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Enhancing bubble bize prediction in flotation processes: a drift flux model accounting for frother type

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Abstract: This communication presents a methodology, based on a modified drift flux model, to determine bubble size distribution in column flotation. The modified drift flux model incorporates a surfactant-type parameter. This parameter considers the impact of surfactant on bubble hydrodynamics. The methodology aims to improve the accuracy of bubble size distribution prediction, which presents deviation depending on surfactant type (i.e. polyglycolic based or alcoholic base). Many authors have proposed different mathematical improvements to reduce de experimental data deviations in the presence of different surfactants. However, from 1988 to 2022, the determination coefficient, or the quality of the adjustments, from the proposed mathematical models is, at the most, 92% (relative error). The proposed methodology improves the quality of the adjustments to 98.6, adding a single parameter for groups of surfactants. This methodology incorporates a single parameter in the terminal velocity calculation that can compensate for the impact of surfactant type in bubble hydrodynamic (bubble skin friction or drag coefficient, bubble wake, bubble shape, bubble rigidity). This parameter is a function of the gas holdup calculated from gas velocity measured and the bubble size distribution calculated (deviated) from gas holdup and gas velocity measured. The methodology is validated with reported experimental results and proposed modifications from various authors. The confidence interval (2 σ) is reduced from 0.11mm to 0.05mm in the case of (Yianatos, Banisi, Ostadrahimi). In the case of the recently reported experimental results from Maldonado and Gomez, the confidence interval is reduced from 0.31 mm to 0.09 mm. These results improve bubble size estimation based on drift flux in column flotation, contributing to a better understanding of surfactant impact on bubble swarm hydrodynamics.

Keywords**:** drift flux flotation, surfactant, bubble terminal velocity

1. Introduction

In flotation processes, bubble size is a critical parameter, significantly influencing efficiency and performance (Yianatos, 2007; Deglon et al., 2000; Reis & Barrozo, 2016; Kracht et al., 2005; Verrelli et al., 2011; Reis et al., 2019; Hassanzadeh et al., 2018). Factors such as superficial gas velocity, impeller speed, and temperature directly affect bubble size distribution (Vinnett et al., 2014; Gorain et al., 1990; Gorain et al., 1999; Han et al., 2002; Zhang, 2014; Wei & Finch, 2014; Shabalala et al., 2011; Quinn et al., 2007). Consequently, accurately predicting and controlling bubble size remains a complex and evolving challenge.

Image analysis techniques are commonly utilized to estimate bubble diameter and describe their hydrodynamic behavior. Despite their wide use, these techniques are primarily limited to diagnostic purposes (Hosseini et al., 2015). Challenges arise, for instance, when bubbles interacting with mineral particles ascend in the viewing chamber, often causing turbidity increases that diminish the clarity and quality of captured images (Tucker et al., 1994; Yianatos et al., 1988; Yianatos, 2005; Leiva et al., 2021). Such turbidity necessitates periodic water replacement in the viewing chamber, rendering the technique

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discontinuous. Furthermore, post-processing collected images is typically offline and time-consuming, resulting in a temporal disconnect between data collection and result analysis (Araya et al., 2014; Wallis, 1969).

Bubble size estimation in flotation processes can be approached using practical techniques like artificial vision and acoustics, which offer alternative avenues for measurement (Sovechles & Waters, 2015; Grau & Heiskanen, 2005; Hernandez-Aguilar et al., 2004; Leiva et al., 2010). Additionally, mathematical methods provide a theoretical framework for estimation, leveraging operational parameters such as gas holdup, superficial gas velocity, and flow density (Leiva et al., 2022; Leiva et al., 2023; Vinnett et al., 2012; Wills & Napier-Munn, 2006; Wills & Finch, 2016).

Responding to the challenges of direct bubble size measurement in flotation plants, a practical and alternative methodology becomes essential. This approach must emphasize accuracy, efficiency, and ease of implementation, facilitating enhanced process control and optimization in industrial applications. As an alternative, the drift flux model has been developed. This model utilizes gas velocity and gas holdup, specifically tailored to the context of column flotation.

Drift flux modeling, a mathematical approach to estimating bubble sizes in column flotation, focuses on the relationships between key parameters: gas velocity, gas holdup, and bubble behavior. This model quantifies the drift velocity, representing the relative motion between gas and liquid phases. It integrates factors like the drag coefficient, slip velocity, and hindered velocity to accurately predict the distribution of bubble sizes, which is essential for optimizing flotation processes.

The development of drift flux analysis for bubble size estimation in column flotation is a consequence of the progressive integration of foundational concepts and subsequent advancements in the field. Initially, Schiller and Naumann's 1933 research on the drag coefficient for particles in fluids laid the groundwork for understanding bubble-liquid interactions in flotation. This early study was crucial for grasping the basics of bubble behavior in such environments. Their work was instrumental in formalizing the theoretical underpinnings of bubble motion in such settings. Similarly, Richardson and Zaki's 1954 exploration of hindered settling in particle-fluid systems contributed significantly to our understanding of bubble motion, particularly in crowded environments like flotation columns where interactions between bubbles and particles are complex and critical.

Wallis's 1969 contributions to two-phase flow further enriched this body of knowledge, providing essential insights for modeling bubble behavior in flotation processes. The refinement of these principles by Dobby et al. in 1987 and 1998, specifically tailored for column flotation, marked a significant advancement in the field, particularly on bubble size estimation. Yianatos et al. 1988 further refined the application of drift flux principles, enhancing the understanding of bubble dynamics in column flotation. Their findings, which revealed variations in bubble behavior with different frother types, indicated the dynamic nature of bubble interactions and the need for continuous refinement of the models (Table 1).

The subsequent work by Ostadrahimi et al. in 2020 simplified the determination of bubble diameter by assuming a constant value for the factor 'm', thereby streamlining the calculation process. This simplification represented a move towards greater efficiency and practicality in bubble size estimation (Table 2).

