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HYBRID PASSIVITY BASED AND FUZZY TYPE-2 CONTROLLER FOR CHAOTIC AND HYPER-CHAOTIC SYSTEMS

Fernando SERRANO[*], Josep M. ROSSELL[**]

[*]Central American Technical University (UNITEC), Tegucigalpa, Zona Jacaleapa, Honduras
[**]Department of Mathematics, Universitat Politècnica de Catalunya (UPC), Av. Bases de Manresa, 61-73 08242-Manresa, Spain

serranofer@eclipso.eu, josep.maria.rossell@upc.edu

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Abstract: In this paper a hybrid passivity based and fuzzy type-2 controller for chaotic and hyper-chaotic systems is presented. The proposed control strategy is an appropriate choice to be implemented for the stabilization of chaotic and hyper-chaotic systems due to the energy considerations of the passivity based controller and the flexibility and capability of the fuzzy type-2 controller to deal with uncertainties. As it is known, chaotic systems are those kinds of systems in which one of their Lyapunov exponents is real positive, and hyper-chaotic systems are those kinds of systems in which more than one Lyapunov exponents are real positive. In this article one chaotic Lorentz attractor and one four dimensions hyper-chaotic system are considered to be stabilized with the proposed control strategy. It is proved that both systems are stabilized by the passivity based and fuzzy type-2 controller, in which a control law is designed according to the energy considerations selecting an appropriate storage function to meet the passivity conditions. The fuzzy type-2 controller part is designed in order to behave as a state feedback controller, exploiting the flexibility and the capability to deal with uncertainties. This work begins with the stability analysis of the chaotic Lorentz attractor and a four dimensions hyper-chaotic system. The rest of the paper deals with the design of the proposed control strategy for both systems in order to design an appropriate controller that meets the design requirements. Finally, numerical simulations are done to corroborate the obtained theoretical results.

Key words: Chaos Synchronization, Passive Control, Type-2 Fuzzy Controller, Chaotic System

1. INTRODUCTION

Chaotic systems have been extensively studied during the last decades, due to their complexity and applications in which this phenomenon is found in chemical, physical, power and mechanical systems just to mention some of them (Effati et al., 2014). Chaos is found in some systems when only one Lyapunov exponent is positive yielding a complex dynamic behaviour. Meanwhile, hyper-chaos is found in those systems which have more than one positive Lyapunov exponent where this phenomenon is mostly found in social and economic systems (Effati et al., 2014). In this article, the control of chaotic and hyper-chaotic systems with a fuzzy type-2 and passivity based control is proposed. The main idea of this hybrid control strategy is to take advantage of the energy considerations for the design of passivity based controllers and the flexibility and capability to deal with uncertainties of the fuzzy type-2 controller. Considering the dynamical complexity of chaotic and hyper-chaotic systems, the fuzzy type-2 controller along with the passivity based controller is a suitable control strategy due to the fuzzy type-2 controller part is designed using the expert knowledge in order to stabilize and improve the chaotic and hyper-chaotic performance systems. Passivity based control has been implemented for the control of different kinds of nonlinear systems. For example, in (Dadras and Momeni, 2013) a passivity based and fractional order integral sliding mode control for uncertain fractional order nonlinear system is proposed, where this controller is designed for some kinds of chaotic systems, specifically, a fractional order Chua circuit and a Van Der Pol oscillator. Meanwhile, in Liu et al. (2013) a passivity based attitude controller is implemented for a rigid spacecraft taking into account the respective energy considerations for an appropriate control design. In(Zhu and Huo (2013) a passivity based controller for a model scaled unmanned helicopter is proposed for trajectory linearization control for this kind of unmanned aerial vehicle, where the energy properties of the systems are considered to design a suitable passivity based control strategy. There is a considerable number of passivity based controllers found in literature, but a complete explanation of this controller can be seen in Haddad and Chellaboina (2008). The references cited before are important because they are fundamental for the design of the passivity based controller part to stabilize the chaotic and hyper-chaotic systems analyzed in this article, so it is important to consider several energy properties of these kinds of systems such as dissipativity in order to stabilize these kinds of systems.

Fuzzy type-2 systems have become an alternative for fuzzy type-1 systems because they have two degrees of freedom, composed by a primary and secondary membership functions. Fuzzy type-2 systems are considered as a generalization of fuzzy type-1 systems, in which uncertainty is considered in the membership functions and not only in the linguistic variables (Castillo and Melin, 2008). Fuzzy type-2 systems process consists in the following steps: a fuzzifier, an inference engine, a type reducer and a defuzzifier (Karnik et al., 1999; Mendel, 2007b; Turksen, 1999). Fuzzy type-2 controllers have been implemented recently and are evolving continuously nowadays. For example, in Fayek et al. (2014) a controller based on optimal fuzzy type-2 systems is proposed, where the gains of the controller are optimized using particle swarm optimization to implement this controller in a real time setup. Different examples of fuzzy type-2 controllers can be found in Castillo and Melin (2014) where some bio-inspired optimization algorithms are implemented to find the parameters and structure of the fuzzy type-2 controllers. Based on this theoretical background, a fuzzy type-2 and a passivity based controller for chaotic and hyper-chaotic systems are proposed, considering the advantages of fuzzy type-2 controllers (Fayek et al., 2014) with the combination of a passivity based controller which have proved to be effective due to its energy consideration. In this article a chaotic Lorentz attractor and a hyper chaotic 4D systems (Effati et al., 2014) are studied to apply the proposed control strategy. Before deriving the proposed controller, an analysis of the stability properties of both systems is performed by studying their eigenvalues and Lyapunov exponents along with their bifurcation diagrams. The fuzzy type-2 part is designed in order to behave as a state feedback controller (Morales-Mata et al., 2008) while the passivity based part is done using an appropriate storage function to find a suitable control law according to the dissipativity properties of the system. Finally to illustrate the theoretical background proposed in this article, some examples are shown with the conclusions of this work.

