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DETERMINING THE DISTRIBUTION OF ACOUSTIC FIELD INTENSITY USING THE SOUND RAY DENSITY METHOD

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The article presents a numerical method for determining the intensity of sound. The method is based on surface density of sound rays. It assumes that a ray of sound carries a specific acoustic power. As a result, it is no longer necessary to determine intensity in caustics, especially given the fact that intensity determined in caustics using the method employed so far has an infinite value.

INTRODUCTION

The range, accuracy of bearing and other sonar operating parameters largely depend on the local conditions of acoustic wave propagation, and in particular on range distribution of sound velocity. The distribution of sound velocities is measured and used as a basis for determining the spatial distribution of acoustic wave intensity. What the distribution shows are areas in which the target can be found and shadow zones in which the intensity is not strong enough for detection to occur.

The most frequent method used to determine the distribution of acoustic field intensity assumes that intensity is inversely proportional to the distance between two adjacent *sound rays* which is computed using geometrical acoustics method [1]. The method yields good results with the exception of particular places (*caustics*) in which the rays of sound cross. In these areas, the intensity computed reaches infinity which is contrary to the energy conservation law and the results of measurements [2].

Presented below is a numerical method for determining the distribution of sound intensity which solves the problem of caustics.

1. DESCRIPTION OF THE METHOD

So far it was assumed that a specific power of the acoustic wave is contained in the cone which is bounded by two neighbouring rays sent from a source. Wave intensity is then inversely proportional to the area of the cone's cross-section. When rays of sound cross, the area of the cross-section is zero and intensity has an infinite value. To avoid the problem of ray crossing, it was assumed that the power of the wave is contained in an infinitely narrow pipe which is represented by a single ray of sound. To enable numerical computations it was also assumed that power is zero beyond the rays. With such assumptions the model of propagation is a *discrete* one, analogue to the discrete model of sound. From the source a certain number of rays is sent in various directions. Each of the rays carries a specific power. The entire power radiated by the source is equal to the sum of power contained in all rays. Wave intensity in a specific point of the centre is equal to the number of rays cutting across the surface having a unit field and stretching around the point. Because the number of rays is limited and the area of the surface is constant and different from zero, the intensity computed in this way will never reach infinity.

The above method could be put to a direct use, if the computations were applied to a three-dimensional space. To do that would require a great – and what is more important – unnecessary number of numerical operations. The number of operations can be radically reduced, if we assume that the distribution of the acoustic field has axial symmetry. This assumption makes the spatial problem a planar one, but requires some modification of the above computational method. Another area to be analysed is the problem of describing the field in rectangular co-ordinates instead of in polar co-ordinates, which would be natural in this case.

Let us consider an element of the surface given in Fig. 1 whose dimensions are:

$$\Delta h(x_1,0)=\Delta\theta\cdot x_1,\qquad \Delta l(x_1,0)=\Delta\varphi\cdot x_1 \tag{1}$$

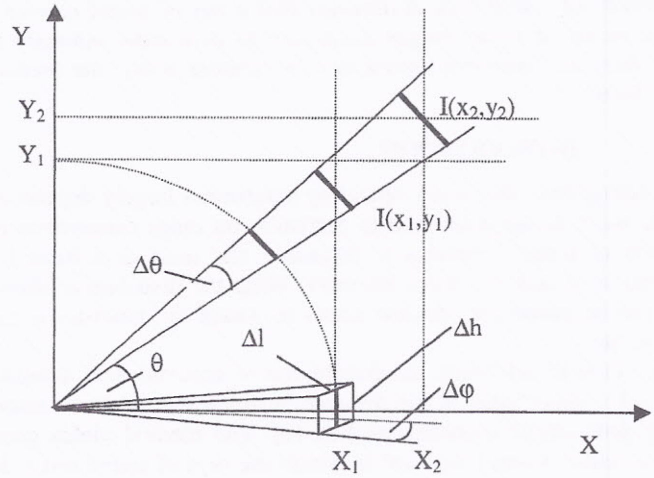


Fig.1. Co-ordinate system for determining the intensity of acoustic field.