Most recently, Gomez and Maldonado in 2022, further adapted the drift flux model, particularly the terminal rise velocity expression, using the Molerus model. This adaptation offered a more nuanced understanding of bubble motion in column flotation, demonstrating these models' ongoing evolution and refinement. Table 1 and 2 shows the different drift flux models.

These cumulative advancements underscore the necessity of continually adapting and refining the models to incorporate the effects of surfactants. The quest for a hydrodynamic link to the molecular structure remains an essential part of this ongoing research, highlighting the need for a comprehensive approach that integrates both the macroscopic and molecular dynamics of bubble behavior in flotation processes. This integrated approach is crucial for continually improving the accuracy and efficacy of bubble size estimation, which is fundamental to optimizing column flotation processes. The enhanced model should include a type of frother parameter directly related to the sliding velocity of bubbles in a swarm and its hydrodynamics effects over holdup (skin friction, aspect ratio, surface tension, viscosity, and density).

Table 1. Yianatos et al and Banisis and Yianatos drift flux models

Yianatos el al	Banisi and Yianatos
$U_{bs}=\frac{J_g}{\varepsilon_g}+\frac{J_l}{(1-\varepsilon_g)}$	$U_{bs}=\frac{J_g}{\varepsilon_g}+\frac{J_l}{(1-\varepsilon_g)}$
$U_{bs}=\frac{g\cdot d_b^2\cdot(\rho_l-\rho_b)\cdot\left(1-\varepsilon_g\right)^{(m-1)}}{18\cdot\mu_l\cdot\left[1+0.15\cdot Re_s^{0,687}\right]}$	$U_{bs}=\frac{g\cdot d_b^2\cdot(\rho_l-\rho_b)\cdot\left(1-\varepsilon_g\right)^{(m-1)}}{18\cdot\mu_l\cdot\left[1+0.15\cdot Re_s^{0,687}\right]}$
$\mathrm{d_b}=\sqrt{\frac{18\cdot\mu_l\cdot U_{bs}\cdot\left[1+0.15\cdot Re_s^{0.687}\right]}{g\cdot(\rho_l-\rho_b)\cdot\left(1-\varepsilon_g\right)^{(m-1)}}}$	$\mathrm{d_b}=\sqrt{\frac{18\cdot\mu_l\cdot U_{bs}\cdot\left[1+0.15\cdot Re_s^{0.687}\right]}{g\cdot(\rho_l-\rho_b)\cdot\left(1-\varepsilon_g\right)^{(m-1)}}}$
$Re_s=\frac{d_b\cdot U_{bs}\cdot\rho_l\cdot(1-\varepsilon_g)}{\mu_l}$ $U_{bs}=U_t\cdot(1-\varepsilon_g)^{m-1}$	$Re_s=\frac{d_b\cdot U_{bs}\cdot\rho_l\cdot(1-\varepsilon_g)}{\mu_l}$ $U_{bs}=U_t\cdot(1-\varepsilon_g)^{m-1}$
$m=\left[4.45+18\cdot\frac{d_b}{d_c}\right]\cdot Re_b^{-0,1}$ $1<Re_b<200$	$m=3$
$m=4.45\cdot Re_b^{-0,1}$ $200<Re_b<500$	$U_t=\frac{g\cdot d_b^2\cdot\rho_l}{18\cdot\mu_l\cdot\left[1+0.15\cdot Re_b^{0,687}\right]}$
$U_t=\frac{g\cdot d_b^2\cdot\rho_l}{18\cdot\mu_l\cdot\left[1+0.15\cdot Re_b^{0,687}\right]}$	

2. Materials and methodology

2.1. Drift flux model proposed

In gas-liquid systems in which there is a net flow of both phases, the slip velocity, U_{bs}, is defined as the mean bubble swarm velocity in Eq. 1.

$$U_{bs}=\frac{J_g}{\varepsilon_g}+\frac{J_l}{(1-\varepsilon_g)} \tag{1}$$

where, U_{bs} is the bubble swarm velocity [cm/s], J_g and J_l are gas and liquid superficial velocities [cm/s], respectively, and ε_g is gas holdup. These parameters are experimentally obtained and essential for studies on drift flux modelling and adjustment proposed over the years.

The drift flux model, an idealized representation of gas-liquid systems, assumes perfect counter-current or co-current flow, simplifying the complex interactions between these phases. However, practical scenarios often diverge from this idealization due to the heterogeneous nature of bubble size distribution, which induces internal circulations within the liquid and among smaller bubbles. These circulations deviate from the model's assumptions, leading to inaccuracies in bubble size estimation, particularly at increased gas flow rates. Additionally, while correction factors for different frothing agents were originally designed to accommodate variations in frother characteristics, they also partially offset the errors stemming from these hydrodynamic deviations. Consequently, these factors, both from modeling assumptions and frother impacts, are reflected in macro properties such as gas holdup, highlighting the need for refined models that can more accurately capture the complex dynamics of gas-liquid systems.

Yianatos et al. (1988) significantly extend the understanding of bubble dynamics. This research adapted and validated Masliyah's (1979) model, initially developed for solid-liquid systems, by applying it to bubble swarms in bi-dimensional columns. Yianatos et al.'s contribution proposes a method to estimate bubble size within a swarm. The adapted drift flux model for bubbling columns was achieved by employing a general expression for the velocity of a bubble swarm. In their approach, the bubbles were considered spherical and rigid, submerged within an aqueous medium. This adaptation and validation mark a critical step in comprehending bubble behavior in complex fluid dynamics

