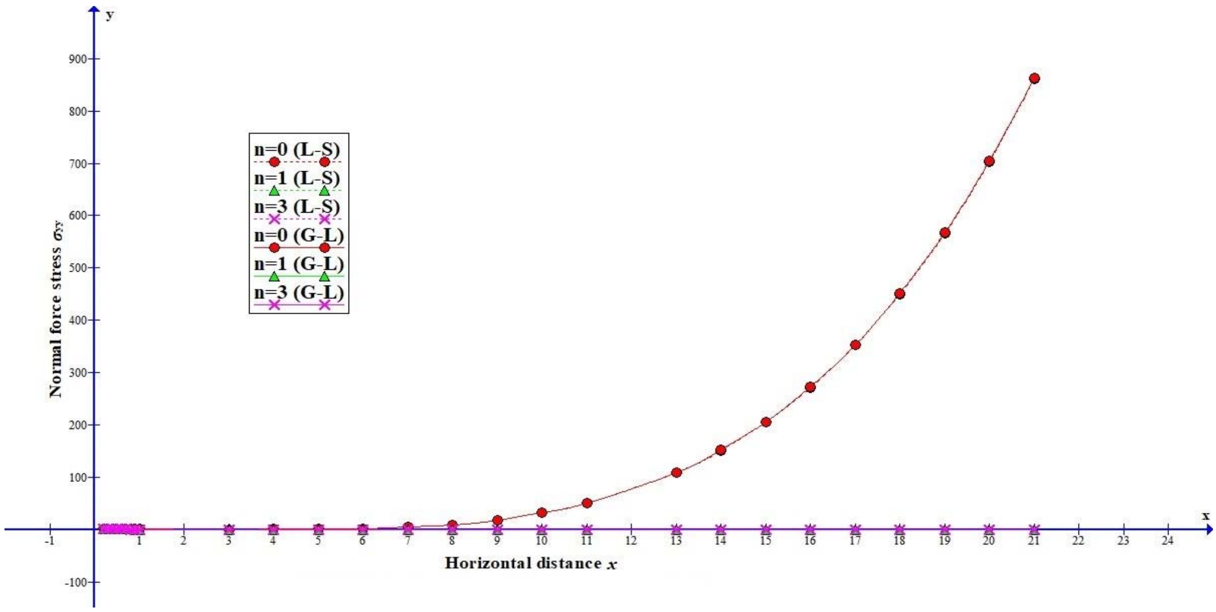

Fig.2. Variation of normal force stress σ_{yy} with distance x.

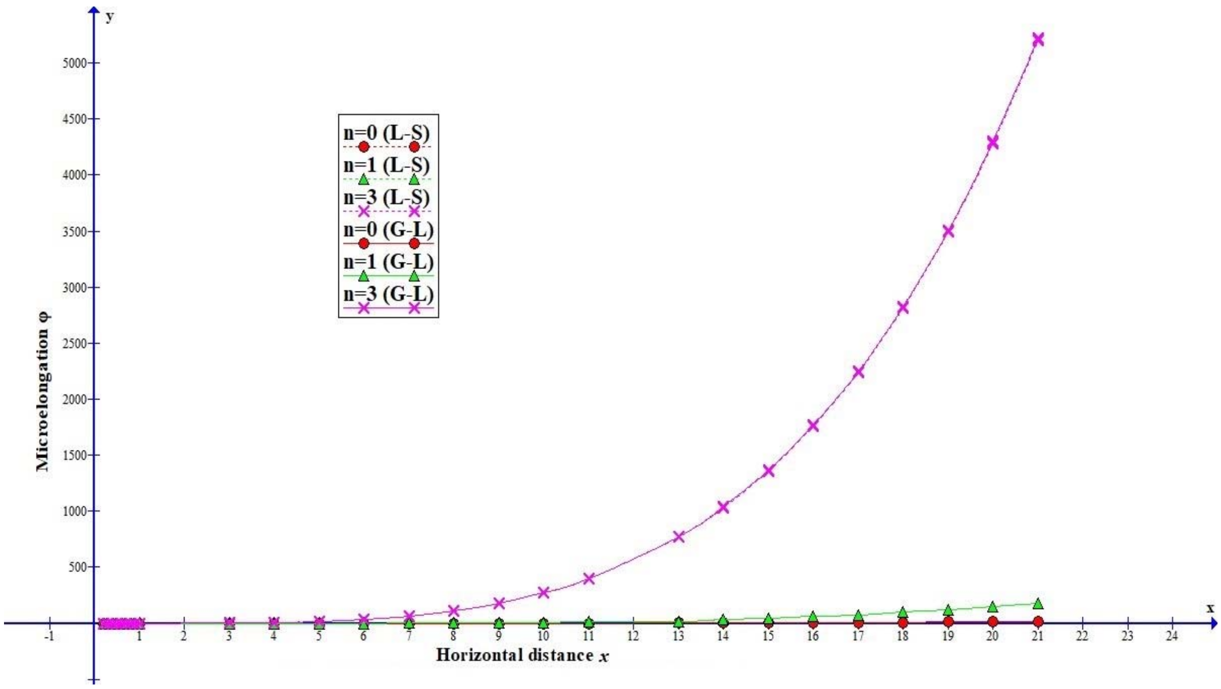

Fig.3. Variation of microelongation φ with distance x .