2. PROBLEM FORMULATION

Before deriving the proposed control strategy, the definitions of the chaotic and hyper-chaotic systems studied in this article are shown in this section. The proposed controller in this paper is designed for different kinds of chaotic and hyper-chaotic systems, but in this case is only considered the stabilization of a chaotic Lorentz attractor (Richter, 2003), (Ontanon-Garcia and Campos-Canton, 2013) and a 4D chaotic system as shown in (Effati et al., 2014), (Zhou and Huang, 2014). A brief analysis of the chaotic Lorentz attractor and hyper-chaotic 4D system stability properties are done in the following subsections where the Lyapunov exponents are analyzed to determine if the system is either chaotic or hyper-chaotic. Apart from this, bifurcation diagrams for both systems are depicted to analyze their chaotic behaviour.

2.1. Stability Properties of the Chaotic Lorentz Attractor

The studied chaotic system, in this case the Lorentz attractor (forced), is shown in (1) (Richter, 2003; Ontanon-Garcia and Campos-Canton, 2013):

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) + u_1 \\ \dot{x}_2 &= \rho x_1 - x_2 - x_1 x_3 + u_2 \\ \dot{x}_3 &= x_1 x_2 - \beta x_3 + u_3 \\ y(x) &= [x_1 \quad x_2 \quad x_3]^T \end{aligned} \tag{1}$$

where $y(x)$ is the measured output and x_1, x_2 and x_3 are the state variables. In order to set the system (1) in chaotic regime, the constants are set as follows: $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$ (Richter, 2003; Ontanon-Garcia and Campos-Canton, 2013) and u_i for $i = 1,2,3$ are the system inputs.

In order to analyze the stability properties of the Lorentz attractor the Lyapunov exponents are shown in Tab. 1.

As it is shown in Tab. 1, only one Lyapunov exponent is positive, obtained by using the methods explained in Yonemoto and Yanagawa (2007), He et al. (1999), and Chen et al. (2006) but more methodologies can be found in (Protasov and Jungers, 2013), (Dieci and Vleck, 1995)

Tab. 1. Eigenvalues and Lyapunov exponents of the Lorentz attractor system

Number	Eigenvalues	Lyapunov exponents
1	11.8277	2.4716
2	-22.8277	-15.9288
3	-2.6667	-15.5890

The phase portrait for the variables x_1 and x_3 is shown in Fig. 1, where as it is noticed these state variables evince the chaotic behaviour of the Lorentz attractor for specific initial conditions. As it is known, the periodicity of this system can be analyzed using Poincare maps (more details can be found in Haddad and Chellaboina (2008)).

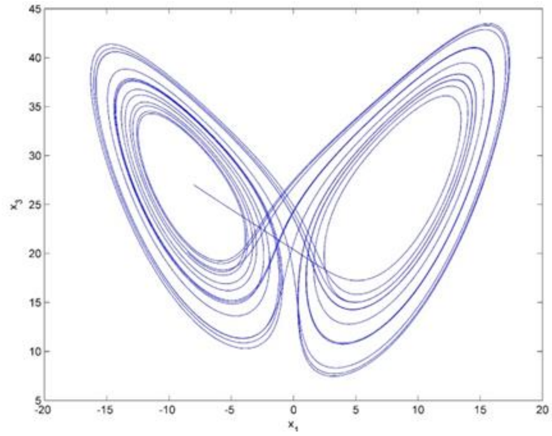

Fig. 1. Phase portrait of the Lorentz attractor

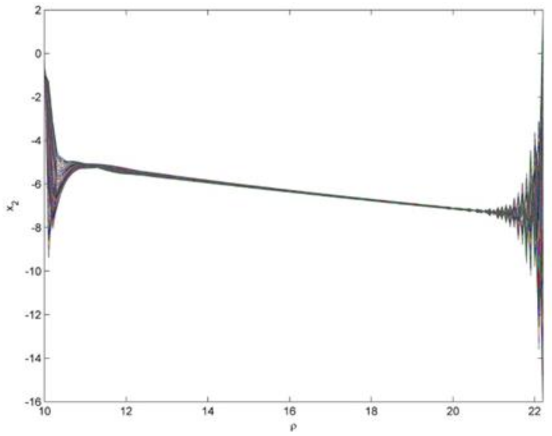

Fig. 2. Bifurcation diagram of the Lorentz attractor

The bifurcation diagram of this system is shown in Fig. 2. It can be noticed that the system (1) is in chaotic regime when $10 \le \rho \le 10.5$ and $21 \le \rho \le 22$. This system is stabilized by the proposed controller as shown in Section 4.

