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Moving cast shadow detection using block nonnegative matrix factorization

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Abstract. In recent years, moving cast shadow detection has become a critical challenge in improving the accuracy of moving object detection in video surveillance. In this paper, we propose two novel moving cast shadow detection methods based on nonnegative matrix factorization (NMF) and block nonnegative matrix factorization (BNMF). First, the algorithm of moving cast shadow detection using NMF is given and the key points such as the determination of moving shadow areas and the choice of discriminant function are specified. Then BNMF are introduced so that the new training samples and new classes can be added constantly with lower computational complexity. Finally, the improved shadow detection method is detailed described according to BNMF. The effectiveness of proposed methods is evaluated in various scenes. Experimental results demonstrate that the method achieves high detection rate and outperforms several state-of-the-art methods.

Key words: moving cast shadow detection, video surveillance, nonnegative matrix factorization, block nonnegative matrix factorization.

1. Introduction

Moving cast shadow detection is a fundamental and critical task in visual application on stationary camera surveillance videos. In detecting the moving objects from a stationary camera video, usually a background subtraction technique is utilized for foreground extraction. However, cast shadows always move with their corresponding objects such that many background subtraction methods cannot separate them accurately. The inaccurate separation might lead to object merging, object shape distortion, and even object losses. Therefore, detecting and eliminating shadow regions is necessary in video processing and motion analysis fields.

In general, existing shadow detection methods can be classified into five categories using different features [1]: chromaticity, geometry, physical, textures and learning.

Chromatic-based methods are simple to implement so they can operate very fast. Chen et al. [2] analyze the shadow property in YUV space. Cucchiara et al. [3] adopted color information in HSV color space for shadow detection to improve object segmentation. The performances in different color spaces are evaluated in literature [4]. Sun et al. [5] propose a novel moving cast shadow detection method based on combined color models. Amato et al. [6] employed local color constancy property to detect both achromatic and chromatic shadows from foreground accurately.

Geometric based methods do not depend on the background reference but need more prior knowledge and scene limitations like illumination source, camera location and object shapes. A Gaussian shadow model [7] is put forward to parameterizing with several features including mean intensity, the orientation and center position of one shadow region. Amato et al. [8] segment each connected component of foreground into candidate regions by local color constancy detection.

Physical-based methods can adapt automatically to complex scene conditions. However, tat requires timely updating of shadow models and user interactions. A physical-based model on a spatio-temporal albedo test and a dichromatic reflection model for moving cast shadow detection are presented by Nadimi et al. [9]. Joshi et al. [10] introduced a shadow detection technique which was implemented by support vector machines (SVMs) based on semi-supervised.

Generally, texture-based shadow detection methods [11–18] assume that background image has similar texture with shadow regions and different texture with moving objects. Javed et al. [11] use color segmentation to get shadow candidate regions. In some cases, the employed color segmentation would make it more sensitive to noise. Edge correlation is used in literature [12] to remove shadows in normal indoor scenes. Ratio edge is first employed by Zhang et al. [13] as the ratio between the intensity of one pixel and its neighboring pixels to detect shadows. Sanin et al. [14] discriminate cast shadows from moving objects by means of gradient information. Xiao et al. [15] propose a method of moving shadow detection based on edge information, which can effectively detect the cast shadow of a moving vehicle in a traffic scene. The limitation of texture-based shadow detection methods is that these methods can detect shadow only for a small region in a frame. Moreover, it is slow as well. To relieve above mentioned problems, Sanin et al. [18] adopt color features for creation of candidate shadow regions.

In the light of the above algorithm, to improve the detection effect, more pattern classification methods can be applied in moving cast shadow detection such as [19, 20]. In this paper,

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NMF based on block idea is adopted to detect the moving cast shadow. In this paper, two novel moving cast shadow detection methods have been presented on the basis of NMF and BNMF. Firstly, the algorithm of moving cast shadow detection based on NMF is introduced. Secondly, the key design processes of our method, such as moving shadow area and discriminant function, are specified. Then the improved shadow detection method is derived on the basis of BNMF. The biggest advantage of our algorithm is its incremental adaptation capability. That is, when new training samples or new categories are added, there is no need to perform retraining using the entire database. Therefore, our method has very low computational complexity. In addition, unlike conventional methods, our algorithm can not only effectively detect moving cast shadow area, but also classify different classes of objects.

