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Parallel computations of the step response of a floor heater with the use of a graphics processing unit. Part 1: models and algorithms

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Abstract. The article presents a method of computing the step response of an air floor heater. The method implements parallel algorithms on a graphics processing unit. In the analyzed concrete slab heating ducts are placed. Hot air is transferred through them, thanks to which the heat penetrates into the slab. Heat transfer into the environment takes place on the top surface of the floor by natural convection and radiation. The bottom surface of the slab is thermally insulated. A two-dimensional heat equation was discretized with the use of the implicit finite difference method. In order to solve the obtained system of equations, the conjugate gradient method was used. Moreover, in order to examine the possibility of shortening the computations time, the algorithm of this method was implemented on a graphics processing unit. A computer program, using the CUDA parallel computing platform and linear algebra libraries CUBLAS and CUSPARSE, was developed.

Key words: air floor heating, step response, implicit finite difference method, parallel computations, GPGPU.

1. Introduction

In recent years, the use of floor heating as a way of providing the heat to rooms has increased [1–3]. Floor heating systems have many advantages, comparing with the traditional ones [3, 4]. They provide higher thermal comfort in rooms (higher temperature in the bottom, and lower - in the upper parts of a room). They also enable using the whole space of a room because there are no outer heaters. Moreover, they do not cause unfavorable air ionization and drying. Another advantage is that they do not generate high investment costs.

In practice, electric and water floor heating systems are the most frequently used [1, 4]. Many scientific works have been devoted to the analysis of thermal fields in those types of floor heaters [1, 2, 5–11]. Whereas, this article presents a method of computing the step response of an air floor heater [1, 12–15]. In this case, warm air, distributed through ducts placed in the floor slab, is a heat source.

The step response is not a typical working state of the heater, which usually works with a regulator [3]. For the following reasons, the step response is though one of the most important dynamic characteristics [5, 16]:

a) it is a convenient connection between the research done on steady and transient states,
b) with the use of the extended version of the Duhamel's theorem [17], it enables determination of the response to any excitation,
c) it is the basis for determining the average time constant of the system, and therefore its equivalent transmittance.

Moreover, the step response models the heating initiation which usually takes longer time in the analyzed systems. The step response may also be a good model of overheating caused by a breakdown of the regulator.

Thermal computations are time-consuming [6]. One of the ways leading to their shortening is implementing parallel computations which can be done on various types of systems: massive parallel computers, clusters, grids, and for the last few years – on graphics processing units. This last approach seems to be very interesting because nowadays each personal computer has a graphics card which enables doing numerical computations which do not involve generating the graphics. This type of technology is called GPGPU – General Purpose Computing on Graphics Processing Unit [18]. The graphics processing unit (GPU) can be equipped with a big number of so called streaming processors. Due to this fact, it is possible to achieve very high performance, exceeding several times the performance of processors used in personal computers. However, it should be stressed that achieving such high performance is not possible with all computational problems.

In order to calculate the step response of a floor heater, a two-dimensional heat equation was discretized in space and time domains with the use of the implicit finite difference method [15, 19–21]. As a result of the discretization, a linear system of algebraic equations was obtained. In order to solve the system of equations, the conjugate gradient method [22, 23] was applied. Its algorithm was implemented on a graphics processing unit. The authors of this article analyzed the conditions in which using the GPU in the conjugate gradient method is beneficial and enables shortening the computations time in comparison with the same work done on a CPU. The research concerned algorithms adapted both to dense and sparse matrices.

2. The analogue model of the step response

Hot air flowing through heating ducts is the source of heat in the analyzed system [13, 15]. The ducts are placed in a

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concrete floor slab (Fig. 1). Their cross-section is a square of the side length L. The distance between the edges of the neighboring ducts also equals L. The ducts are placed symmetrically according to top and bottom surface of the floor slab. The bottom surface is thermally insulated. The air is heated by an electric heater and reaches temperature T_H on the central axis of the duct. Circulation of the hot air in the ducts is done by fans. The air temperature far from the floor equals T_{amb}.