The power cutting across the surface amounts to:

$$P(x_1)=I(x_1,0)\cdot\Delta h(x_1,0)\cdot\Delta l(x_1,0)\;=\;P(x_1)=I(x_1,0)\cdot\Delta s(x_1,0) \tag{2}$$

The same power cuts across every elementary surface placed on the surface of a sphere with radius x_1 which matches the beam's upward deflection by angle θ or a side one by angle φ. The power also cuts across the elementary surface placed on the cylinder lateral surface of radius x_1. The surface is equal to:

$$\Delta S(x_2,y_2)=\Delta h(x_2,y_2)\cdot\Delta l(x_2,y_2), \tag{3}$$

where:

$$\Delta l(x_2,y_2)=\Delta\varphi\cdot\sqrt{x_2^2+y_2^2}=\Delta\varphi\cdot x_2\cdot\sqrt{1+\mathrm{tg}^2\theta}=\Delta\varphi\cdot x_2\cdot\frac{1}{\cos\theta}, \tag{4}$$

because for small $\Delta\varphi$ we have $\operatorname{tg}\frac{\Delta\varphi}{2} \cong \frac{\Delta\varphi}{2} = \frac{\Delta l}{2\cdot\sqrt{x_2^2+y_2^2}}$.

By analogy we get:

$$\Delta h(x_2,y_2) = x_2\left[tg(\theta+\Delta\theta)-tg\,\theta\right] = \Delta\varphi\sqrt{x_2^2+y_2^2} = x_2\frac{\sin\Delta\theta}{\cos(\theta+\Delta\theta)\cdot\cos\theta} = x_2\frac{\Delta\theta}{\cos^2\theta} \quad (5)$$

because: $\sin\Delta\theta \cong \Delta\theta$ and $\cos(\theta+\Delta\theta)\cong\cos\theta$

Sound intensity at the point of co-ordinates x_2, y_2 is equal to:

$$I(x_2,y_2) = \frac{P(x_1)}{\Delta S(x_2,y_2)\cdot\cos\theta} \quad (6)$$

after substituting relations (4) and (5) to the above formula we get:

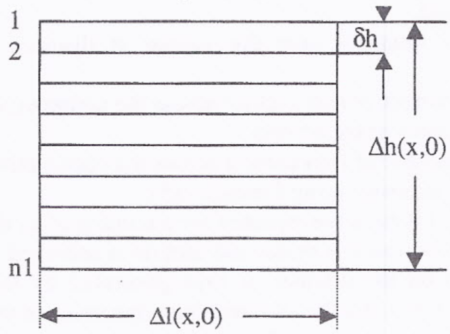
$$I(x_2,y_2) = \frac{I(x_1,0)\cdot\Delta\theta\cdot\Delta\varphi\cdot x_1^2\cdot\cos^3\theta}{\Delta\varphi\cdot x_2\cdot\Delta\theta\cdot x_2\cdot\cos\theta} = \frac{I(x_1,0)\cdot x_1^2\cdot\cos^2\theta}{x_2^2} \quad (7)$$

hence we get:

$$\frac{I(x_2,y_2)}{I(x_1,0)} = \frac{x_1^2\cdot\cos^2\theta}{x_2^2}$$

Let us divide surface $\Delta S(x_1,0)$ into n_1 surfaces as given in Fig. 2.

Fig.2. Division of surface ΔS into elementary surfaces.



Per one surface there is power p_1 which is equal to:

$$p_1 = \frac{P(x)}{n_1} \quad \text{where } n_1 = \frac{\Delta h(x,0)}{\delta h} \quad (8)$$

Therefore, intensity in point x_1 is:

$$I(x,0) = \frac{P(x)}{\Delta h(x,0)\cdot\Delta l(x,0)} = \frac{p_1\cdot n_1}{\Delta h(x,0)\cdot\Delta l(x,0)} \quad (9)$$