2. Nonnegative matrix factorization

In 1999, Lee and Seung proposed the notion of NMF [21, 22], as a way to recover hidden nonnegative structures or patterns from nonnegative data. Given n data points with m features, we denote the input data by the matrix $\mathbf{X} \in \mathbb{R}^{m \times n}$. Here, the symbol $\mathbf{X} \in \mathbb{R}$ means the real data sets with nonnegative elements. This collection of data is expected to be categorized or partitioned into c groups. Nonnegative matrix factorization (NMF) aims to find two non-negative matrices $\mathbf{W} \in \mathbb{R}^{m \times r}$ and $\mathbf{H} \in \mathbb{R}^{r \times n}$, such that the product of them approximates the original data matrix as much as possible, i.e.,

$$\mathbf{X} \approx \mathbf{WH}, \tag{1}$$

where $\mathbf{W}$ is regarded as a basis matrix and $\mathbf{H}$ is a coefficient matrix. In practical applications, the reduced dimension r is generally much lower than the rank of $\mathbf{X}$, i.e., $r \ll m, r \ll n$. Each $\mathbf{h}_j (j = 1, 2, \cdots, n)$ which is the column vector of the efficient matrix $\mathbf{H}$ is treated as the low-dimensional data representation of the data point $\mathbf{w}_j$ under the new basis. NMF approximates it by a linear combination of r "basis" columns in $\mathbf{W}$ if each column of $\mathbf{X}$ represents an object. This method has found a variety of real-world applications in the areas such as pattern recognition, dimensionality reduction, clustering etc.

3. NMF based moving shadow detection

3.1. Determination of moving shadow areas. Before using NMF based detection algorithm, some key points such as the determination of moving shadow areas and the choice of parameter r and discriminant function must be considered. Normally, the distinction between shadow points and non-shadow points is on the basis of the features of brightness, color and so on. These features in matrix $\mathbf{X}$ will be reflected by the matrixes $\mathbf{W}$ and $\mathbf{H}$ after NMF. However, the dimensions of $\mathbf{W}$ and $\mathbf{H}$ are different with $\mathbf{X}$. The classification is meaningless if the shadow points in original image can not be effectively located in the detection process. As a result, we propose the detailed solution as follows:

Firstly, we divide image matrix into several image pieces with the same size. Then each piece is decomposed by NMF. According to the classification process, the corresponding image piece is treated as shadow area or non-shadow area. The key point of the second solution is the size of the image piece. If the size is too large, the detection error becomes large. On the contrary, if the size is too small, the calculation becomes huge. In our experiment, original matrix is divided into small pieces with $N \times N$ size. Then shadow is detected by NMF algorithm. If some piece can not be classified correctly, we divide it into smaller piece with $(N/2) \times (N/2)$ size and shadow detection is executed again. Above process are continually implemented until all pieces are classified properly. According to a large number of pre-experiments, we adopt the piece with size 8×8 generally in the detection process generally.

3.2. Discriminant function. There is no ideal solution about how to choose the parameter r. the value of r represents the dimension of the feature subspace. If r and the feature space dimension of actual dataset have the equal size, the feature subspace of the NMF would have the best press from both sides to the data. In fact, we choose the most suitable r according to multiple tests.

The choice of discriminant function is very important in the classification process. Generally, the discriminant function is given as $g'(\mathbf{x}, L_i) = \|\mathbf{H}_i - \tilde{\mathbf{H}}_i\|_2$. In our method, considering the characteristic of NMF, we choose the discriminant function as

$$g(\mathbf{x}, L_i) = k_1 \left\| \mathbf{x} - \mathbf{W}^i \mathbf{H}_i \right\|_2 + k_2 \left\| \mathbf{H}_i - \tilde{\mathbf{H}}_i \right\|_2, \tag{2}$$

where $$\tilde{\mathbf{H}}_i = \mathbf{W}_{i^+} x = \left((\mathbf{W}_i)^T \mathbf{W}_i \right)^{-1} (\mathbf{W}_i)^T x, \tag{3}$$

x represents the test sample and L_i represents the subspace of i-th class. In the process of shadow detection, we divided the objects in the scene into different classes such as building, shadow, person and so on.