Table 2. Ostadrahimi and Molerus drift flux models

Ostradahimi	Molerus
$U_{bs}=\frac{J_g}{\varepsilon_g}+\frac{J_l}{(1-\varepsilon_g)}$	$U_{bs}=\frac{J_g}{\varepsilon_g}+\frac{J_l}{(1-\varepsilon_g)}$
$U_{bs}=\frac{g\cdot d_b^2\cdot(\rho_l-\rho_b)\cdot(1-\varepsilon_g)^{(m-1)}}{18\cdot\mu_l\cdot[1+0.15\cdot Re_s^{0,687}]}$	$U_{bs}=\frac{g\cdot d_b^2\cdot(\rho_l-\rho_b)\cdot(1-\varepsilon_g)^{(m-1)}}{18\cdot\mu_l\cdot[1+0.15\cdot Re_s^{0,687}]}$
$\mathrm{d_b}=\sqrt{\frac{18\cdot\mu_l\cdot U_{bs}\cdot[1+0.15\cdot Re_s^{0.687}]}{g\cdot(\rho_l-\rho_b)\cdot(1-\varepsilon_g)^{(m-1)}}}$	$\mathrm{d_b}=\sqrt{\frac{18\cdot\mu_l\cdot U_{bs}\cdot[1+0.15\cdot Re_s^{0.687}]}{g\cdot(\rho_l-\rho_b)\cdot(1-\varepsilon_g)^{(m-1)}}}$
$Re_s=\frac{d_b\cdot U_{bs}\cdot\rho_l\cdot(1-\varepsilon_g)}{\mu_l}$	$Re_s=\frac{d_b\cdot U_{bs}\cdot\rho_l\cdot(1-\varepsilon_g)}{\mu_l}$
$U_{bs}=U_t\cdot(1-\varepsilon_g)^{m-1}$	$U_{bs}=U_t\cdot(1-\varepsilon_g)^{m-1}$
$m=4$	$m=\left[4.45+18\cdot\frac{d_b}{d_c}\right]\cdot Re_b^{-0,1}$ $1<Re_b<200$
$U_t'=\frac{g\cdot d_b^2\cdot(1-\varepsilon_g)^{(n-1)}}{18\cdot\mu_l\cdot[1+0,15\cdot Re_s^{0,687}]}$	$m=4.45\cdot Re_b^{-0,1}$ $200<Re_b<500$
$n=\left[4,45+18\cdot\frac{d_b}{d_c}\right]\cdot Re_s^{-1}$	$m=2.39$ $500<Re_b$
	$\frac{r_0}{\delta}=\frac{1}{(\varsigma/\sqrt[3]{\varepsilon_g}-1)}$
	$\beta^3=18\left[1+0.341\left\{\frac{r_0}{\delta}+\frac{1}{2}\left(\frac{r_0}{\delta}\right)^2\right\}\right]Re+3\left[1+0.07\left(\frac{r_0}{\delta}\right)^{1.5}\right]Re^{1.5}+\left[0.3+\frac{0.68}{Re^{0.1}}\left(\frac{r_0}{\delta}\right)\right]Re^2$ $\beta^3=\frac{g\rho_l(\rho_l-\rho_g)}{\mu_l^2}d_b^3$
	$Re=\frac{\rho_l U_{bs} d_b}{\mu_l}$

scenarios (Eq.2a). The bubble size can be estimated from Eq.2b, which requires interactive calculation for the swarm Reynolds number.

$$U_{bs}=\frac{g\cdot d_b^2\cdot(\rho_l-\rho_b)\cdot(1-\varepsilon_g)^{(m-1)}}{18\cdot\mu_l\cdot[1+0.15\cdot Re_s^{0,687}]} \tag{2a}$$

$$\mathrm{d_b}=\sqrt{\frac{18\cdot\mu_l\cdot U_{bs}\cdot[1+0.15\cdot Re_s^{0.687}]}{g\cdot(\rho_l-\rho_b)\cdot(1-\varepsilon_g)^{(m-1)}}} \tag{2b}$$

where, g is gravitational acceleration [cm/s^2]; $\mathrm{d_b}$ is bubble diameter [cm]; m is a factor according to the Reynolds number of a bubble; ρ_l and ρ_b are liquid and bubble density [g/cm^3], respectively; μ_l is liquid viscosity [g/(cm · s)]; and Re_s is defined as the Reynolds number of bubbles in a swarm, expressed by Eq. 3.

$$Re_s=\frac{d_b\cdot U_{bs}\cdot\rho_l\cdot(1-\varepsilon_g)}{\mu_l} \tag{3}$$

An expression relates rising velocity (U_{bs}) to the terminal rise velocity of a single bubble *(U_t)*, expressed in Eq. (19).

$$U_{bs}=U_t\cdot(1-\varepsilon_g)^{m-1} \tag{4}$$

This expression was adapted for predicting the diameter of bubbles in a swarm since it is practically equivalent to a Reynolds number lower than 500. Hence, factor m is estimated with Eq. 5 and 6, according to the corrresponding Reynolds number interval.

$$m=\left[4.45+18\cdot\frac{d_b}{d_c}\right]\cdot Re_b^{-0,1} \qquad 1<Re_b<200 \tag{5}$$

$$m=4.45\cdot Re_b^{-0,1} \qquad 200<Re_b<500 \tag{6}$$

where, d_c is column diameter [cm] and $\mathrm{Re_b}$ is defined as the Reynolds number of a particular bubble, expresssed in Eq. 7.

$$Re_b = \frac{d_b \cdot U_t \cdot \rho_l}{\mu_l} \tag{7}$$

Finally, applying the relationship of velocities in Eq. 4, Eq. 8, which generalizes the terminal rise velocity of a single bubble, is obtained.

$$U_t = \frac{g \cdot d_b^2 \cdot \rho_l}{18 \cdot \mu_l \cdot [1 + 0.15 \cdot Re_b^{0,687}]} \tag{8}$$

In this expression, the Reynolds number is fitted for a single bubble (Re_b) unlike Eq. 2a, which uses Reynolds number for bubbles in a swarm (Re_s).