We assume that from every small surface a ray carrying power p_1 comes out. The ray moves in straight lines or curves in a vertical plane exclusively. Let us now assume that the intensity at a random point is proportional to number n(x,y) of rays cutting across the surface of a cylinder $\Delta h(x_1,0)$ high. Let number A be the proportionality coefficient. This can be written as:

$$I(x,y) = n_2\cdot A \quad (10)$$

The power cutting across surface $(x_1,0)$ high is:

$$P(x,y) = n_2\cdot p_1, \quad (11)$$

and wave intensity is equal to:

$$I(x,y)=\frac{P(x,y)}{\Delta h(x,0)\cdot \Delta l(x,y)\cdot \cos\theta}=\frac{p_1\cdot n_2}{\Delta h(x,0)\cdot \Delta l(x,y)\cdot \cos\theta}. \quad (12)$$

After substituting expression (8) we get:

$$I(x,y)=\frac{n_2}{n_1}\frac{\Delta h(x,0)\cdot \Delta l(x,0)\cdot I(x,0)}{\Delta h(x,0)\cdot \Delta l(x,y)\cdot \cos\theta}=\frac{n_2}{n_1}\cdot I(x,0)\cdot\frac{\Delta l(x,0)}{\Delta l(x,y)\cdot \cos\theta} \quad (13)$$

After considering expressions (1) and (4) we then have:

$$I(x,y)=\frac{n_2}{n_1}\cdot I(x,0)\cdot\frac{\Delta\varphi\cdot x_1\cdot\cos\theta}{\Delta\varphi\cdot x_2\cdot\cos\theta}=\frac{n_2}{n_1}\cdot I(x,0)\cdot\frac{x_1}{x_2}, \quad (14)$$

hence the final relation which binds field intensity with the number of rays cutting across a unit surface adopts the form:

$$\frac{I(x,y)}{I(x,0)}=\frac{n_2}{n_1}\cdot\frac{x_1}{x_2} \quad (15)$$

Following the above derivation, the methodology of computing field intensity is to:

- generate a certain number of rays coming out of the source at equal angular distances,
- divide the depth of the water area into a number of layers of the same thickness which will be a number that will guarantee the desired resolution of the field intensity computed,
- determine field intensity near the source at distance x_1, assuming spherical propagation,
- determine the number of rays cutting across the consecutive layers at distance $x_{1,}$
- determine the routes of sound rays,
- determine the number of rays cutting across the consecutive layers at distance $x_{2,}$
- calculate sound intensity using formula (15).

Computations following this pattern are repeated for a number of x_2 distances until the desired resolution of intensity distribution in a horizontal section is achieved. The general accuracy of the computations depends on the number of rays generated by the source. The selection criterion for the number of rays could be an interesting dynamics of the changes in intensity in the area being computed. The dynamics is determined from formula (15) by inserting $n_2=1$.

2. RESULTS OF COMPUTATIONS OF ACOUSTIC FIELD INTENSITY

Fig. 3 shows the distribution of acoustic field intensity which was determined using a theoretical distribution of acoustic velocity. The X-axis shows the distance from the source while the Y-axis shows the depth. Shades of grey illustrate wave intensity with black showing the level of intensity equal to 0 dB and white the level of intensity 40 dB less than when 3 m away from the source. It was assumed that sound velocity on the surface is 1500 m/s and dropping linearly to 1400 m/s at 40 m deep to increase linearly and reach 1475 m/s at a depth of 70 m. The source of a conic beam is placed at 35 m deep. The computed sound intensity in caustic area is 6.5 dB higher than the intensity which would occur in this place in spherical propagation.