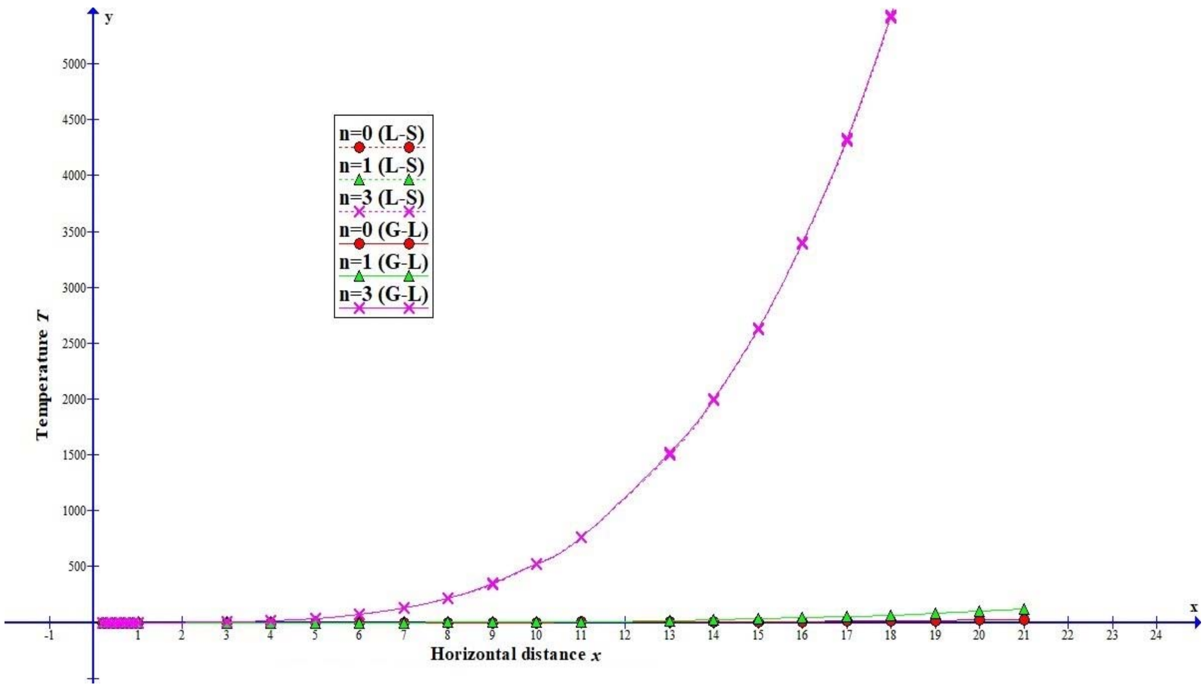

Fig.4. Variation of temperature T with distance x.

8. Conclusion:

The two-dimensional deformation in a functionally graded thermoelastic microelongated solid has been investigated and the components of displacement, stress, temperature and microelongation have been evaluated subject to thermal and mechanical boundary conditions. The results demonstrate that:

1. The variations of physical quantities are similar in nature for both thermal and mechanical boundary conditions.
2. The variations are also similar in nature for the Lord-Shulman (L-S) and Green- Lindsay (G-L) theories of thermoelasticity.
3. The effect of non-homogeneity is observed on all the quantities.
4. While the values of temperature distribution, microelongation and normal displacement increase with an increase in value of parameter n, the values of normal force stress decrease with parameter n.

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Nomenclature:

$\beta_1,\beta_0,\ \lambda_0,\lambda_1,a_0$	– microelongational constants
C^*	– specific heat at constant strain
K^*	– thermal conductivity
$\alpha_{t_1},\alpha_{t_2}$	– coefficient of linear thermal expansion
T	– temperature
T_0	– reference temperature
j_0	– microinertia
t_0,t_1	– thermal relaxation times
ρ	– density
u_i	– displacement vector
ϕ	– microelongational scaler
λ,μ	– Lame's elastic constants
σ	– microelongational stress tensor
δ_{kl}	– Kronecker delta
s	– components of stress tensor

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