2.2. Stability Properties of a Hyper-Chaotic 4D System

The studied 4D hyper-chaotic system (forced) (E ffati et al., 2014) used in this paper for stabilization purposes is shown in (2)

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2x_3 + u_1 \\ \dot{x}_2 &= x_1x_3 - bx_2 + u_2 \\ \dot{x}_3 &= cx_1x_2 - dx_3 + gx_1x_4 + u_3 \\ \dot{x}_4 &= kx_4 - hx_2 + u_4 \\ y(x) &= [x_1 \quad x_2\, x_3 \quad x_4]^T \end{aligned} \tag{2}$$

where $y(x)$ is the measured output and x_1, x_2, x_3 and x_4 are the state variables. In order to yield a hyper - chaotic behaviour, the constants are set as a=8, b=40, c=2, d=14, g=5, h=0.2 and k=0.05; u_i are the system inputs for i=1,2,3,4. The Lyapunov exponents and eigenvalues of the system are shown in Tab. 2.

Tab. 2. Eigenvalues and Lyapunov exponents of the 4D hyper-chaotic system

Number	Eigenvalues	Lyapunov exponents
1	8.0000	7.3246
2	0.0500	0.0110
3	-40.0000	-30.3907
4	-14.0000	-0.5755

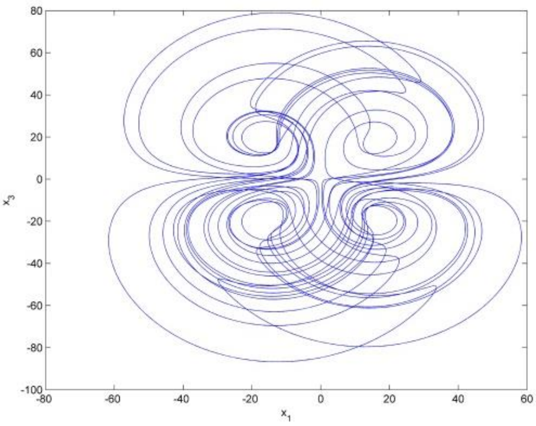

Fig. 3. Phase portrait of the 4D hyper-chaotic system

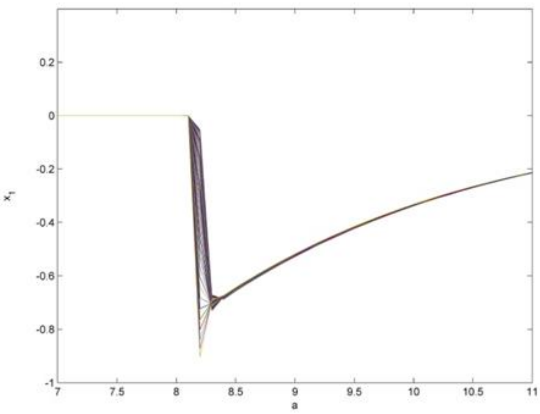

Fig. 4. Bifurcation diagram of the 4D hyper-chaotic system

As it is noticed in Tab. 2, there are two positive eigenvalues and Lyapunov exponents that set the system in hyper-chaotic regime. In Fig. 3 the phase portrait for x_1 and x_3 is shown.

Finally, in Fig. 4 the bifurcation diagram is shown and it can be noticed that the system is stable in the range $7 \le a \le 8.2$ and in the range $8.2 \le a \le 8.4$ the system is in hyper-chaotic regime. This is the studied hyper - chaotic system that will be stabilized by the proposed control strategy.

3. CONTROLLER DESIGN

In this section the design of the fuzzy type-2 passivity based controllers is shown. In this case, both controllers to stabilize chaotic and hyper-chaotic systems of several kinds and dimensions and later this controller is implemented for the stabilization of the chaotic Lorentz attractor and the 4D hyper-chaotic systems. The control law of both systems is defined as follow:

$$u = u_{pb} + u_{ft2} \tag{3}$$

where u_{pb} is the passivity based component and u_{ft2} is the fuzzy type-2 controller component.

In order to derive both controllers, consider the following chaotic and/or hyper-chaotic system:

$$\begin{aligned} \dot{x}(t) &= F(x(t), u(t)) \\ y(t) &= H(x(t)) \end{aligned} \tag{4}$$

where $x(t) \in D \subseteq \mathbb{R}^n$, $u(t) \in U$, $y(t) \in Y \subseteq \mathbb{R}^l$, $F: D \times U \to \mathbb{R}^n$ and $H: D \times U \to Y$ with the initial condition $x(t_0) = x_0$ (Haddad and Chellaboina, 2008). In Subsection 3.1 a brief description of fuzzy type-2 controllers is shown first, and then the derivation of the control law of this controller is depicted in this subsection. Then in Subsection 3.2, the passivity based controller is evinced based on the energy and dissipativity considerations of the system.