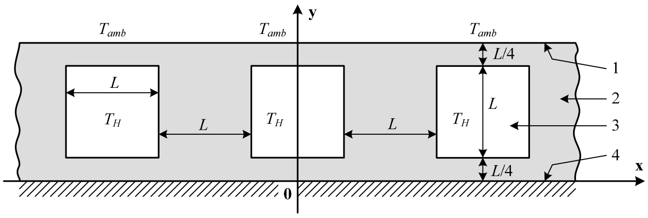

Fig. 1. Cross-section of the floor heater: 1 – top surface of the floor, 2 – concrete, 3 – duct with circulating hot air, 4 – insulation (after Ref. 15)

It is assumed that the length of the system is significantly greater than the dimensions of its cross-section (which occurs in heating ducts of high rooms, e.g. stations and halls). In such case the thermal field of the slab is plane-parallel (which means that it depends on two spatial variables x, y, and a time variable – t). The system is low-temperature. This means that the interval $\langle T_{amb}, T_H \rangle$ is not wide. In this range the values of:

1. heat transfer coefficients (in the ducts – α_H, and from the floor – α_F),
2. concrete parameters (specific heat – c, mass density – ρ, thermal conductivity – λ),

can be then averaged.

The assumption concerning step excitation of the system means taking constant value of air temperature T_H in the cross-section of the ducts for $t \geq 0$.

In the middle of the distance between the ducts, the temperature $T(x,y,t)$ will obtain its minimum (e.g. for $x = \pm L$ – Fig. 2). The segment between the straight lines $x = \pm L$ is also symmetrical in terms of thermal conditions in relation to $x = 0$ axis. For this reason, the maximum of temperature is at $x = 0$. This can be expressed by the following equations:

$$\left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=-L} = 0 \quad \text{for} \quad 0 \leq y \leq 1.5L, \ t \geq 0, \tag{1a}$$

$$\left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=0} = 0 \quad \text{for} \quad y \in \langle 0,\, L/4 \rangle \cup \langle 5L/4,\, 3L/2 \rangle,\ t \geq 0. \tag{1b}$$

Equations (1a,b) implicate that during determination of the spatio-temporal step response, the analysis can be limited to the area of the slab distincted in Fig. 2 (the striped area). In other parts of the system, the field distributions are mirror images and repeat.

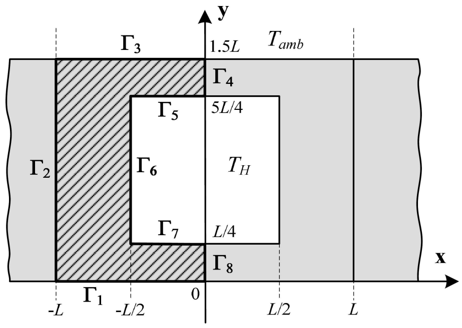

Fig. 2. The analyzed fragment of the floor heater

In the analyzed model, the system thermal response $h(x,y,t)$ to letting the hot air into the ducts at the zero time moment ($t = 0$) is its step characteristics. The previous assumption about the plane-parallel temperature distribution leads to the field description by a two-dimensional heat equation [15, 19, 20, 24]

$$\frac{\partial^2 h(x,y,t)}{\partial x^2} + \frac{\partial^2 h(x,y,t)}{\partial y^2} - \frac{1}{\chi}\frac{\partial h(x,y,t)}{\partial t} = 0, \tag{2}$$

where $\chi = \lambda/(c\rho)$ – concrete diffusivity.

In moment $t = 0$, the system is in the steady state and its all points are in ambient temperature T_{amb}:

$$h(x,y,t=0) = T_{amb}. \tag{3}$$

On the top surface of the floor slab (contour Γ_3 in Fig. 2), the heat transfer into the environment occurs by natural convection and radiation, which is described by the Hankel's boundary condition with the total heat transfer coefficient α_F:

$$-\lambda\left.\frac{\partial h(x,y,t)}{\partial y}\right|_{y=1.5L} = \alpha_F\left[h(x,y=1.5L,t) - T_{amb}\right] \quad \text{for} \quad -L \leq x \leq 0 \ \text{and} \ t \geq 0. \tag{4}$$