According to the Eq. (2), the discriminant function $g'(\mathbf{x}, L_i)$ takes into account not only the decomposition accuracy of NMF in the part of $\|\mathbf{x} - \mathbf{W}^i \mathbf{H}_i\|_2$, but also the classification condition of shadow and non-shadow in the part of $\|\mathbf{H}_i - \tilde{\mathbf{H}}_i\|_2$. As a consequence, the detection effect using $g'(\mathbf{x}, L_i)$ is better than that using $g'(\mathbf{x}, L_i)$ inevitably.

3.3. NMF based shadow detection. We introduce a method of shadow detection using NMF. Shadow detection can be regarded as a recognition problem. It can be realized as long as the shadow region is recognized. So we propose the following algorithm.

Algorithm 1. NMF based shadow detection method
1) Divide the training images into blocks with the same size and form the training image block set.
2) According to the different objects in the blocks, classify the training block set into c class (in this paper, we divide the training samples into 2 classes: shadow block and non-shadow block.)

3) Extract the color and brightness features of each training sample. Obtain the each class sample feature $\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_c$ according to simple feature weighted fusion, where c is the number of class.
4) Obtain the subspace basis matrix $\mathbf{W}_1, \mathbf{W}_2, \cdots, \mathbf{W}_c$ and the corresponding coefficient matrix $\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_c$ of the each class training samples $\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_c$ using basic NMF decomposition algorithm [21, 22].
5) Denote $\mathbf{W}_{i^+}$ be the generalized inverse matrix of $\mathbf{W}_i$ where $\mathbf{W}_i = \left[\mathbf{W}_i^1, \mathbf{W}_i^2, \cdots, \mathbf{W}_i^r\right]$ is the vector space basis of samples in *i-th* class.
6) Calculate the coefficient matrix $\tilde{\mathbf{H}}_i$ of the test samples x with respect to the corresponding vector space basis using Eq. (3)
7) Compute the Euclidean distance of test sample x with respect to each class according to the discriminant function as shown in Eq. (2)
8) According to the following formula, determine the class which x belongs to
$j = \arg\min g(\mathbf{x}, L_i),\ C(x) = j$

4. Block NMF (BNMF) based moving shadow detection

4.1. Block NMF. In order to overcome the disadvantages of NMF, Block NMF (BNMF) is introduced. According to BNMF, the training samples in the same class are collected to form a block matrix. Then, every block matrix is factorized by NMF. We can get the final NMF using NMF of all block matrices.

We define $\mathbf{X} = \left[\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_c\right]$, where $\mathbf{X}_i = \left[\mathbf{x}_1^{(i)}, \mathbf{x}_2^{(i)}, \mathbf{x}_{n_i}^{(i)}\right]$, $\mathbf{x}_j^{(i)}$ is the training sample belong to the *i-th* class, n_i is the number of training samples in the i-th class, c is the number of class, $i = 1, 2, \cdots, c, j = 1, 2, \cdots, n_i$.

Suppose: n_0 is set up as the number of the training sample in each class. The total number n of the training samples is cn_0. According to NMF, we can factorize each $\mathbf{X}_i$ as follows:

$$(\mathbf{X}_i)_{m\times n_0} \approx (\mathbf{W}_i)_{m\times r_0}(\mathbf{H}_i)_{r_0\times n_0};\ i = 1, 2, \cdots, c$$

Then we have

$$\left[\mathbf{X}_1\ \ \mathbf{X}_2\ \cdots\ \mathbf{X}_c\right] \approx \left[\mathbf{W}_1\ \ \mathbf{W}_2\ \cdots\ \mathbf{W}_c\right]\begin{bmatrix} \mathbf{H}_1 & & & \\ & \mathbf{H}_2 & & \\ & & \ddots & \\ & & & \mathbf{H}_c \end{bmatrix}. \quad (4)$$