3.1. Fuzzy type-2 Controller Component

Before explaining the fuzzy type-2 controller part, a brief explanation on fuzzy type-2 system is depicted. Consider a fuzzy type-2 set X represented by:

$$\tilde{A} = \int_{x\in X} \mu_{\tilde{A}}(x)/x = \int_{x\in X} \left(\int_{u\in J_x} f_x(u)/u\right)/x \tag{5}$$

where $J_x \subseteq [0,1]$, $\mu_{\tilde{A}}$ is the membership grade of $x \in X$ which is a type-1 fuzzy set in $[0,1]$; J_x is the primary membership function of x and a secondary membership function of x in $\tilde{A}$ denoted as $f_x(u)$ (Wang and Yu, 2011). The fuzzy type-2 set is in a region bounded by an upper membership function and a lower membership function denoted as $\bar{\mu}_{\tilde{A}(x)}$ and $\underline{\mu}_{\tilde{A}(x)}$ and is called the footprint of uncertainty FOU (Castillo and Melin, 2008; Karnik et al., 1999; Mendel, 2007b; Wang and Yu, 2011).

$$FOU(\tilde{A}) = U_{x\in X}\left[\underline{\mu}_{\tilde{A}(x)}, \bar{\mu}_{\tilde{A}(x)}\right] \tag{6}$$

The fuzzy type-2 process consists in the following steps: fuzzification, inference engine, type reducer and defuzzification. The inference process is done as follow according to the firing strength for the k rule as specified in Karnik et al. (1999):

$$F^k = \left[\underline{f}^k, \bar{f}^k\right] \tag{7}$$

where:

$$\begin{aligned} \underline{f}^k &= \min\left[\underline{\mu}_{E_1^k}(x_1), \dots, \underline{\mu}_{E_p^k}(x_p)\right] \\ \bar{f}^k &= \min\left[\bar{\mu}_{E_1^k}(x_1), \dots, \bar{\mu}_{E_p^k}(x_p)\right] \end{aligned} \tag{8}$$

for the $p-\text{th}$ input (Mendel, 2007a; Mendel and Wu, 2007; Zhou et al., 2009). Then the type reduction is done by finding the centroid of a fuzzy type-2 systems as shown below (Mendel, 2007a;

Mendel and Wu, 2007; Zhou et al., 2009; Mendel, 2005; Singh and Gupta, 2007):

$$y_l = min_{\theta_i \in \left[\underline{f}^k(y_i), \overline{f}^k(y_i)\right]} \frac{\sum_{i=1}^{N} y_i\theta_i}{\sum_{i=1}^{N} \theta_i}$$
$$y_r = max_{\theta_i \in \left[\underline{f}^k(y_i), \overline{f}^k(y_i)\right]} \frac{\sum_{i=1}^{N} y_i\theta_i}{\sum_{i=1}^{N} \theta_i} \quad (9)$$

for N samples. Then the defuzzified output is obtained by:

$$y = \frac{y_l + y_r}{2} = \frac{1}{2}(\underline{\xi}_r \quad \underline{\xi}_l)(\underline{\Theta}_r \quad \underline{\Theta}_l) = \xi^T\Theta \quad (10)$$

where ξ and Θ are the defuzzified parameters used in the design of our proposed fuzzy type-2 MIMO controller component for the stabilization of chaotic and hyper-chaotic systems as explained in the following paragraph (Morales-Mata et al., 2008; Kang and Vachtsevanos, 1992) .

Consider the fuzzy type-2 MIMO controller part with p inputs and q outputs, according to the chaotic and hyper-chaotic systems to be stabilized. The controller output (system input) is defined as Castillo and Melin (2014), Martino and Sessa (2014):

$$u_{ft2} = \begin{bmatrix} \xi_1^T\Theta_1 \\ \vdots \\ \xi_q^T\Theta_q \end{bmatrix} = \begin{bmatrix} u_{ft2,1} \\ \vdots \\ u_{ft2,q} \end{bmatrix} \quad (11)$$

Therefore the outputs of the fuzzy type-2 controller to stabilize the chaotic and hyper-chaotic systems are given by

$$u_{ft2,j}(y_j) = \frac{\sum_{i=1}^{N} y_{i,j} f_u(y_{i,j})}{N} \quad (12)$$

where N is the number of samples of the universe of discourse of input y_j for $j = 1, \ldots, q$ where $f_u(y_{i,j}) = 1$ considering that the secondary membership value is 1 (Wang and Yu, 2011). It is important to consider that if $y_{1,j} = y_{l,j}$, $y_{N,j} = y_{r,j}$, and N=2 then (10) is obtained. The fuzzy type-2 controller component for the control of the chaotic and hyper-chaotic system is given by Wang and Yu (2011), Castillo and Rico (2006) and Hagras (2004).

With these derivations the fuzzy type-2 controller component

$$u_{ft2,j}(y_j) = \frac{\sum_{i=1}^{N} y_{i,j}}{N} \quad (13)$$

is established. Then, the passivity based controller component is explained in the following subsection.

3.2. Passivity Based Controller Component

Even when there are some control strategies found in literature for chaotic and hyper-chaotic systems such as Effati et al. (2014) and Zhou and Huang (2014), passivity based control is a suitable control strategy for both kinds of systems, and with the addition of a fuzzy type-2 controller the performance is augmented significantly. In order to obtain the passivity based controller component for the two kinds of systems to be stabilized, the following definition is necessary (Liu et al., 2013; Haddad and Chellaboina, 2008)

Definition 1: The system (4) is passive if there exist a storage function V_s such that:

$$u^T y \geq \dot{V}_s = \frac{\partial V_s}{\partial x} f(x, u) \quad (14)$$

With this definition the passivity based control law component u_{pb} can be designed in order to meet the passivity condition. With this requirement the controlled chaotic and hyper-chaotic systems are stabilized according to their energy properties. The results obtained in this section are used to stabilize the chaotic Lorentz attractor and 4D hyper-chaotic systems; it is important to consider that the fuzzy type-2 control law u_{ft2} and the passivity based control law u_{pb} are designed independently in order to stabilize the two kinds of systems explained in Section 2 and improve the performance of the controlled systems. The appropriate membership functions and rules for the MIMO fuzzy type-2 controller are selected along with the passivity based controller for the two studied systems in this article.