The heat penetrates into the floor slab from a thermal duct. This phenomenon can be described by an equation similar to (4):

$$-\lambda\frac{\partial h(x,y,t)}{\partial \eta} = \alpha_H\left[T_H - h(x,y,t)\right] \quad \text{for} \quad (x,y) \in \Gamma_5 \cup \Gamma_6 \cup \Gamma_7,\ t \geq 0, \tag{5}$$

where $\partial h(x,y,t)/\partial\eta$ is a derivative in the normal direction to contour $\Gamma_5 \cup \Gamma_6 \cup \Gamma_7$ (Fig. 2). On the other edges of the model, the heat transfer does not occur because of its symmetry (contours Γ_2, Γ_4, Γ_8 – Eqs. (1a,b)) or ideal insulation (contour Γ_1):

$$\frac{\partial h(x,y,t)}{\partial \eta} = 0 \quad \text{for} \quad (x,y) \in \Gamma_1 \cup \Gamma_2 \cup \Gamma_4 \cup \Gamma_8,\ t \geq 0, \tag{6}$$

where the derivative in the normal direction is calculated with respect to contour $\Gamma_1 \cup \Gamma_2 \cup \Gamma_4 \cup \Gamma_8$.

Equations (2)–(6) define the initial-boundary value problem whose solution means the determination of the step response of the system.

3. The discrete model of the step response

The two-dimensional heat Eq. (2) was discretized in space and time domains with the use of the implicit finite difference method [15, 19]. The model of a floor heater, presented in Fig. 2, was covered with a finite difference mesh. The fragment of the mesh used and the method of nodes numbering, are presented in Fig. 3a and Fig. 3b, respectively.

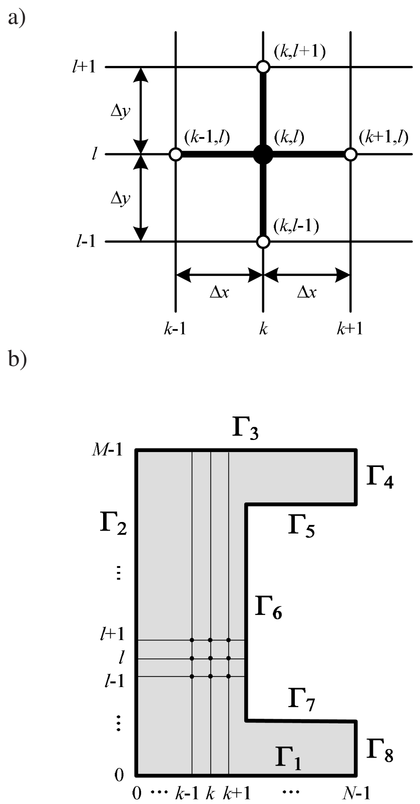

Fig. 3. Fragment of the finite difference mesh (a) and the method of nodes numbering (b), where N – the number of nodes along the x axis, M – the number of nodes along the y axis

Replacing in (2) the second spatial derivatives ($\partial^2 h(x,y,t)/\partial x^2$ and $\partial^2 h(x,y,t)/\partial y^2$) with central difference quotients, and the first order time derivative ($\partial h(x,y,t)/\partial t$) with a forward difference quotient, a two-dimensional heat equation in the differential form was obtained:

$$\frac{h_{k+1,l}^{n+1}+h_{k-1,l}^{n+1}-2h_{k,l}^{n+1}}{(\Delta x)^2}+\frac{h_{k,l+1}^{n+1}+h_{k,l-1}^{n+1}-2h_{k,l}^{n+1}}{(\Delta y)^2} -\frac{1}{\chi}\frac{h_{k,l}^{n+1}-h_{k,l}^{n}}{\Delta t}=0, \tag{7}$$

where subscripts k, l determine the location of a mesh node, and the superscript n is the number of time moment.