Let $\mathbf{W}_{m\times r} = \left[\mathbf{W}_1\ \ \mathbf{W}_2\ \ \mathbf{W}_c\right]$, $\mathbf{H}_{r\times n} = diag(\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_c)$, where $r = cr_0$. According to Eq. (4), we can get the block non-negative matrix factorization (BNMF) $\mathbf{X}_{m\times n} \approx \mathbf{W}_{m\times r}\mathbf{H}_{r\times n}$. By the Eq. (4), it is easy to see that the column vectors of different class in $\mathbf{H}_{r\times n}$ are orthogonal and it can eliminate the correlation between the different classes. As a result, the features of different class can be extracted perfectly by BNMF algorithm. If the new classes or samples are added, we can get the decomposed results of BNMF according to the following rules.

1) If new T samples $\mathbf{x}_{i1}, \mathbf{x}_{ij}, \cdots, \mathbf{x}_{iT}$ are added into the *i-th* class, we just need to calculate $\tilde{\mathbf{X}}_i \approx \tilde{\mathbf{W}}_i\tilde{\mathbf{H}}_i$ according to NMF, where $\tilde{\mathbf{X}}_i = \left[\mathbf{X}_i, \mathbf{x}_{i1}, \mathbf{x}_{ij}, \cdots, \mathbf{x}_{iT}\right]$ is the new *i-th* class. Then the result of BNMF is given as

$$\tilde{\mathbf{X}} \approx \tilde{\mathbf{W}}\tilde{\mathbf{H}} = \left[\mathbf{W}_1\ \cdots\ \tilde{\mathbf{W}}_i\ \cdots\ \mathbf{W}_c\right]\begin{bmatrix} \mathbf{H}_1 & & & & \\ & \ddots & & & \\ & & \tilde{\mathbf{H}}_i & & \\ & & & \ddots & \\ & & & & \mathbf{H}_c \end{bmatrix}. \quad (5)$$

2) If new class $\mathbf{X}_{c+1}$ is added into sample space, the training sample matrix changes to $\tilde{\mathbf{X}} = \left[\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_c, \mathbf{X}_{c+1}\right]$. We just need to calculate $\mathbf{X}_{c+1} \approx \mathbf{W}_{c+1}\mathbf{H}_{c+1}$ according to NMF. Then the result of BNMF is given as

$$\tilde{\mathbf{X}} \approx \tilde{\mathbf{W}}\tilde{\mathbf{H}} = \left[\mathbf{W}_1, \cdots, \mathbf{W}_c, \mathbf{W}_{c+1}\right]\begin{bmatrix} \mathbf{H}_1 & & & \\ & \ddots & & \\ & & \mathbf{H}_c & \\ & & & \mathbf{H}_{c+1} \end{bmatrix}. \quad (6)$$

4.2, Improved shadow detection algorithm. From the Eq. (6) and (7), the main advantage of BNMF based method is that the new training samples and new classes can be added constantly with lower computational complexity in the detection process. The new training samples will help to improve the accuracy of detection. The new classes in detection process will contribute to multi-class classification. We can detect the other classes of objects such as persons, buildings, etc in detection process. Algorithm 2 shows the outline of our approach, which we refer to as BNMF based moving shadow detection algorithm. Meanwhile, the flowchart of this method is given in Fig. 1.