4. PROPOSED CONTROL STRATEGIES FOR THE STUDIED CHAOTIC AND HYPER-CHAOTIC SYSTEMS

In this section the proposed control strategy is designed for the chaotic (Lorentz attractor) and hyper-chaotic system (4D system) studied in this paper. In the following subsections the MIMO fuzzy type-2 component is derived first showing the structure of this part of the control law including the membership functions and rules for the two systems analyzed in this article. Then, the passivity based controller is designed according to the energy considerations shown in Definition 1 in a separate fashion.

4.1. Proposed Control Strategy for the Chaotic Lorentz Attractor

In order to design the proposed control strategy for the Lorentz chaotic attractor shown in (1), the results obtained in Section 3 are implemented deriving first the MIMO fuzzy type-2 controller component and then the passivity based controller component shown in that section.

The MIMO fuzzy type-2 component for the chaotic Lorentz attractor is represented by (15) according to the results shown in (10) and (11) for q=3 due to the dimension of the system is 3 (three inputs and three outputs).

$$u_{ft2} = \begin{bmatrix} \xi_1^T\Theta_1 \\ \xi_2^T\Theta_2 \\ \xi_3^T\Theta_3 \end{bmatrix} = \begin{bmatrix} u_{ft2,1} \\ u_{ft2,2} \\ u_{ft2,3} \end{bmatrix} \quad (15)$$

The membership functions for the inputs of the MIMO fuzzy type-2 controller are shown in Tab. 3 with the membership functions N (negative), Z (zero), and P (positive).

Meanwhile in Tab. 4 the output membership functions of the fuzzy type-2 controller component are shown; where N (negative), Z (zero), P (positive) are the membership functions for each output. In Fig. 5 the respective membership functions for input and output 1 of the fuzzy controller component are shown.

Tab. 3. Input membership functions of the fuzzy type-2 controller component

Input variable	mf1	mf2	mf3
x_1	N	Z	P
x_2	N	Z	P
x_3	N	Z	P

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Tab. 4. Output membership functions of the fuzzy type-2 controller component

Output variable	mf1	mf2	mf3
$u_{ft2,1}$	N	Z	P
$u_{ft2,2}$	N	Z	P
$u_{ft2,3}$	N	Z	P

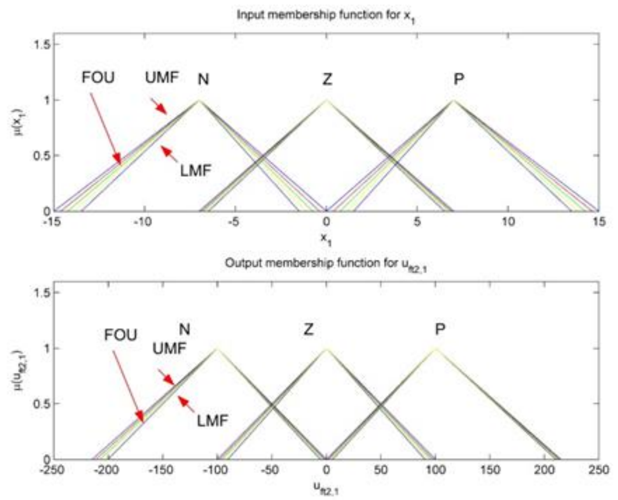

Fig. 5. Membership functions for the input and output 1

The rules of the MIMO fuzzy type-2 controller component for the chaotic Lorentz attractor are in the form:

$$\textit{IF } x_1 = N \textit{ AND } x_2 = N \textit{ AND } x_3 = N \textit{ THEN } u_{ft2,1} = Z \\ \vdots \tag{16}$$

so, the passivity based controller component is defined as follow based on the energy considerations explained in Definition 1, designing the passivity based controller independently, so the input u defined in (1) is $\mathrm{u} = \mathrm{u}_{\mathrm{pb}}$. The passivity based control law is derived according to the following theorem:

Theorem 1: A suitable passivity based control law u_{pb} component for the chaotic Lorentz attractor is found, according to the requirements of Definition 1, if there exists an appropriate storage function such as $\mathrm{V_s}(0) = 0$.

Proof: Consider the following storage function and system input $\mathrm{u}_{\mathrm{pb}}(\mathrm{t}) = [\mathrm{u}_{\mathrm{pb1}} \quad \mathrm{u}_{\mathrm{pb2}} \quad \mathrm{u}_{\mathrm{pb3}}]^{\mathrm{T}}$, as shown below:

$$V_s(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}x_3^2 \tag{17}$$

Now deriving (17) along the trajectory of (1) and reorganizing yields:

$$\dot{V}_s(x) = \begin{bmatrix} \sigma x_1(x_2 - x_1) \\ \rho x_1 x_2 - x_2^2 - x_1 x_2 x_3 \\ x_1 x_2 x_3 - \beta x_3^2 \end{bmatrix} + y^T u_{pb} \tag{18}$$

Then, in order to make the closed loop system passive, the following control law is chosen:

$$u_{pb} = \begin{bmatrix} -\sigma(x_2 - x_1) \\ -\rho x_1 + x_2 + x_1 x_3 \\ -x_1 x_2 + \beta x_3 \end{bmatrix} + v \tag{19}$$

where v is the input of the passivity based controller. Substituting (19) in (18) yields:

$$\dot{V}_s(x) = y^T v \leq y^T v \tag{20}$$

So the y system is passive by implementing the control law (19), where is defined in (1).

