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Developing a High-Performance System to Strengthen Construction Structures Against Mechanical Fatigue Using Shape Memory Materials

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ABSTRACT

To construct resilient structures, systems and sustainable buildings, capable of enduring fatigue, inclement weather, and seismic activity, researchers are actively seeking effective solutions to minimize vibrations and cyclic loading. Although these factors may not have immediate effects, they contribute to residual deformation in structures that gradually grows over time. For this reason, shape memory alloy (SMA) can be used as a perfect damper to dissipate the mechanical load in structures construction and buildings. The SMA actuators characterized by several thermo-mechanical functions, they are generally used in different applications as Mechatronics, Biomedical, Mechanical engineering and building systems. This study aims to adapt SMA actuator with structures for construction and buildings, in order to ensure a high displacement and vigilance taking into account fatigue phenomena to repulse mechanical fatigue and fretting. Accordingly, a thermomechanical analysis has been developed using finite element techniques to describe shape memory alloys' behavior and can integrate these material as a thermomechanical actuator dampers in building engineering systems. Furthermore, the suggested model elucidates the actuator's thermomechanical response, showcasing its adaptable behavior to both superelasticity and the shape memory effect within the desired structure in the building. Thus, the numerical findings affirm the efficacy of the proposed design that based on shape memory materials in addressing thermomechanical fatigue within buildings, concurrently enhancing structural resilience against mechanical fatigue. The primary outcome of this study is the successful preservation of the Ni-Ti superelastic response within the proposed system. This preservation is validated through cycling variations of up to 7.6% strain, significantly surpassing the requirements typically mandated for applications in earthquake engineering.

Keywords: shape memory alloys, SMA damper, fatigue, vibrations, structures, constructions.

INTRODUCTION

The industrial development has emphasized the importance of integrating multifunctional materials into various structures due to their high performance and adaptability to the surrounding environment, ensuring increased safety and prolonged lifespan of different systems [1, 2]. Over the past few years, SMAs have exhibited impressive flexibility, showcasing their adaptability across a wide range of sectors such as medicine, mechatronics, robotics, and construction [3, 4]. Indeed, the SMA can be designed in diverse shapes in order to provide more flexibility to its surrounding for instance the most famous forms are the springs, wires and films [5]. Specifically, the SMA spring exhibits a unique capability in responding phase change points with contraction and expansion, which can generate significant strain in response to mechanical force. Shape memory alloys owe their distinctiveness to martensitic transformation, a phase transition enabling them to adapt to mechanical forces and temperature shifts. Consequently, SMA actuators can be activated by heat or mechanical force. This dynamic behavior ensures the SMA actuator to switch between austenite prevalent at elevated temperatures, and martensite phases dominant at lower temperatures, regardless of stress levels. SMA materials exhibit superelastic behavior, with significant strain recovery under high stress during loading. Increasing stress in the austenite phase induces a phase change to oriented martensite, causing notable strain, which reverses upon unloading. Martensitic transformation and austenitic reversion in SMAs dissipate energy, known as damping, during load-unload cycles, making SMA actuators effective in attenuating vibrations in structures. However, the non-linear and unpredictable nature of SMAs, mainly due to hysteresis, presents challenges in actuator control. Hence, numerous studies have been conducted to enhance the reliability and performance of SMA-based systems in various applications within the built environment [6, 7]. Recently, the utilization of SMA actuators in regions experiencing intense stress can induce substantial deformations, often leading to residual deformations that exacerbate material fatigue over prolonged use. Recent advancements in research have aimed to address these challenges. For instance, Ju et al. [8] proposed a comprehensive model to describe phenomena such as fatigue and thermomechanical behavior in Ni-Ti shape memory alloys, while Petrini et al. [9] introduced a macroscopic approach for cyclically loaded device design using finite element simulations. These advancements offer promising avenues for improving SMA performance and reliability. Internal factors representing the build-up of inelastic stresses owing to fatigue, plasticity, and deformation amplitude are involved in the proposed model, which defines the cyclic behavior of SMA.