Algorithm 2. BNMF based moving shadow detection method
1) Divide the training images into blocks with the same size and form the training image block set.
2) According to the different objects in the blocks, classify the training block set into c class (in this paper, we divide the training samples into 3 classes: shadow block, non-shadow block, and the building.)
3) Extract the sample features of shadow areas, non-shadow areas and other classes such as building, person and so on. For gray images, the sample features are gray values. For color images, the samplefeatures are the fusion of brightness features and color features. Then each class samples are represented by matrix $\mathbf{X}_i$, where i is the number of class and $i = 1, \cdots, c$.
4) Decompose each $\mathbf{X}_i$ by NMF and compute the corresponding $\mathbf{W}_i$ and $\mathbf{H}_i$.
5) Obtain the matrixes $\tilde{\mathbf{W}}$ and $\tilde{\mathbf{H}}$ using the Eq. (4) and (5) if new samples or classes are added.
6) Compute the coefficient matrix of the test sample x with respect to the corresponding vector space basis $\tilde{\mathbf{W}}$ using the formula as $\breve{\mathbf{H}} = \tilde{\mathbf{W}}^{+}x = \left(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}\right)^{-1}\tilde{\mathbf{W}}^T x$ where x represents the test sample, $\tilde{\mathbf{W}}^{+}$ is the generalized inverse matrix of $\tilde{\mathbf{W}}$.

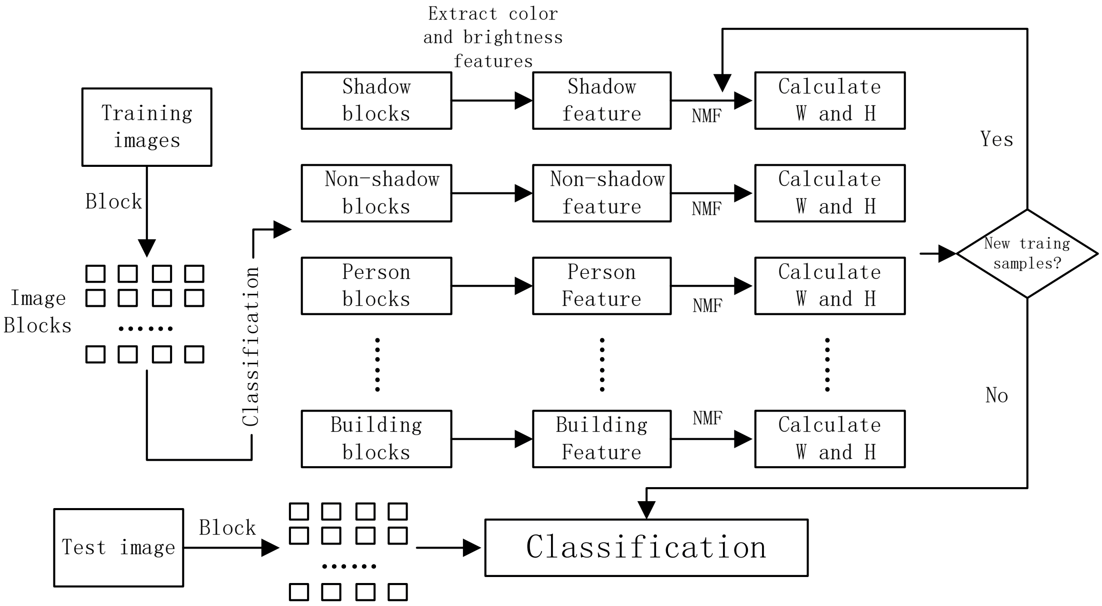

Fig. 1. The flowchart of the Algorithm 2

7) Calculate the distances between the test sample x and the training samples of each class using the formula as

$$d(\hat{\mathbf{H}}_i, \breve{\mathbf{H}}) = \|\hat{\mathbf{H}}_i - \breve{\mathbf{H}}\|_2,$$

where $\hat{\mathbf{H}}_i = \tilde{\mathbf{W}}^{+}\bar{x}_i, \cdots, c.\ \bar{x}_i$ is the sample mean of the i-th class.

5. Experimental results

In this section, we evaluate the proposed method with real data sets, both qualitatively and quantitatively. In order to evaluate the performance of our method effectively and systematically, we compare our method with several state-of-the-art methods [16–18] to prove the superiority from the aspects of quality and quantity. The test sequences and related ground truths in experiment are all publicly available1 and used in algorithm evaluation frequently. In each sequence, we randomly choose 20 frames as the training samples to learn the model of the objects and shadows. In each training sample, we get the objects and shadows in the same class manually. In the test process, all frames in each sequence are implemented by NMF and BNMF to evaluate the performance of detection rate and operation speed. The sample frames of these test sequences are showed in Fig. 1. In our experiments, the methods labeled Proposal 1 and Proposal 2 represent the shadow detection algorithms based on NMF and BNMF, respectively.