Assuming that the mesh is spread evenly towards the x and y axes ($\Delta x = \Delta y$) [15], and introducing the Fourier's number:

$$Fo=\frac{\chi\Delta t}{\Delta x^2} \tag{8}$$

and rearranging, the following equation was obtained:

$$(1+4Fo)h_{k,l}^{n+1}-Fo(h_{k+1,l}^{n+1}+h_{k-1,l}^{n+1} +h_{k,l+1}^{n+1}+h_{k,l-1}^{n+1})=h_{k,l}^{n}. \tag{9}$$

Equation (9) enables computing the value of the step response at the (k,l) nodal point for the time moment $n+1$, according to the value of this function in neighboring nodes $(k+1,l)$, $(k-1,l)$, $(k,l+1)$, $(k,l-1)$ in the same $n+1$ time moment and the value of the response in the (k,l) node from the previous moment n.

Equation (9) concerns only the nodes located inside the analyzed model and having 4 neighboring nodes. For the nodes situated on the edges of the model, new equations concerning boundary conditions should be worked out. For this purpose Eq. (9) can be applied, but the step response values in non-existing nodes must be eliminated.

In terms of the nodes situated on a plane surface, from which the heat is transferred into the environment (contour Γ_3), the $(k,l+1)$ nodal point should be eliminated from Eq. (9). Boundary condition (4) was used for this reason, after having been transformed to the differential form. After replacing the spatial derivative of the first order by a forward difference quotient, condition (4) will take the following form:

$$-\lambda\frac{h_{k,l+1}^{n+1}-h_{k,l-1}^{n+1}}{2\Delta y}=\alpha_F\left[h_{k,l}^{n+1}-T_{amb}\right]. \tag{10}$$

After transforming (10), the equation describing the value of the step response $h_{k,l+1}^{n+1}$ was obtained. It should be eliminated from (9):

$$h_{k,l+1}^{n+1}=h_{k,l-1}^{n+1}-2Bi_F(h_{k,l}^{n+1}-T_{amb}), \tag{11}$$

where Bi_F is the Biot's number for the heat transfer coefficient from the floor (α_F):

$$Bi_F=\frac{\alpha_F\Delta x}{\lambda}, \quad \text{for} \quad \Delta x=\Delta y. \tag{12}$$

After having replaced (11) for (9) and rearranging, the final form of the differential equation was obtained for the nodes situated on the edge Γ_3:

$$\Gamma_3: \quad (1+2Fo(2+Bi_F))h_{k,l}^{n+1} -Fo(h_{k+1,l}^{n+1}+h_{k-1,l}^{n+1}+2h_{k,l-1}^{n+1}) =h_{k,l}^{n}+2Bi_F Fo T_{amb}. \tag{13}$$

The eliminated $(k,l+1)$ nodal point is shown in Fig. 4.

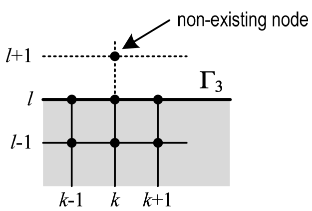

Fig. 4. The finite difference mesh with non-existing nodal point $(k,l+1)$

In the case of the nodes situated on surfaces into which the heat penetrates (contours Γ_5, Γ_6, Γ_7), the procedure should be identical (boundary condition (5) in the differential form should be used). The final form of the obtained equations takes the following form:

$$\Gamma_5: \quad (1+2Fo(2+Bi_H))h_{k,l}^{n+1} - Fo(h_{k+1,l}^{n+1}+h_{k-1,l}^{n+1}+2h_{k,l+1}^{n+1}) = h_{k,l}^{n}+2Bi_H FoT_H, \tag{14a}$$

$$\Gamma_6: \quad (1+2Fo(2+Bi_H))h_{k,l}^{n+1} - Fo(h_{k,l+1}^{n+1}+h_{k,l-1}^{n+1}+2h_{k-1,l}^{n+1}) = h_{k,l}^{n}+2Bi_H FoT_H, \tag{14b}$$

$$\Gamma_7: \quad (1+2Fo(2+Bi_H))h_{k,l}^{n+1} - Fo(h_{k+1,l}^{n+1}+h_{k-1,l}^{n+1}+2h_{k,l-1}^{n+1}) = h_{k,l}^{n}+2Bi_H FoT_H, \tag{14c}$$

where Bi_H is the Biot's number for the heat transfer coefficient in the ducts (α_H):

$$Bi_H = \frac{\alpha_H \Delta x}{\lambda}, \quad \text{for} \quad \Delta x = \Delta y. \tag{15}$$