On the other hand, many researchers are implanting SMA wires in flexible plates in order to develop active systems for construction [10]. Andrew [11] provides an empirical work using advanced laminated plates based on fiber-reinforced composite combined with SMA wires in order to establish a novel approach for rapidly achieving the requisite wire temperature and tip deflection. Recently, Gideon [12] conducted a comparison between fibres and powder of Nitinol that integrated cement composite to achieved ductilities, concluding that Nitinol powder can be used as a ductile cement composite that has a good performance as the usage of fibres. Indeed, Abraik et al. [6] presents a special reinforced concrete wall with SMA that can be slowed down the motion of vibrating building by dissipating energy through plastic strain. Therefore, the SMA actuator can reduce significant permanent damage in buildings located in active seismic areas. Sourav Das [7] proposed to integrate SMA spring inserted in mid the column and the outrigger beam in order to control the vibration of tall timber buildings that can disperse the seismic activity energy and decrease abnormal load demand on the column. As a result, many studies has been conducted for adapting the shape memory alloy and describe their behavior respond to cyclic stress in order to integrate the SMA actuator in construction as a damper that assures and protects the building from the fretting and seismic activities.

The present study shows great potential by utilizing a robust model for SMA actuators to enhance material responsiveness. This methodology represents a notable advancement in SMA actuator technology, promising improved performance and durability in real-world applications. The primary goal of this contribution is to enhance structure systems' resilience against physical stressors like fatigue, vibrations, and severe weather conditions. The innovation lies in integrating a novel actuator, based on a shape memory alloy spring, into structure systems to prolong their lifespan and enhance construction safety against fatigue and vibrations.

PROPOSED SYSTEMS

In order to improve structures and constructions systems against physical threats as fatigue, vibration, and seismic activity, a novel actuator based on a shape memory alloy spring is proposed to ensure safety and optimize the lifespan of various construction elements. SMA springs are deemed ideal for damping due to their energy dissipation, recovery capabilities, and elastic flexibility. Consequently, an advanced SMA actuator (depicted in Fig. 1) is introduced, comprising four SMA springs functioning as damper actuators to mitigate physical threats, fatigue, and seismic activities. The proposed actuator is designed to coordinate four similar Ni-Ti actuators in opposition: when one SMA spring is active, the others remain inactive.

The cyclic loading provides significant variable stress that activates the Martensite phase change. Hence, the SMA can store the mechanical energy absorbed as work reacts to the variable stress. Consequently, the loading-unloading cycle can be repeated as presented in Figure 2: the system that is suggested in this study is made up of