Recognizing gas holdup's dependency on bubble swarm Reynolds number, this methodology proposes a singular, encompassing parameter (C_L) in the model to reflect these complex interactions between surfactant and bubble hydrodynamics (skin friction, drag, shape, rigidity)

The proposed factor in the drift flux model is designed to encapsulate the influence of surfactants on bubble hydrodynamics, including aspects like skin friction, drag coefficient, wake, shape, and rigidity. This factor, grounded in empirical evidence, aims to accurately reflect the resultant variations in gas holdup, thereby enhancing the model's predictive accuracy in surfactant-influenced systems (Eq.9)

$$U'_{bs} = \frac{g \cdot {d_b}^2 \cdot C_L \cdot (1 - \varepsilon_g)^{m-1} \cdot [\rho_l - \rho_b]}{18 \cdot \mu_l \cdot [1 + 0.15 \cdot {Re_s}^{0.687}]} \tag{9}$$

where, C_L is a non-dimensional average parameter possibly depending on variables related to the frother, such as type, concentration, and dosage.

Eq. 10 expresses the calculated quadratic difference between the rising velocity of the drift flux model and the one proposed in this study, as described in Eq. 1 and 9, respectively.

$$\Delta U_{bs} = \left(U_{bs\,(Drift\ flux)} - U'_{bs\,(Proposed)} \right)^2 \tag{10}$$

For estimating parameter (C_L) a mean squared error (MSE) range is defined, minimizing the squared difference of velocities (ΔU_{bs}). The iterative process of the mathematical model proposed for drift flux analysis is shown in Fig. 1, which estimate a gas holdup as close as the gas holdup measured with experimental data on liquid velocity, gas velocity, and bubble diameter measured.

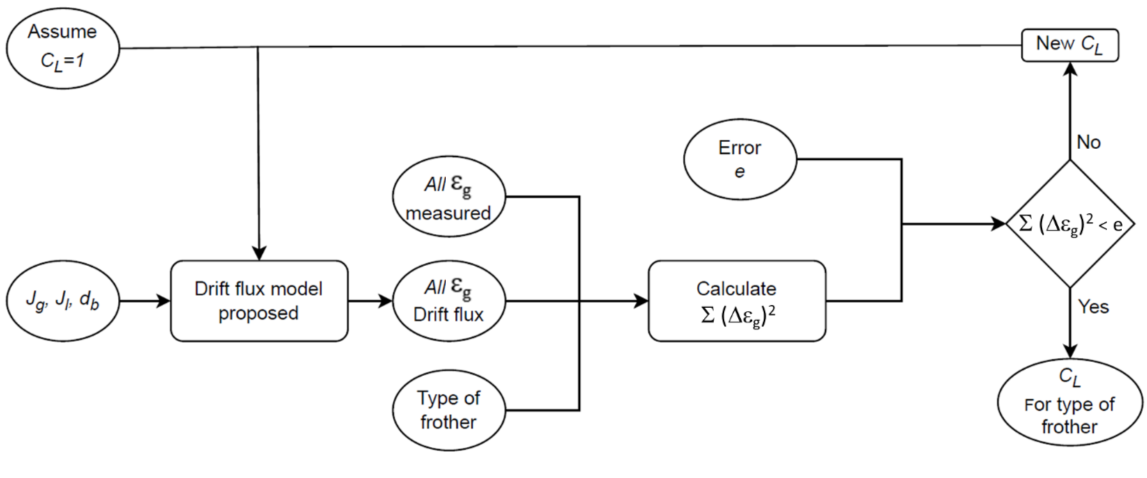

Fig. 1. Iterative model proposed for estimating parameter (C_L)

For estimating bubble diameter (d_b) a mean squared error (MSE) range is defined, minimizing the squared difference of velocities (ΔU_{bs}), as shown in Eq. 10. The iterative process of the mathematical model proposed for drift flux analysis is shown in Fig. 2, which estimate a bubble diameter as close as the bubble diameter measured with experimental data on liquid velocity, gas velocity, and gas holdup. The model is particularly related to lab tests conducted in countercurrent flotation columns. Once the analysis of the models is conducted, it is observed that, although their fits are acceptable, they show significant errors in estimating bubble diameter.

First, an initial value is assumed for bubble diameter (d_b). Rising velocity (U_{bs}) is calculated with Eq. 1, corresponding to the drift flux model of a countercurrent flotation system using operational data

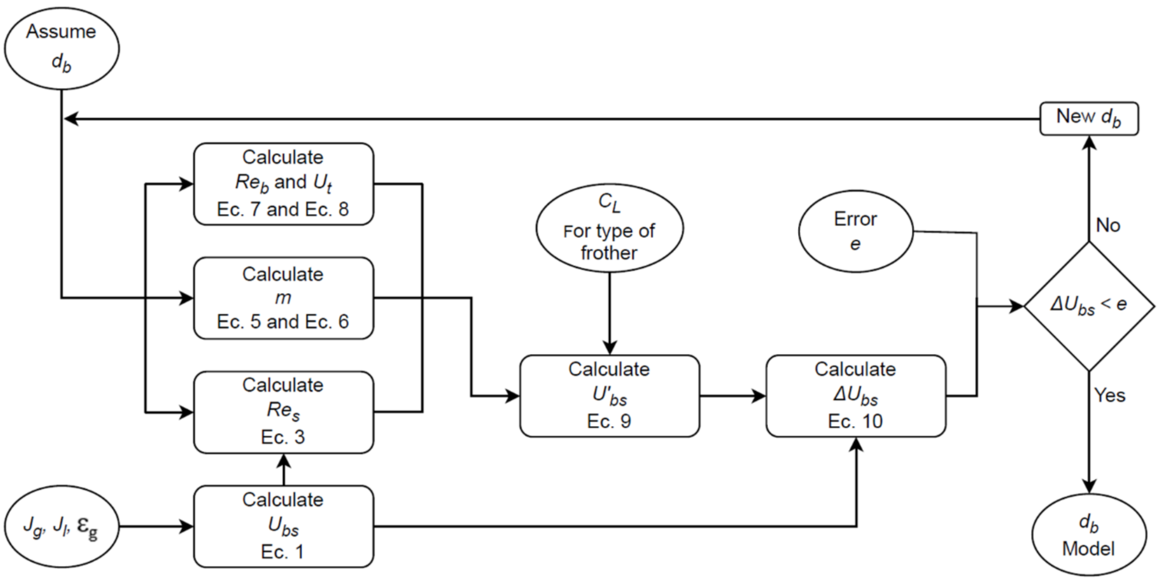

Fig. 2. Iterative model proposed for estimating bubble diameter (d_b)

for gas superficial velocity (J_g), liquid superficial velocity (J_l), and gas holdup (ε_g) from Yianatos et al. (1998). Reynolds number for bubbles in a swarm (Re_s) is calculated with Eq. 3.