The bottom part of the floor slab is perfectly insulated. The differential equation for the nodes situated on the Γ_1 edge can be determined by eliminating (k,$l-1$) nodal point from (9). For this reason boundary condition (6) takes the following differential form:

$$\frac{h_{k,l-1}^{n+1}-h_{k,l+1}^{n+1}}{2\Delta y} = 0. \tag{16}$$

Transforming (16) into the form:

$$h_{k,l-1}^{n+1} = h_{k,l+1}^{n+1} \tag{17}$$

and substituting it to (9), the following form of equation is obtained:

$$\Gamma_1: \quad (1+4Fo)h_{k,l}^{n+1} - Fo(h_{k+1,l}^{n+1}+h_{k-1,l}^{n+1}+2h_{k,l+1}^{n+1}) = h_{k,l}^{n}. \tag{18}$$

Equations for the nodes situated on the adiabatic surfaces are determined in the same way:

$$\Gamma_2: \quad (1+4Fo)h_{k,l}^{n+1} - Fo(h_{k,l+1}^{n+1}+h_{k,l-1}^{n+1}+2h_{k+1,l}^{n+1}) = h_{k,l}^{n}, \tag{19a}$$

$$\Gamma_4 \cup \Gamma_8: \quad (1+4Fo)h_{k,l}^{n+1} - Fo(h_{k,l+1}^{n+1}+h_{k,l-1}^{n+1}+2h_{k-1,l}^{n+1}) = h_{k,l}^{n}. \tag{19b}$$

Corner nodes are another problem. They require forming some other equations. There are three types of such nodes:

- nodes joining two adiabatic contours,
- nodes joining an adiabatic contour with a contour of heat transfer,
- nodes joining two contours of heat transfer.

The equations were derived with the use of the energy balance method [15, 19]. In terms of corner nodes joining two adiabatic contours, the equations are as follows:

$$\Gamma_1 \cap \Gamma_2: \quad (1+4Fo)h_{k,l}^{n+1} - 2Fo(h_{k,l+1}^{n+1}+h_{k+1,l}^{n+1}) = h_{k,l}^{n}, \tag{20a}$$

$$\Gamma_1 \cap \Gamma_8: \quad (1+4Fo)h_{k,l}^{n+1} - 2Fo(h_{k,l+1}^{n+1}+h_{k-1,l}^{n+1}) = h_{k,l}^{n}. \tag{20b}$$

The following differential equations concern corner nodes joining an adiabatic contour with a contour of heat transfer:

$$\Gamma_2 \cap \Gamma_3: \quad (1+2Fo(2+Bi_F))h_{k,l}^{n+1} - 2Fo(h_{k+1,l}^{n+1}+h_{k,l-1}^{n+1}) = h_{k,l}^{n}+2Bi_F FoT_{amb}, \tag{21a}$$

$$\Gamma_3 \cap \Gamma_4: \quad (1+2Fo(2+Bi_F))h_{k,l}^{n+1} - 2Fo(h_{k-1,l}^{n+1}+h_{k,l-1}^{n+1}) = h_{k,l}^{n}+2Bi_F FoT_{amb},) \tag{21b}$$

$$\Gamma_4 \cap \Gamma_5: \quad (1+2Fo(2+Bi_H))h_{k,l}^{n+1} - 2Fo(h_{k-1,l}^{n+1}+h_{k,l+1}^{n+1}) = h_{k,l}^{n}+2Bi_H FoT_H, \tag{21c}$$

$$\Gamma_7 \cap \Gamma_8: \quad (1+2Fo(2+Bi_H))h_{k,l}^{n+1} - 2Fo(h_{k-1,l}^{n+1}+h_{k,l-1}^{n+1}) = h_{k,l}^{n}+2Bi_H FoT_H. \tag{21d}$$