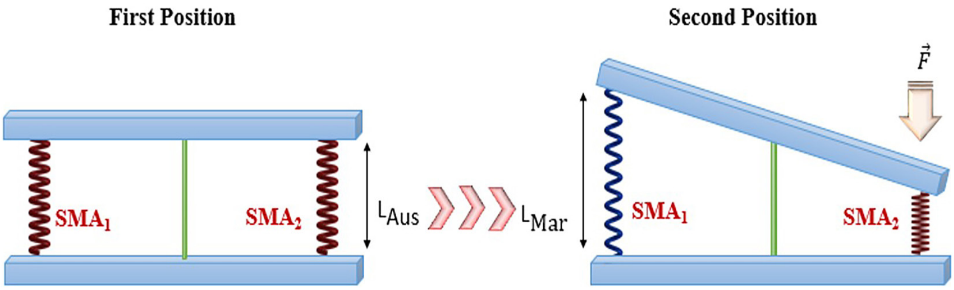

Figure 1. Functional model of the proposed system

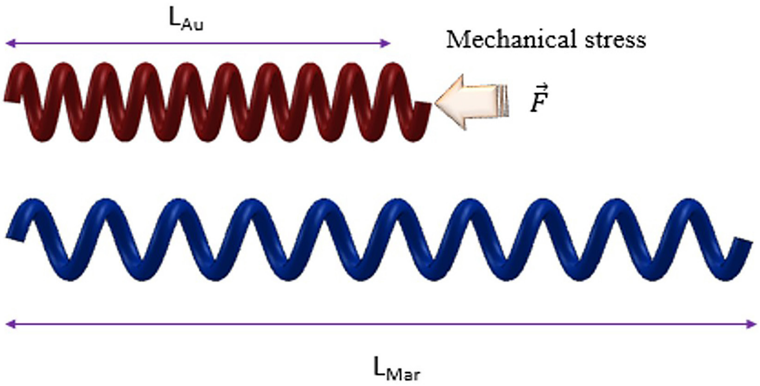

Figure 2. Functional principle of SMA spring under mechanical stress

four actuators that are linked together to operate as an additional support that consolidates the primary reinforcements and gradually absorbs various cycle loads (Fig. 3a). Thus, the actuator is made up of shape memory alloy springs that function as a damper to deflect and absorb irregular stresses, whether they be periodic or random (Fig. 3b). At normal temperature, which is the austenite phase, the SMA spring expands, helps the attached part to absorb vibration and return to its original shape after deformation (Fig. 3c). On the other hand, the spring will be compressed and absorbs the

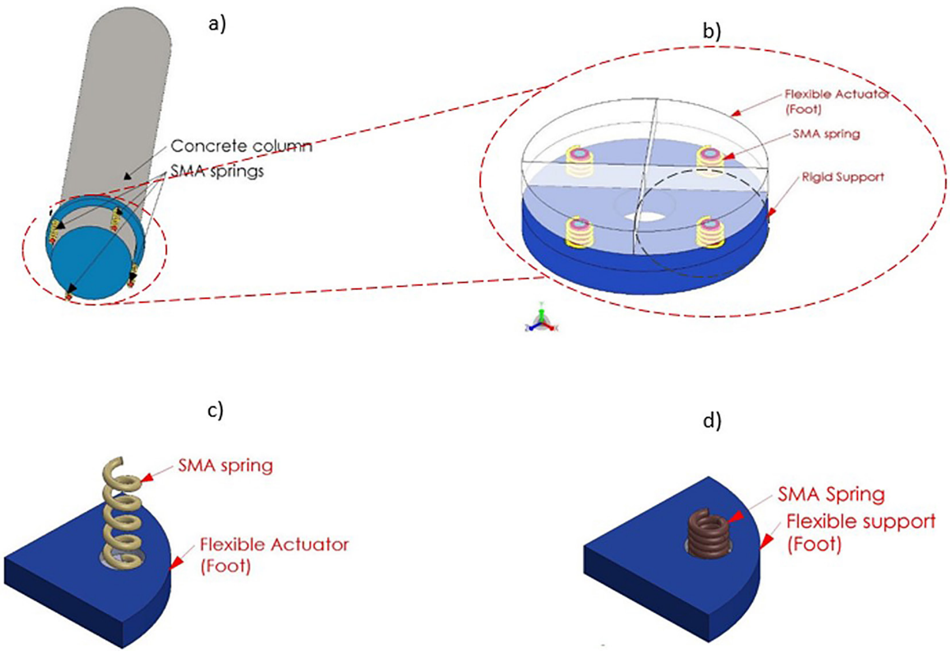

Figure 3. The proposed SMA Actuator integrated in concrete column

variable stress if the temperature is under the martensite start (Fig. 3d), however the structure will be back to the original form when the temperature increases [13].