On the other hand, using the bubble diameter assumed, the equation system is solved for the Reynolds number of a single bubble (Re_b) and the terminal rise velocity with Eq. 7 and 8. Once Re_b is obtained, factor m is determined with Eq. 5 or 6, according to the corresponding range conditions.

A (C_L) parameter from Fig. 1 and the adjusted rising velocity (U'_{bs}) is calculated using Eq. 9. Next, the square difference between both velocities (ΔU_{bs}) is determined with Eq. 10. Finally, e range is defined and compared with the squared differences of velocities. If the squared difference calculated is greater than the range defined ($\Delta U_{bs} > e$), the iterative process is repeated from the beginning with the new d_b value. On the contrary, if the squared difference calculated is smaller than or equal to the range defined ($\Delta U_{bs} \leq e$), the iterative process ends, obtaining bubble diameter and the drift flux model parameter.

2.2. Measuring bubble diameter

Operational parameters were the same used by Yianatos et al. (1988). For their development, fluid density (ρ_l) was considered as 1 [g/cm^3] and viscosity (μ_l) as 0.01 [g/cm·s]. Yianatos et al. (1988) conducted lab tests, distributing different frothers and types of columns, whose characteristics and dimensions are shown in Table 3.

Table 3. Flotation column characteristics (Yianatos et al, 1988)

Test number	Type of frother	Height [cm]	Diameter [cm]	Type of injector	Column shape
1-5	DOW	200	3.81	Ceramics	Circular
6-7	DOW	180	2.5x10	Steel	Rectangular
8-13	DOW	450	5.71es	Ceramics	Circular
14-18	TEB	200	3.81	Ceramics	Circular
19-23	MIBC	200	3.81	Ceramics	Circular

DOW, Dowfroth 250C (polypropylene glycol methyl ether); TEB, triethoxy butane; MIBC, methylisobutylcarbinol (methylamyl alcohol)

Manometers were used for calculating gas holdup (ε_g) via pressure decrease, while gas and liquid superficial velocities (J_g and J_l, respectively) were measured with fluxometers, (Yianatos et al, 1988). The bubbles were introduced via ceramics and stainless steel injectors. Bubble size was controlled with frothing agents (DOW, TEB, and MIBC). Between 400 and 600 bubbles were quantified both naturally and using an automatic digitizer. A plexiglass box full of water was placed around the system to reduce optical distortion due to column curvature.

3. Results and discussion

3.1. Drift flux model application

An adding a parameter to the iterative routine is proposed to obtain an adjustment for each test and type of frother, as shown in Table 5. As can be seen using the dimensionless factor by type of frother C_L, an R^2 of 98.62 is obtained. Table 4 shows the parameter C_L for each type of frother.

Table 4. C_L for each type of frother

Type of frother	C_L
DOWN	1.003
TEB	0.973
MIBC	1.202

Table 5. Results of the drift flux model proposed

N°	Type of frother	ppm	Measured	Estimated		
			d_m [mm]	C_L	d_{bp} [mm]	$\lvert\Delta d\rvert$ [mm]
1		5	1.20		1.22	0.02
2		10	0.86		0.85	0.01
3	DOW	15	0.77		0.77	0.00
4		20	0.69		0.65	0.04
5		25	0.73		0.72	0.01
6	DOW	10	1.51		1.57	0.06
7		15	1.13	1.003	1.15	0.02
8		15	0.62		0.66	0.04
9		15	0.67		0.69	0.02
10	DOW	15	0.70		0.71	0.01
11		15	0.74		0.74	0.00
12		15	0.81		0.80	0.01
13		15	0.88		0.90	0.02
14		5	0.97		0.95	0.02
15		10	0.85		0.83	0.02
16	TEB	15	0.85	0.973	0.85	0.00
17		20	0.82		0.86	0.04
18		25	0.71		0.74	0.03
19		20	0.78		0.82	0.04
20		30	0.75		0.77	0.02
21	MIBC	45	0.80	1.202	0.85	0.05
22		60	0.73		0.77	0.04
23		75	0.67		0.69	0.02
	$\overline{\lvert\Delta d\rvert}$ [mm]			-	-	0.01
	R^2: Determination coef. [%]			-	98.62	-

The data fitting improvement is due to adding parameter (C_L), which considers the effect of the frother on the bubble size variation due to bubble hydrodynamic (bubble skin friction or drag coefficient, bubble wake, bubble shape, bubble rigidity).

Fig. 3 shows the comparison between bubble diameters measured (d_{bm}) and estimated (d_{bp}). It can be seen in the Fig. that the fit of the model is R^2 is 0.9862. The model allows a better adjustment if the type of frother used is considered as a parameter.