Finally, the input of the passivity based controller v is selected as $\mathrm{v} = -[\mathrm{x}_1 \quad \mathrm{x}_2 \quad \mathrm{x}_3]^{\mathrm{T}}$. With these results the hybrid control law u shown in (3) can be implemented with (19) as u_{pb}.

4.2. Proposed Control Strategy for the Hyper-Chaotic 4D System

The design of the proposed strategy for the control of the 4D hyper-chaotic system shown in (2) is done by following the procedure explained in Section 3. In this subsection a similar methodology as the implemented in the controller design for the chaotic Lorentz attractor is followed. First, the fuzzy type-2 controller component is derived and then the passivity based controller is derived.

The fuzzy type-2 controller part is given below:

$$u_{ft2} = \begin{bmatrix} \xi_1^T \Theta_1 \\ \xi_2^T \Theta_2 \\ \xi_3^T \Theta_3 \\ \xi_4^T \Theta_4 \end{bmatrix} = \begin{bmatrix} u_{ft2,1} \\ u_{ft2,2} \\ u_{ft2,3} \\ u_{ft2,4} \end{bmatrix} \tag{21}$$

The membership functions for the inputs of the MIMO fuzzy type-2 controller component of the 4D hyper-chaotic system is shown in Table 5, where they are called as N (negative), Z (zero) and P (positive).

Tab. 5. Input membership functions of the fuzzy type-2 controller component

Input variable	mf1	mf2	mf3
x_1	N	Z	P
x_2	N	Z	P
x_3	N	Z	P
x_4	N	Z	P

In Tab. 6 the output membership functions of the fuzzy type-2 controller component for the hyper-chaotic system are shown, where they are denoted as N (negative), Z (zero) and P (positive).

Tab. 6. Output membership functions of the fuzzy type-2 controller component

Output variable	mf1	mf2	mf3
$u_{ft2,1}$	N	Z	P
$u_{ft2,2}$	N	Z	P
$u_{ft2,3}$	N	Z	P
$u_{ft2,4}$	N	Z	P

The fuzzy type-2 membership functions for the input and output 1 of this controller component are shown in Fig. 6.

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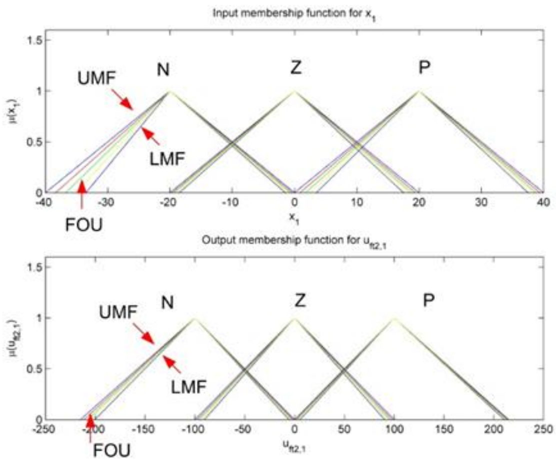

Fig. 6. Membership functions for the input and output 1

The rules of the MIMO fuzzy type-2 controller component implemented for the stabilization of the 4D chaotic system are in the form:

IF $x_1 = N$ *AND* $x_2 = P$ *AND* $x_3 = Z$ *AND* $x_4 = N$

then

$$u_{ft2,1} = Z \tag{22}$$

⋮

so, the passivity based controller component is defined as follow based on the energy considerations explained in Definition 1, designing the passivity based controller independently, so the input u defined in (2) is $\mathrm{u} = \mathrm{u_{pb}}$. The passivity based control law is derived according to the following theorem:

Theorem 2: A suitable passivity based control law $\mathrm{u_{pb}}$ component for the hyper-chaotic 4D system is found, according to the requirements of Definition 1, if an appropriate storage function such as $\mathrm{V_s}(0) = 0$ is selected.

Proof: Consider the storage function $\mathrm{V_s(x)}$ and system input vector, $\mathrm{u_{pb}(t)} = [\mathrm{u_{pb1}} \quad \mathrm{u_{pb2}}\ \mathrm{u_{pb3}} \quad \mathrm{u_{pb4}}]^{\mathrm{T}}$, as shown below:

$$V_s(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}x_3^2 + \frac{1}{2}x_4^2 \tag{23}$$

By taking the derivative of (23) along (2) and reorganizing the following result is obtained:

$$\dot{V}_s(x) = \begin{bmatrix} ax_1^2 - x_1x_2x_3 \\ x_1x_2x_3 - bx_2^2 \\ cx_1x_2x_3 - dx_3^2 + gx_1x_3x_4 \\ kx_4^2 - hx_2x_4 \end{bmatrix} + y^T u_{pb} \tag{24}$$

Therefore the following control law makes the closed loop system passive:

$$u_{pb} = \begin{bmatrix} -ax_1 + x_2x_3 \\ -x_1x_3 + bx_2 \\ -cx_1x_2 + dx_3 - gx_1x_4 \\ -kx_4 + hx_2 \end{bmatrix} + v \tag{25}$$

Then, substituting (25) in $\dot{\mathrm{V}}_\mathrm{s}(\mathrm{x})$ given in (24) with y defined in (2):

$$\dot{V}_s(x) = y^T v \leq y^T v \tag{26}$$

where the controller input v is selected as $\mathrm{v} = -[\mathrm{x_1} \quad \mathrm{x_2}\ \mathrm{x_3} \quad \mathrm{x_4}]^{\mathrm{T}}$. Then $\mathrm{u_{pb}}$ is given by (25) and can be substituted in the hybrid control law (3).