MODELLING

The structures are designed perfectly for constructions systems and building which are involved with each other that no displacement is possible under ideal circumstances. However, under real conditions the structures can undergo different physical and chemical attacks. These phenomena capable of mobilizing the structure and creating a random stress that causes accompanying damages over time then sudden fatigue. Over the previous years, diverse models have integrated SMAs in order to adapt their non-linear behavior with structures. In this study, our goal is to adapt the behavior of SMA springs commonly used with various structures, particularly in this type of application. Hence, to adapt SMA effectively to proposed structure, it's essential to study and understand the different types of deformation that SMA springs undergo. Consequently, the total deformation can exhibit two primary types of deformation that possibly divided into inelastic deformation ϵ^{il} and elastic one ϵ^{el}:

$$\epsilon \ = \epsilon^{il} + \epsilon^{el} \tag{1}$$

Inelastic strain is the permanent deformation beyond the elastic limit of the SMA spring, while elastic strain is the reversible deformation within its elastic limit. Thus, the total deformation can be expressed:

$$\epsilon \ = {}^{\sigma}/_{\mathrm{E}} + b.\Delta T + \textstyle\sum_{n=1}^{24} \mathrm{f}^n R^n + \sum_{n=1}^{24} \gamma^n R^n \tag{2}$$

where: E – represents the Young modulus, σ denotes the stress, T denotes the temperature and b signifies the expansion coefficient. Where $\varkappa$ denotes the martensitic fraction and γ denotes the slippage of the friction slip system and R denotes the orientation tensor.

The suggested model of the SMA can be described using the total Helmholtz free energy that captures the energy associated with different previous deformations expressed as:

$$\begin{aligned} \Psi\left(T, \epsilon^{el}, \epsilon^{tr}, \epsilon^{p}, x,\right) &= \frac{1}{2\rho}\, \epsilon^{el}\!: E\!: \epsilon^{el} + \\ + C_v[\,(T - T_0) - T\, ln \frac{T}{T_0})\,] &+ b.(T - T_0)f + \\ + \textstyle\int_0^t \sum_{n=1}^{24} Y^p \dot{f}^n\, dt &+ \textstyle\int_0^t \sum_{n=1}^{12} R^n |(1 - f)\dot{\gamma}^n|\, dt \end{aligned} \tag{3}$$

where: T_0 – denotes the equilibrium temperature, C_v – signifies the heat capacity.

Incorporating force and stress into the thermo-mechanical model of a SMA spring involves considering their effects on the elastic energy and phase transformation behavior of the material. Force and stress play a crucial role in determining the mechanical response of the SMA spring to external loads and in driving phase transformations during the SME. The force applied to the spring actuator, as described in reference [14], can be computed using:

$$F_L = \frac{\left(\frac{d^4 S_{max}}{8D^3} G_l\right)\left(\frac{1}{n}\right) + F_{sm}\left(\frac{G_l}{G_h}\right)}{(G_l - G_h)} \tag{4}$$

where: F_L, F_{sm} – the restoring and driving load respectively, S_{max} – the maximum deformation of the spring, D – a characteristic dimension of the spring, d 2 the diameter of the spring wire, n is a material constant related to the material's properties, G_l, G_h – the low-cycle and the high-cycle fatigue endurance limit of the material respectively. The normal stress can be expressed as:

$$\sigma_{nor} = \frac{F}{S} = \frac{4F}{\pi d^2} \tag{5}$$

The fluctuation of various parameters can induce variable stress, which is characterized by a mean stress as demonstrated in the equation below:

$$\sigma_m \ = \frac{1}{2}(\sigma_{max} - \sigma_{min}) \tag{6}$$

where: σ_{max}, σ_{min} – represent, respectively, the maximum positive and minimum negative stresses experienced during each loading cycle.