5.1. Quantitative evaluation. The performance is estimated quantitatively to obtain a systematic and objective evaluation of the proposed algorithm. Commonly, any one metric cannot be competent to evaluate the overall detection performance. According to literature [23], detection rate η and shadow discrimination rate ξ are employed. In our paper, we calculate the average of shadow detection rate and shadow discrimination rate simultaneously.

The quantitative comparison results are presented in Table 1, in which the best experimental results are emphasized in bold.

Table 1
Quantitative comparison results

Sequence	Highway		Room		Lab		Campus	
method	η	ζ	η	ζ	η	ζ	η	ζ
SNP	44.87	69.88	83.87	73.78	61.29	74.52	82.04	70.28
DNM	87.30	54.92	83.99	76.20	86.54	80.28	84.19	77.61
ICF	83.11	59.61	83.50	82.58	79.58	86.37	80.15	85.33
CCM	90.60	39.44	85.10	83.22	89.56	91.05	89.27	87.25
Proposal 1	93.22	81.69	93.18	87.27	92.14	93.13	92.64	91.39
Proposal 2	94.55	85.19	95.36	91.37	96.23	95.21	95.10	94.24

From Table 1, it can be easily concluded that proposed methods perform better that others, achieving the highest shadow detection rate and the highest shadow discrimination rate. In a word, our methods show the best detection performance, followed by CCM. Other methods provide relatively worse detection performance.

From the experiments, we can also find that the detection results of BNMF method is better than that of the NMF method. The reason is that the decomposition results of BNMF include basis matrixes and coefficient matrixes of all class which can be seen from (5–7). The coefficient matrix of BNMF is a diagonal matrix which elements are the coefficient matrixes of all classes. So the column vectors of various coefficient matrices are mutually orthogonal and the relevance of different class is eliminated. As a result, the recognition effect is improved.

5.2. Qualitative evaluation. To prove the effectiveness of the proposed algorithms, qualitative evaluation is implemented by four state-of-the-art shadow detection methods and our methods for comparisons clearly.

The qualitative evaluation is performed in four scenes: (i) Highway I, (ii) Room, (iii) Lab, (iv) Campus, which are representations of outdoor scenes with camouflages, indoor scenes with more than one source lights, indoor scenes with one light, and outdoor scenes with a lot of shadows respectively.

We illustrate the visual comparison results in Fig. 2 in which each row shows detection result obtained by different methods and our two proposed methods. For comparison, the detection results of our algorithms on the four diverse applied scenes are given in rows (d) and (e). The corresponding results of other three state-of-the-art methods are also presented in rows (a-c) of Fig. 2. As shown in the figure, moving cast shadows can be detected correctly by all methods in a certain degree. Obviously, it can be seen that the proposed method performs better on the four diverse applied scenes.

(a)
(b)
(c)
(d)
(e)