5. NUMERICAL EXAMPLES

In this section two numerical examples to evince the effectiveness of the proposed hybrid control strategy are shown in the following subsections.

5.1. Example 1: Hybrid Controller Strategy for the Chaotic Lorentz Attractor

The Lorentz attractor used in this example is (1) with initial conditions $\mathrm{X}(0) = [-8 \quad 8 \quad 27]^{\mathrm{T}}$. All the simulations were performed in MATLAB implementing the Runge-Kutta method to solve the chaotic and hyper-chaotic system.

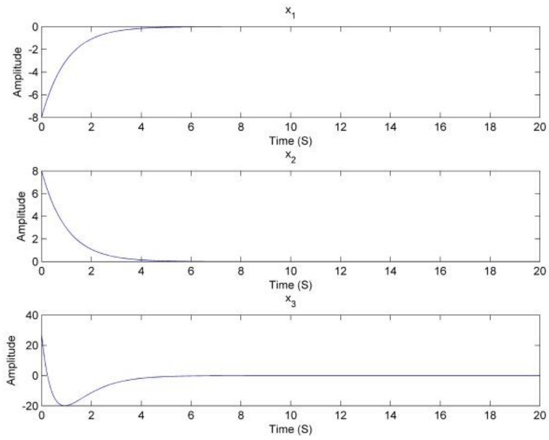

Fig. 7. State variables response of the Lorentz attractor

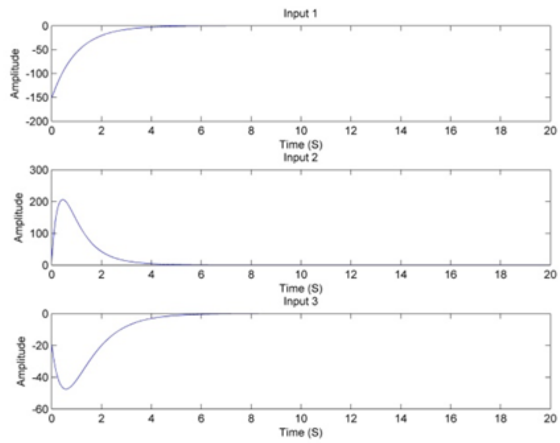

Fig. 8. Hybrid control inputs of the Lorentz attractor

In Fig. 7 the state variables of the Lorentz attractor are shown, where as it is noticed all the variables reach the origin proving that the system is stabilized by the proposed control law. With this example is corroborated numerically the theoretical results presented in the previous sections, so fuzzy type-2 and the passivity based hybrid technique is a suitable control strategy for different kinds of chaotic systems and not only for the Lorentz attractor analyzed in this article. In Fig. 8 the hybrid control input u for the Lorentz attractor are shown and as can be noticed the control effort of all inputs is significantly small, so the requirements of the controlled system are met.

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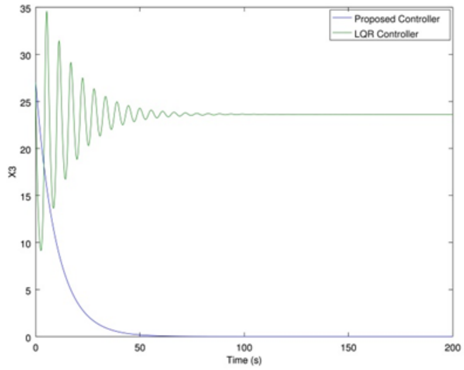

Fig. 9. Comparison between the proposed strategy and a linear quadratic regulator

In Fig. 9. a comparison between the proposed strategy and a linear quadratic regulator is shown and as can be noticed the response of the variable x_3 with the LQR strategy shows an oscillatory behaviour even when it stabilizes in other equilibrium point in comparison with the proposed control strategy, so the state variable response with the proposed approach is superior than the LQR strategy.

5.2. 5.2 Example 2: Proposed Hybrid Controller for the 4D Hyper-Chaotic System

In this subsection a numerical example implementing the hybrid controller to stabilizes the 4D hyper-chaotic system (2) is shown with initial conditions $X(0) = [-8 \quad 8\,27 \quad 0]^T$. In Fig. 10 the system response is shown, where as it is noticed all the variables reach the origin corroborating the theoretical results obtained in the previous sections. The hybrid fuzzy type-2 and passivity based control strategy is effective for the stabilization of any kind of hyper-chaotic systems of any dimension and not only the 4D hyper - chaotic system studied in this article.