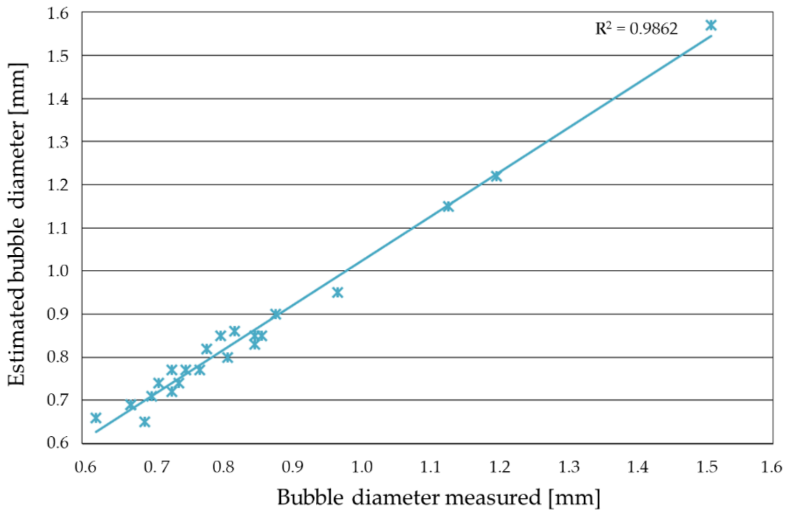

Fig. 3. Fit model proposed

3.2. Drift flux models comparison

The results of the model proposed were compared with those of the model studied, as shown in Table 6.

Tabla 6. Fit model results

N°	Type of frother	Measured	Yianatos et al. (1988)		Banisi and Finch (1994)		Ostadrahimi et al.(2020)		Estimated	
		d_m [mm]	d_b [mm]	$\|\Delta d\|$ [mm]	d_b [mm]	$\|\Delta d\|$ [mm]	d_b [mm]	$\|\Delta d\|$ [mm]	d_b [mm]	$\|\Delta d\|$ [mm]
1	DOW	1.20	1.11	0.09	1.14	0.06	1.14	0.06	1.22	0.02
2		0.86	0.87	0.01	0.88	0.02	0.89	0.03	0.85	0.01
3		0.77	0.76	0.01	0.77	0.00	0.78	0.01	0.77	0.00
4		0.69	0.77	0.08	0.78	0.09	0.79	0.10	0.65	0.04
5		0.73	0.74	0.01	0.75	0.02	0.76	0.03	0.72	0.01
6	DOW	1.51	1.40	0.11	1.47	0.04	1.48	0.03	1.57	0.06
7		1.13	1.11	0.02	1.14	0.01	1.15	0.02	1.15	0.02
8	DOW	0.62	0.55	0.07	0.54	0.08	0.55	0.07	0.66	0.04
9		0.67	0.64	0.03	0.64	0.03	0.65	0.02	0.69	0.02
10		0.70	0.69	0.01	0.70	0.00	0.71	0.01	0.71	0.01
11		0.74	0.74	0.00	0.75	0.01	0.77	0.03	0.74	0.00
12		0.81	0.80	0.01	0.83	0.02	0.85	0.04	0.80	0.01
13		0.88	0.87	0.01	0.93	0.05	0.95	0.07	0.90	0.02
14	TEB	0.97	0.98	0.01	1.00	0.03	1.00	0.03	0.95	0.02
15		0.85	0.86	0.01	0.87	0.02	0.88	0.03	0.83	0.02
16		0.85	0.81	0.04	0.83	0.02	0.83	0.02	0.85	0.00
17		0.82	0.72	0.10	0.72	0.10	0.74	0.08	0.86	0.04
18		0.71	0.65	0.06	0.66	0.05	0.67	0.04	0.74	0.03
19	MIBC	0.78	0.86	0.08	0.88	0.10	0.88	0.10	0.82	0.04
20		0.75	0.86	0.11	0.87	0.12	0.88	0.13	0.77	0.02
21		0.80	0.84	0.04	0.86	0.06	0.86	0.06	0.85	0.05
22		0.73	0.78	0.05	0.79	0.06	0.80	0.07	0.77	0.04
23		0.67	0.72	0.05	0.73	0.06	0.74	0.07	0.69	0.02
$\overline{\|\Delta d\|}$ [mm]			-	0.04	-	0.05	-	0.05	-	0.02
R^2 [%]			92.2	-	92.2	-	92.4	-	98.6	-

Although the mean differences of the bubble diameters and determination coefficients of the models studied show a good fit, they can be improved by adapting, with a factor by frother types. Also, the models show similar statistical results, without significant variations, despite changes and simplifications of mathematical calculations, with about 92% representation of real data. The model proposed had a better adjustment with a determination coefficient of 0.986.

The comparison of bubble diameters resulting from the adjustments above and the experimental ones obtained by Yianatos et al. (1988) is shown in Fig. 4, where dashed lines represent a ±15% confidence interval, according to the bubble diameter measured.

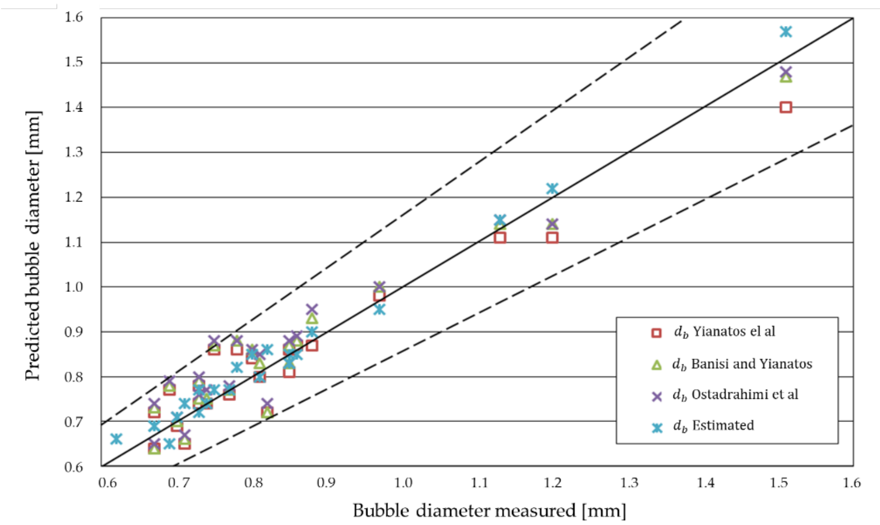

Fig. 4. Fit model comparison

On the other hand, errors were identified in nomenclature and the use of measurement units, maybe due to writing in the publications analyzed, which created confusion and ambiguity. Finally, it is assumed that the model proposed by Yianatos et al. (1988); Banisi and Finch (1994); and Ostadrahimi et al., (2020), can be used in a multi-species system, including bubbles in a swarm in a flotation column.