The last type of corner nodes concerns two contours of heat transfer:

$$\Gamma_5 \cap \Gamma_6: \quad \left(1+4Fo\left(1+\frac{1}{3}Bi_H\right)\right)h_{k,l}^{n+1} - \frac{2}{3}Fo(h_{k+1,l}^{n+1}+2h_{k-1,l}^{n+1}+2h_{k,l+1}^{n+1}+h_{k,l-1}^{n+1}) = h_{k,l}^{n}+\frac{4}{3}Bi_H FoT_H, \tag{22a}$$

$$\Gamma_6 \cap \Gamma_7: \quad \left(1+4Fo\left(1+\frac{1}{3}Bi_H\right)\right)h_{k,l}^{n+1} - \frac{2}{3}Fo(h_{k+1,l}^{n+1}+2h_{k-1,l}^{n+1}+h_{k,l+1}^{n+1}+2h_{k,l-1}^{n+1}) = h_{k,l}^{n}+\frac{4}{3}Bi_H FoT_H. \tag{22b}$$

Putting Eqs. (9), (13), (14), (18)–(22) together for each node of a finite difference mesh, we obtain the $N \cdot M - (N-1) \cdot (2 \cdot M - 5)/6$ system of linear algebraic equations. The meaning of N and M is explained in Fig. 3b. The part of the formula after the sign of subtraction, $(N-1) \cdot (2 \cdot M - 5)/6$, results from the fact that doing the calculations in thermal ducts is not necessary. The computation of the step response of a floor heater requires solving the above system of equations for chosen time moments Δt, $2\Delta t$, $3\Delta t$, $\ldots$ from the analyzed time interval (Δt – assumed time step). In addition the initial condition (3) is also taken into consideration. During the computation of the step response in discrete time moment $n+1=1$, the initial temperature distribution for $n=0$, that is for $t=0$, is used (on the right side of (9), (13), (14), (18)–(22)).

The computer program which was then created, as well as the applied algorithms, are described in the following chapter of this paper.

4. Algorithm of the discrete model solving

An original computer program was created in order to solve the discrete model and to compute the step response of a floor heater. The program was written in C++ programming language. Its flow chart and the most important modules are presented in Fig. 5.

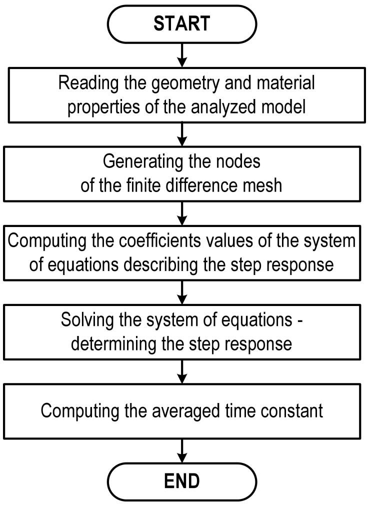

Fig. 5. The computer program flow chart

The program consists of 5 main modules. The first one reads the geometry of the analyzed model, its material properties and the parameters of numerical simulation. The second module generates the nodes of the finite difference mesh on the basis of the data read by the first module. Each node has its type stated, considering its location in the model (interior node, edge node or corner node). Taking in consideration the types of nodes, the following module computes the coefficients values of the system of equations describing the dynamics of the floor heater. The aim of the fourth module is to determine the step response of the heater. The fifth module computes its averaged time constant.

Determining the step response of the heater is the most time-consuming part of the program, as it requires multiple solution of the system of equations for the following time moments. The order of operations conducted in this module is presented in Fig. 6.