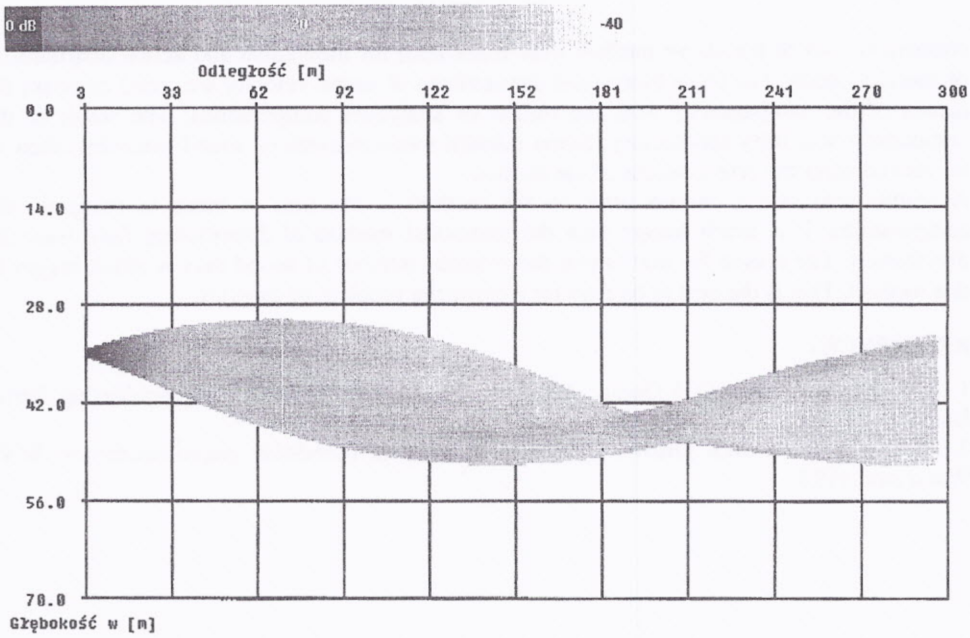


Fig.3. Distribution of sound intensity in media with acoustic duct. The axis of the duct is at 40 m deep, the source is at 35 m deep

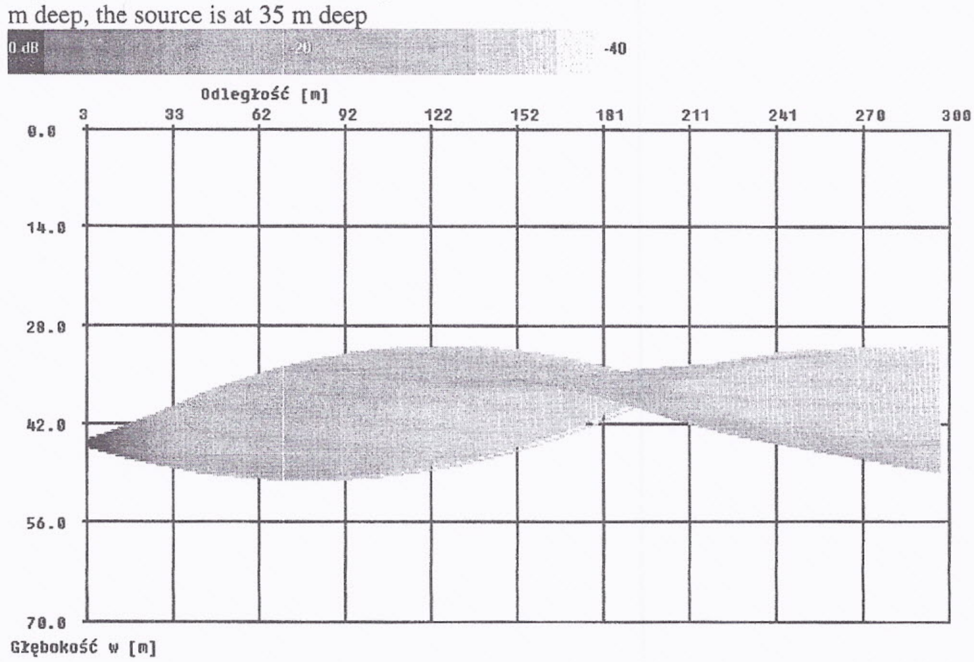


Fig.4. Distribution of sound intensity in media with acoustic duct. The axis of the duct is at 40 m deep, the source is at 45 m deep

3. CONCLUSIONS

Numerical tests of the above method were made both for theoretical and actual distributions of sound velocity. For some theoretical distributions of sound velocity we could compare the results of the computations with the results of analytical computations. The result of the comparison was fully satisfactory. Some random measurements of sound intensity taken so far also confirm the effectiveness of the method.

A feature that sets a certain limit on the method is the time it takes to complete the computations. It is much longer than the traditional method of determining field intensity distribution. The reason for that is that the required number of sound rays is much bigger in this method. This is the cost to be paid for solving the problem of caustics.

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