Fig. 5 shows the comparison of the errors obtained by frother type between the three mentioned models and the proposed model. It can be seen that using the parameter by type of frother reduces the errors in the proposed model.

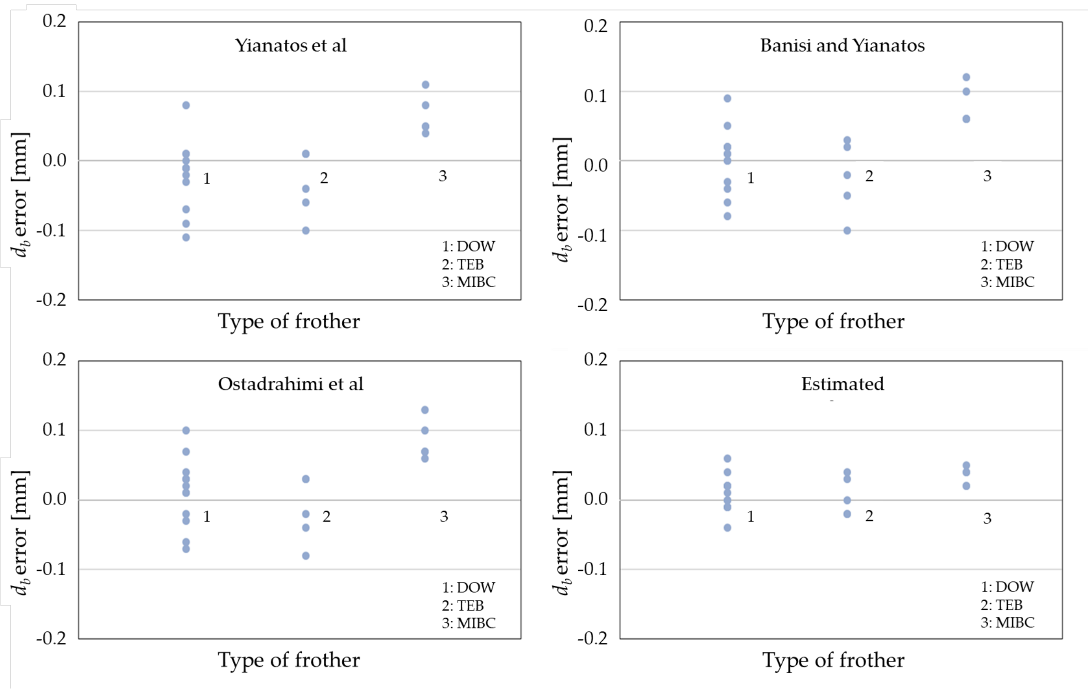

Fig. 5. Error by frother type

Fig. 6 shows the histogram of the errors obtained by frother type between the three mentioned models and the proposed model. It can be seen that using the parameter by type of frother improve the bubble size in the proposed model.

The analysis carried out on the data from previous investigations was compared with the data of mathematical model proposed by Gómez and Maldonado (2022), as shown Table 7. it is worth mentioning that the same procedure was carried out taking into in consideration frother type parameter.

The comparison of bubble diameters resulting from the adjustments above and the experimental ones obtained by Gómez and Maldonado (2022) is shown in Fig. 7, where dashed lines represent a ±15% confidence interval, according to the bubble diameter measured.