RESULTS AND DISCUSSION

After describing how SMA behavior adapts to different structures, capturing the thermo-mechanical behavior of the SMA actuator in response to variable stress, a numerical study is conducted to validate the suggested model. This study defines the model's characteristics and the thermomechanical parameters of the suggested system under various loading conditions, temperatures, and timeframes. The strain responds to temperature fluctuations, which can trigger the Martensite transformation, as illustrated in Figure 4:

The proposed model effectively anticipates the thermomechanical behavior of SMA

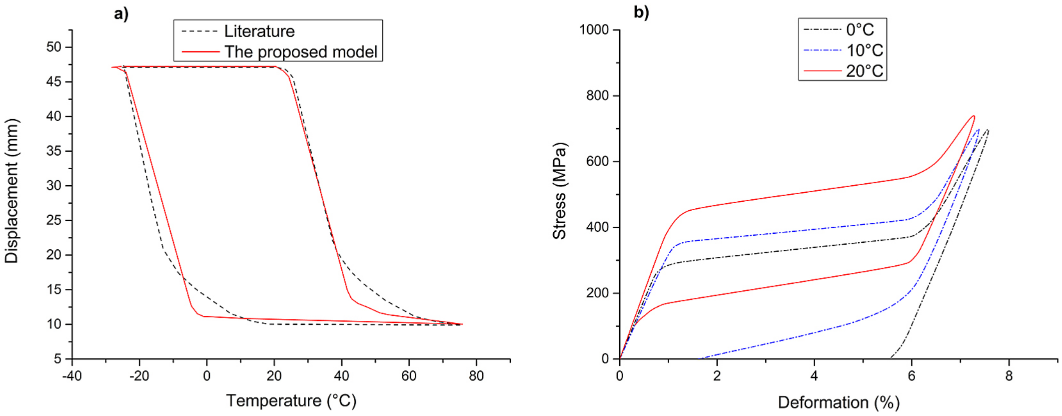

Figure 4. (a) Temperature evolution versus strain and stress; (b) Evolution stress versus strain

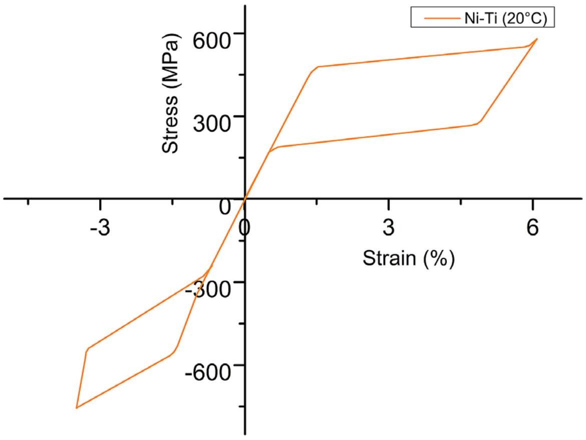

Figure 5. Strain-stress curve for tensile and compression load

materials under diverse conditions. It is observed that changes in temperature trigger the martensitic transformation, transitioning the material between the austenite phase and the self-accommodating martensite phase, and vice versa (Fig. 4a). Conversely, stress variations above the austenite temperature induce a transformation from austenite to oriented martensite, with strain disappearing upon unloading (Fig. 4b). These findings underscore the model's capability to capture complex interactions in SMAs. The current model effectively captures the core characteristics of the designated behavior. A thorough literature review, led by Karamooz-ravari, focuses on cyclic modeling, particularly regarding the shape memory and superelasticity behavior of NiTi fabricated via laser melting. This recent review has identified essential features of SMA for numerical simulation. These findings demonstrate promising concordance results and exhibit encouraging alignment with the intended outcomes [15].

The study subjected the SMA material to varying stress, resulting in significant tensile and compression strains, as depicted in Figure 5. It illustrates the system reaction to tensile-compression loading, displaying an asymmetric stress-strain response. Compression tests reveal an upper transformation start stress but a lower transformation strain, typically around 4%, compared to tensile tests which reach strains of approximately 6%. These differences arise from accumulated defects such as dislocations and twins induced during loading cycles. These defects create additional stress fields, influencing the material's behavior and hindering complete inverse transformations. As a result, residual martensite gradually accumulates over cycles, a phenomenon corroborated by previous studies [16, 17].These findings are reinforced by comparing our results with existing literature, which indicates variations in tensile and compression deformations. On the other hand, Alipour, [18] provide relevant insights through their investigation into the design, analysis, and manufacture of a tension-compression self-centering damper utilizing energy dissipation from pre-stretched superelastic shape memory alloy wires. Their findings align well with ours, further validating the current research.