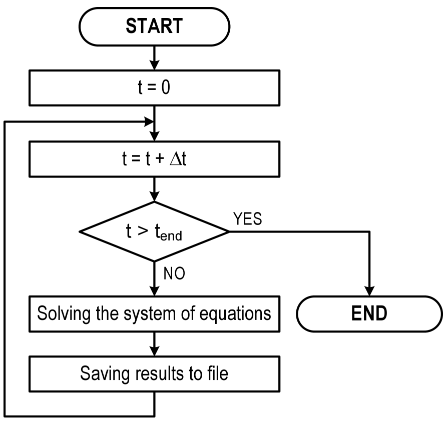

Fig. 6. The flow chart of the module computing the step response

The conjugate gradient method [22, 23] was applied to solve the system of linear equations. In the first step (for $t = \Delta t$) the ambient temperature T_{amb} is taken as the initial approximation of the solution. After the system of equations has been solved, the obtained results are saved in a file. In the following steps, the values of solutions from the previous step are taken as the initial approximation. The computations are conducted until the end of a time interval ($t = t_{end}$) in which the step response is elaborated.

Solving a system of equations with the use of the conjugate gradient method requires doing some basic operations on vectors and matrices, e.g.: matrix-vector multiplication, dot product. Shortening the time needed to do the above operations is possible while a graphics processing unit is used. In such case the program is executed simultaneously by a traditional processor (CPU) and a graphics processing unit (GPU). To some extent, the graphics card serves as a math coprocessor which conducts time-consuming algebraic operations. They are done as special programs, called threads. A modern graphics processing unit consists of a large number (several hundreds) of so called streaming processors, on which the threads are executed. Memory accesses in the graphics card, initiated by running threads, are very slow. They can cause significant delays in threads executing. This problem was solved by sending to initiation on the GPU a significantly greater number of threads than the number of accessible streaming processors. When certain threads initiate memory access, the control system of the GPU suspends their executing until the memory operations have been finished, and it starts to execute other threads. Threads switching is done without time delay. Such way of program executing on the GPU requires that the threads are able to be done in any order, independently from each other. It influences the solved problem. Obtaining high performance (maximum utilization of the GPU resources) is only possible when the problem can be divided into sub-problems done in any order and independently from each other. Among tasks of this type, there are operations on vectors and matrices occurring in the algorithm of the conjugate gradient method.

The CUDA (Compute Unified Device Architecture) [18, 25, 26] was applied in order to create the computer program. CUDA is a parallel computing platform enabling writing programs in C/C++ languages, their compilation and executing on a GPU made by NVIDIA Corporation. Together with the CUDA, two implementations of numerical library BLAS (Basic Linear Algebra Subprograms) are delivered. They are used to perform basic linear algebra operations on vectors and matrices. In CUBLAS library [27] procedures for dense matrices are implemented, and in CUSPARSE library [28] – for sparse matrices.

Using the GPU for computations which are not related to generating the graphics is a technique introduced just a few years ago, and for this reason – not well known. After the analyzed problem was discretized with the use of the implicit finite differences method, a system of equations with a sparse matrix of coefficients was obtained. Despite that, in order to test the conditions, under which using a GPU is profitable, two implementations of the algorithm of conjugate gradient, were examined: for sparse and dense matrices. The elaborated programs differed in the method of storing matrices, and therefore – in the method of performing algebraic operations on matrices.

In the case of the first program, adapted to dense matrices, all elements of a matrix were stored in the computer memory (also those of zero value). During the analysis of the conjugate gradient method algorithm it occurred, that the most time-consuming operation is a matrix-vector multiplication. Performing this operation was then implemented on a GPU. The cublasSgemv(...) function from CUBLAS library was used for this purpose. Other algebraic operations of conjugate gradient method were done on a CPU.

In the second program the method of storing matrix elements was changed. The CSR method (Compressed Sparse Row) [23] was applied, in which non-zero elements are stored in memory line by line. In the case of vectors, all their elements were stored in the computer memory. Similarly to the first program, only the multiplication of sparse matrix by a dense vector was performed on a GPU. The cusparseScsrmv(...) function from CUSPARSE library was used for this purpose. The division of the created program into five modules caused that the change of the method of matrix storing required only some modifications in the third and the fourth modules of the program.

5. Summary

The article presents theoretical basis of computing the step response of a floor air heater. It also describes algorithms applied in the created computer program which uses a GPU for numerical computations. The second part of the article will contain the results of computations, their numerical verification and the evaluation of the applied algorithms.

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