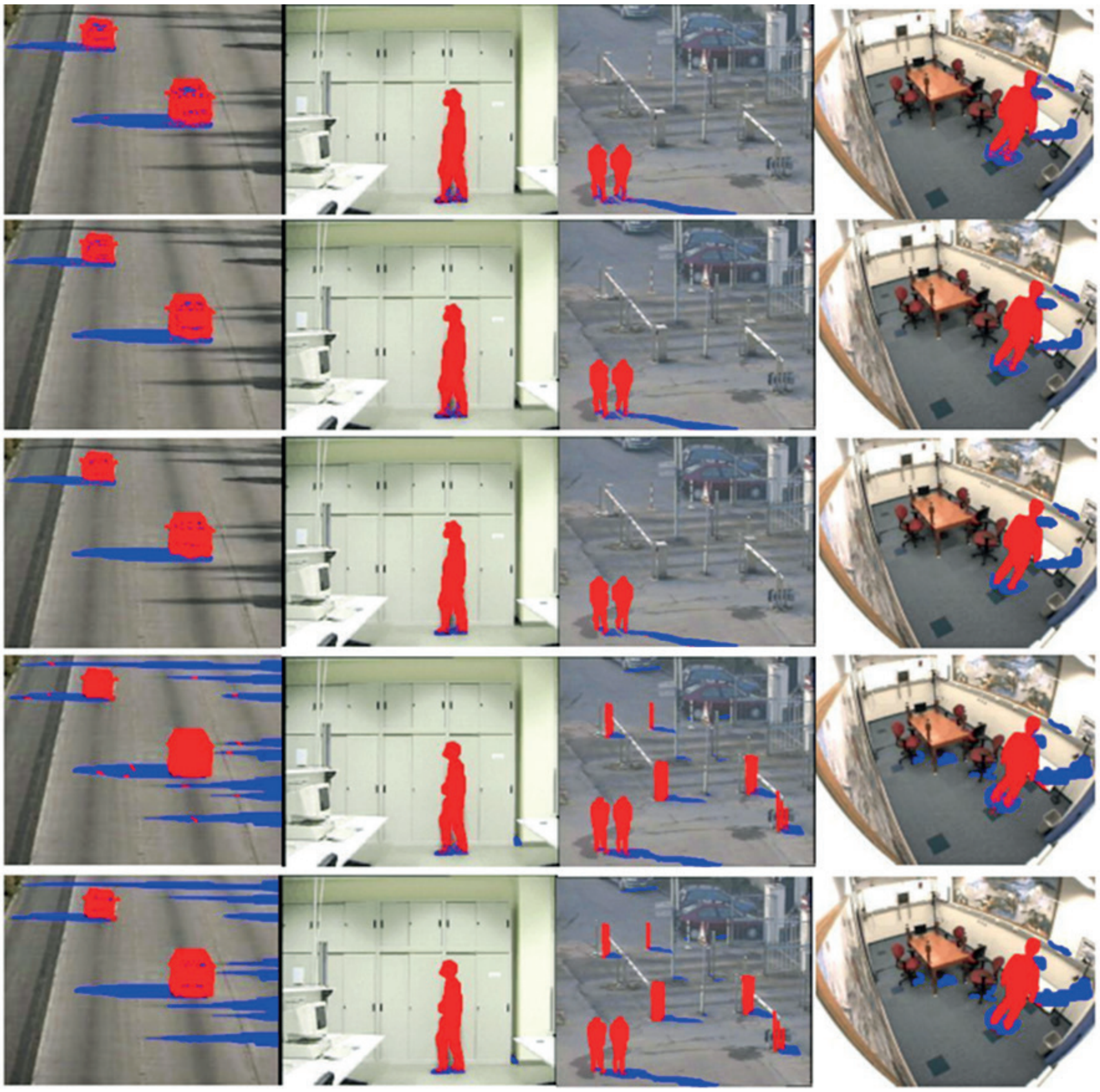

Fig. 2. The visualized comparison results of two frames in different 4 videos (a) [16], (b) [17], (c) [18], (d) Proposal 1, (e) Proposal 2

6. Conclusions

In this paper, two novel moving cast shadow detection methods have been presented on the basis of NMF and BNMF. Firstly, the algorithm of moving cast shadow detection based on NMF is introduced. Then the key points of our method such as the determination of moving shadow areas and the choice of discriminant function are specified. Finally, the improved shadow detection method is derived according to BNMF. The biggest advantage of our algorithm is that when new training samples are added into a certain class or a new class is added into database, BNMF need not to be re-executed so that it can get lower computational complexity. In addition, unlike conventional methods, our algorithm can not only effectively detect moving cast shadow area, but also classify different classes of objects. In a word, the BNMF based method is that the new training samples and new classes can be added constantly with lower computational complexity in the detection process. The new training samples will help to improve the accuracy of detection. The new classes in detection process will contribute to multi-class classification. We can detect the other classes of objects such as person, building and so on in detection process.

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