Cyclic study for SMA actuator

Understanding the fatigue behavior of SMA actuators is vital for their practical utilization, especially in scenarios involving repetitive loading cycles throughout their operational life. Despite enduring cyclic loading, SMA actuators demonstrate robust resistance against fatigue, indicating minimal material degradation over repeated loading and unloading cycles.

The investigation generated simulation stress-strain curves illustrating strain evolution during cyclic loading. These curves are essential for comprehending SMA actuator performance in practical loading conditions, offering guidance for designing and implementing SMA-based systems in structures that need resilience against dynamic loading scenarios. In Figure 6, there's no clear discrepancy between the both stress responses compared to the hysteresis loops under static load. The simulation indicates that SMA can withstand cyclic loading, exhibiting minimal fatigue after numerous cycles. However, fatigue resistance may decrease due to multiple factors that influence the observed hysteresis curves. Thus, the current model effectively captures the fundamental characteristics of the designated thermomechanical behavior, placing particular emphasis on thermomechanical cyclic stress, a key feature within its promising framework. In support of this, parallel findings are echoed by Ravari [19], who developed a microplane constitutive model tailored for shape memory alloys, with specific attention to tension considerations. Moreover, these findings have been subsequently refined to encompass the cyclic shape memory and superelasticity exhibited by NiTi fabricated through selective laser melting.

The proposed model describes the SMA material for building applications which is captures correctly the super-elastic behavior and it in good agreement with experiments. In Figure 7, the stress plotted against deformation for SMA material precisely captures its superelastic behavior, consistent with both numerical and experimental findings. As cyclic tensile loading increases, there's a noticeable decrease in forward transformation stress. This underscores how cyclic stress affects SMA actuators, gradually causing microscopic residual deformation over time and ultimately shortening the material's lifespan. The literature echoes similar findings, as highlighted by Huang [20], who studied the impact of aspect ratio on the elastocaloric effect and cyclic stability of nanocrystalline NiTi. Conversely, Ashrafi's research [21] presented constitutive modeling techniques for shape memory alloys under cyclic loading, considering the effects of permanent strain. These findings reflect comparable work to our results. However, in the Figure 7 demonstrates that the proposed actuator can withstand various cyclic stresses due to its design, which compensates for different spring behaviors. This design feature enables the actuator to protect the structure: if one part of the design experiences high stress, other parts can handle the excess stress, restoring the structure and absorbing the stress [32].

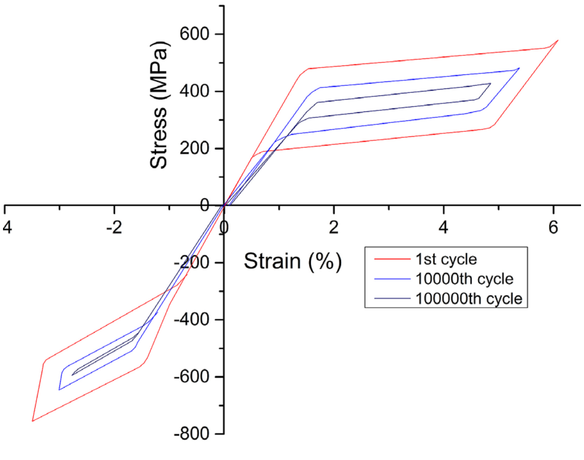

Figure 6. The stress-strain evolution under cyclic stress

To thoroughly investigate the performance of the proposed actuator, we conducted a comprehensive numerical study using COMSOL software. This powerful tool, grounded in the finite element method and equipped with an extensive material library encompassing shape memory alloys, provides an ideal platform for analysing the behaviour of these materials across diverse conditions.