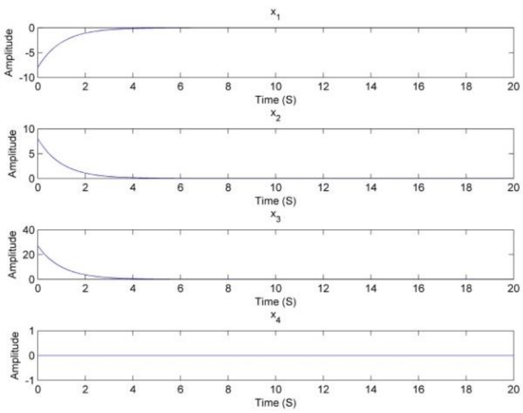

Fig. 10. State variables response of the 4D hyper-chaotic system

In Fig. 11 the hybrid control inputs of the 4D hyper-chaotic system are shown where as it is noticed the control effort for all the inputs is considerably small in order to stabilize the studied 4D hyper-chaotic system.

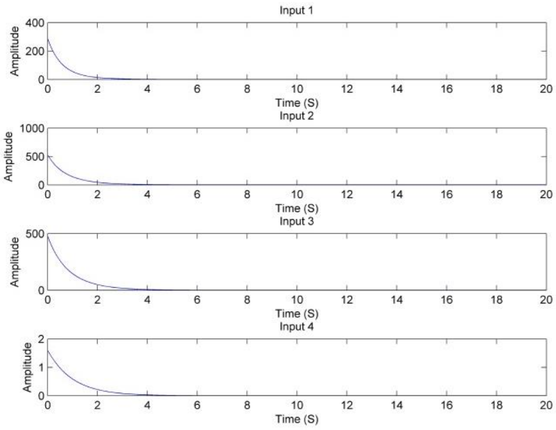

Fig. 11. Hybrid control inputs of the 4D hyper-chaotic system

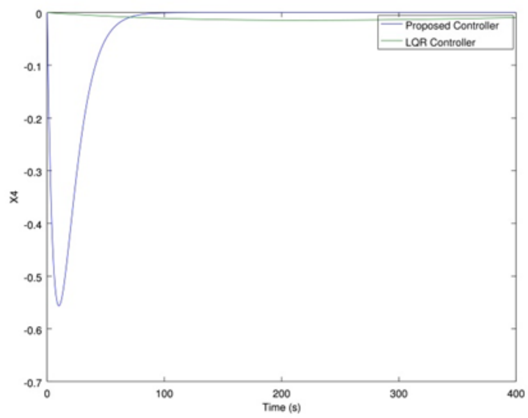

Fig. 12. Comparison between the proposed strategy and a linear quadratic regulator

Finally, in Fig. 12. a comparative analysis of the variable x_4 is shown where as can be noticed the response of this variable with the proposed control strategy in comparison with the LQR strategy is superior considering that this variable reaches the equilibrium point with no error instead of the convergence error of the variable response yielded by the LQR strategy.

6. DISCUSSION

According to the theoretical results obtained in this paper, it is proved that a hybrid fuzzy type-2 and passivity based control strategy is effective for the synchronization of chaotic and hyper-chaotic systems of different kinds. Due to the effectiveness of fuzzy type-2 controllers to deal with uncertainties and the capability of passivity based controllers to deal with the energy properties of nonlinear systems, a novel control strategy for chaotic and hyper-chaotic systems is developed to stabilize these kinds of systems independently of the initial conditions. Even when in this paper only 3D chaotic and 4D hyper-chaotic systems are considered, the proposed control strategy can be implemented for these kinds of systems of any dimension. The fuzzy type-2 membership functions and rules of this controller component can be easily obtained either by the expert knowledge or by any parameter adjustment methodology, such as genetic algorithms and particle swarm optimization, for example. The passivity based controller part is obtained by selecting an appropriate storage function in order to obtain the appropriate control law for the two kinds of systems analyzed in this study. The passivity based and the fuzzy type-2 controller components are designed independently, due to the inference properties of fuzzy type-2 controller so the stability of chaotic and hyper-chaotic systems of any dimension are assured.

7. CONCLUSION

In this paper a hybrid fuzzy-type 2 and passivity based controller for the stabilization of chaotic and hyper-chaotic system is shown. Even when the two analyzed systems are a chaotic Lorentz attractor and a hyper-chaotic 4D systems, the proposed control strategy can be implemented satisfactorily in some kinds of chaotic and hyper-chaotic systems of any dimension. This study begins with the analysis of a chaotic Lorentz attractor and a hyper-chaotic 4D system, in which their stability properties such as eigenvalues and Lyapunov exponents are analyzed along with their bifurcation diagrams to find when both systems are in stability, chaotic or hyper-chaotic behaviour respectively. Then, a general fuzzy type-2 and passivity based controller approach for some kinds of chaotic and hyper-chaotic systems of any dimension is presented that can be used in several kinds of systems and not only the analyzed ones in this article.

The proposed control strategy is developed by an independent design of the two components of the controller, the MIMO fuzzy type-2 component and passivity-based component. The MIMO fuzzy type-2 controller part is designed considering the expert knowledge and exploiting the advantages of fuzzy type-2 systems such as noise rejection and the capability to deal with uncertainties and finally, the passivity based controller component is designed considering the energy properties of the two studied systems. Passivity based control is implemented due to its effectiveness and suitability to stabilize different kinds of nonlinear systems, and when it is implemented along with a fuzzy type-2 controller the performance of the controlled systems are improved including other advantages such as noise and disturbance rejection.

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