The finite element method in this study breaks down complex structures into smaller, more manageable elements that achieve a mesh comprising approximately 13000 elements, ensuring detailed resolution for accurate analysis. Figure 8 illustrates the successful simulation of the thermo-mechanical behavior of SMAs. The SMA actuator demonstrates superelastic behavior, reverting to its initial state after stress removal (Fig. 8h). Additionally, the simulation predicts that the middle section is most prone to fatigue over time after multiple cycles.

The results suggest a response time of around 8 seconds, indicating an improvement of approximately 40% for the SMA actuator compared to results reported in the literature. This enhancement is attributed to the proposed design's capability to detect initial vibrations and reacting accordingly. Thanks to previous studies, upon which we relied to enhance our findings. The numerical study's results exhibit promising alignment with previous researches particulary that has been conducted by Varkani et al. [22], focusing on the mathematical modeling and dynamic response of concrete frames incorporating shape memory alloys under seismic loading conditions.

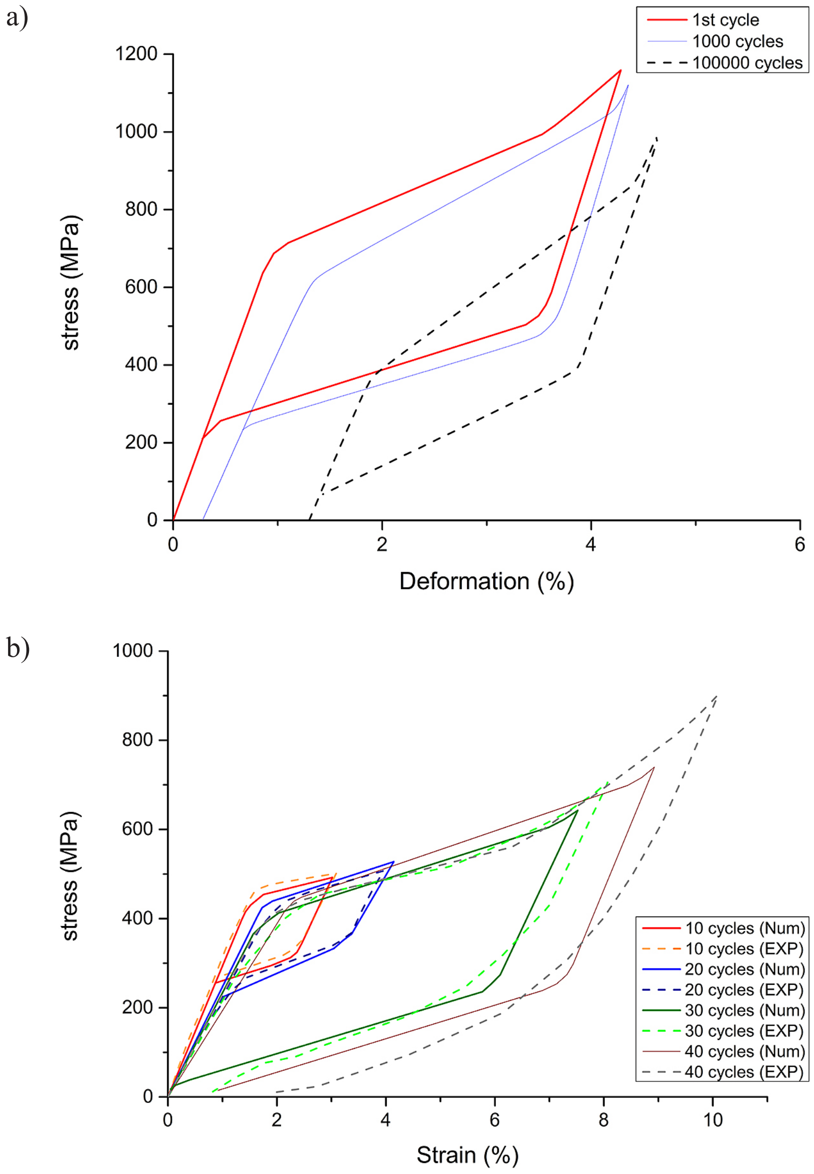

Figure 7. (a) Comparison between numerical and experimental results, (b) super-elastic behavior for a numerous repetitive cycles

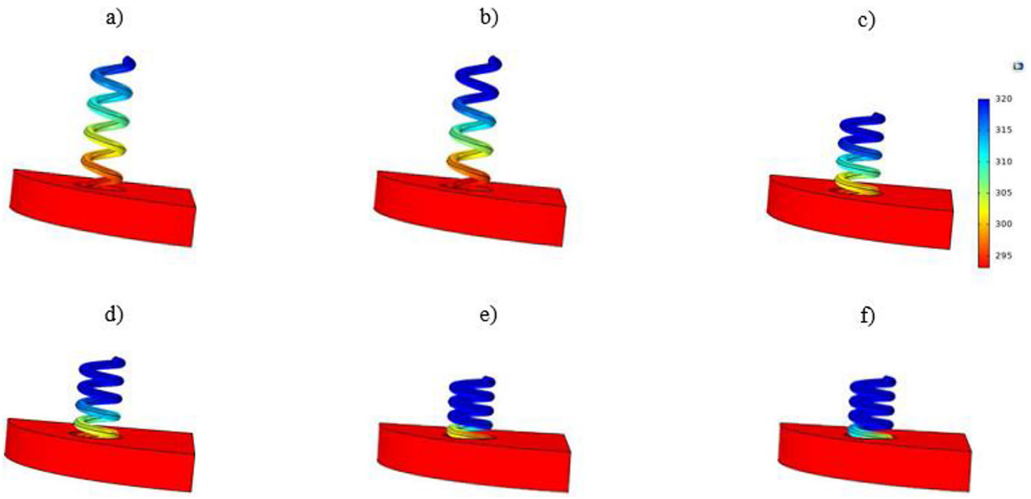

Figure 8. The hat transfers in the SMAA during phase transformation.

CONCLUSIONS

In order To extend the lifespan and improve the safety of structures facing cyclic stresses, physical attacks, and seismic events, a thermomechanical design utilizing shape memory springs has been proposed. This design aims to strengthen building structures and mitigate mechanical damages. Hence, the proposed design incorporates four SMA springs, functioning as damper, which are engineered to ensure that when one SMA spring is activated, the others remain inactive, effectively preventing and absorbing physical attacks, fatigue, and vibration activities. Furthermore, this study utilizes a mathematical model to capture the thermomechanical behavior of SMA actuators, revealing their shape memory effects and superelasticity under cyclic stress. Therefore, numerical investigations provide insights into the system's response to diverse loading scenarios. The efficacy of SMA actuators in dampening mechanical vibrations increases with greater deformations, attributed to their ability to store and release more energy. In fact, SMAs exhibit significant resilience, maintaining superelastic behavior even under strains of up to 7.6% and undergoing phase transitions across temperatures ranging from 30 °C to 100 °C. Consequently, quantitative data affirm the utility of SMAs in construction, demonstrating their capability to attenuate vibrations by 25%, and bolster structural stiffness by 20%, reduce building motion during seismic events by up to 30%. Despite inherent challenges in control and predictability, with error margins ranging from 5% to 15%, SMAs exhibit robust fatigue resistance, enduring up to 1,000 cycles. In fact, the response time is crucial for promptly stabilizing vibrations, thus ensuring the effectiveness of the damping system. Therefore, the key innovation lies in preserving the superelastic properties of Ni-Ti even during prolonged strain cycles, in addition to enhancing the actuator's responsiveness. Thus, enhancements in the actuator's response time can lead to an impressive reduction in vibrations around 40%. Such a significant advancement bolsters structural integrity and longevity, effectively reducing the propagation of cracks and averting sudden ruptures over time.

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