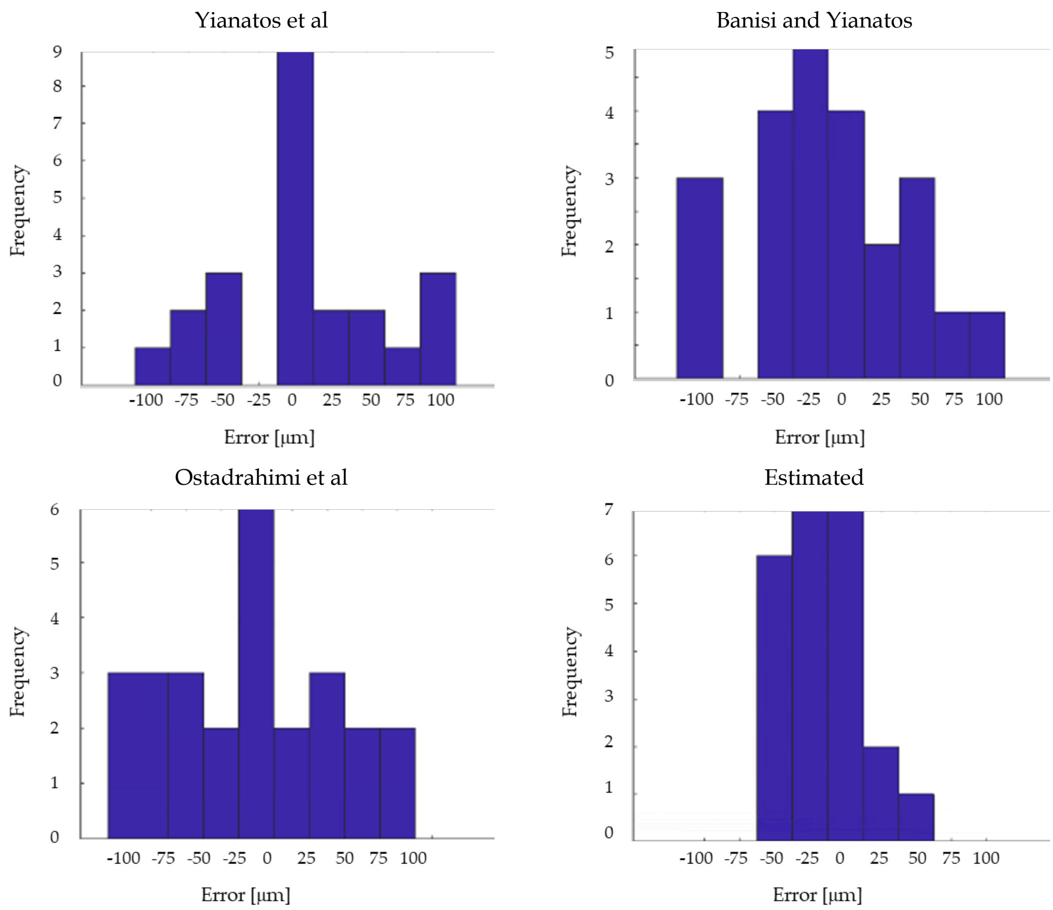

Fig. 6. Histogram of error by frother type

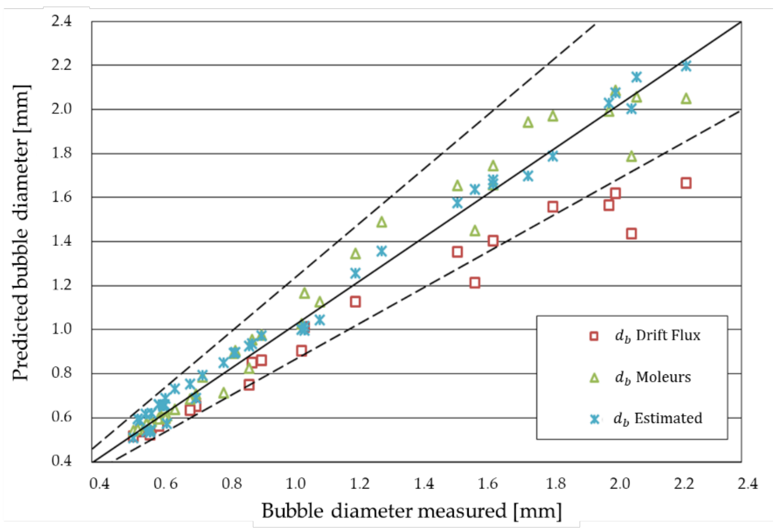

Fig. 7. Fit model comparision

Table 7. Comparative Gomez and Maldonado (2022) with method proposed

N°	Type of frother	Measured	Drift flux		Molerus		Estimated	
		d_m [mm]	d_b [mm]	$\|\Delta d\|$ [mm]	d_b [mm]	$\|\Delta d\|$ [mm]	d_b [mm]	$\|\Delta d\|$ [mm]
1	F140	2.063	1.437	0.626	1.787	0.276	2.004	0.059
2		1.581	1.214	0.367	1.451	0.130	1.636	0.055
3		1.045	0.903	0.142	1.028	0.017	1.002	0.043
4		0.884	0.749	0.135	0.824	0.060	0.926	0.042
5		0.721	0.655	0.066	0.707	0.014	0.688	0.033
6		0.607	0.567	0.040	0.599	0.008	0.662	0.055
7		0.584	0.556	0.028	0.584	0.000	0.620	0.036
8	F150	1.993	1.567	0.426	1.993	0.000	2.030	0.037
9		1.527	1.355	0.172	1.654	0.127	1.576	0.049
10		1.055	1.011	0.044	1.168	0.113	0.999	0.056
11		0.894	0.850	0.044	0.954	0.060	0.936	0.042
12		0.621	0.599	0.022	0.639	0.018	0.654	0.033
13		0.547	0.538	0.009	0.565	0.018	0.588	0.041
14		0.527	0.517	0.010	0.541	0.014	0.508	0.019
15	F160-05	2.012	1.618	0.394	2.086	0.074	2.077	0.065
16		1.821	1.558	0.263	1.972	0.151	1.789	0.032
17		1.212	1.128	0.084	1.345	0.133	1.256	0.044
18		0.922	0.862	0.060	0.976	0.054	0.973	0.051
19		0.703	0.634	0.069	0.686	0.017	0.755	0.052
20		0.607	0.565	0.042	0.597	0.010	0.662	0.055
21		0.579	0.523	0.056	0.548	0.031	0.534	0.045
22	F160-10	2.231	1.667	0.564	2.051	0.180	2.197	0.034
23		1.637	1.404	0.233	1.659	0.022	1.680	0.043
24		1.101	0.976	0.125	1.128	0.027	1.045	0.056
25		0.807	0.661	0.146	0.714	0.093	0.851	0.044
26		0.62	0.588	0.032	0.626	0.006	0.655	0.035
27		0.582	0.552	0.030	0.581	0.001	0.545	0.037
28		0.566	0.543	0.023	0.568	0.002	0.617	0.051
29	F160-13	2.079	1.622	0.457	2.057	0.022	2.148	0.069
30		1.637	1.415	0.222	1.746	0.109	1.662	0.025
31		1.054	1.010	0.044	1.166	0.112	1.016	0.038
32		0.841	0.811	0.030	0.903	0.062	0.897	0.056
33		0.655	0.600	0.055	0.639	0.016	0.731	0.076
34		0.569	0.545	0.024	0.575	0.006	0.537	0.032
35		0.541	0.523	0.018	0.548	0.007	0.595	0.054
36	F173	1.744	1.557	0.187	1.943	0.199	1.699	0.045
37		1.292	1.240	0.052	1.491	0.199	1.358	0.066
38		0.837	0.796	0.041	0.894	0.057	0.894	0.057
39		0.741	0.716	0.025	0.785	0.044	0.793	0.052
40		0.625	0.600	0.025	0.641	0.016	0.660	0.035
41		0.63	0.574	0.056	0.609	0.021	0.573	0.057
42		0.627	0.567	0.060	0.600	0.027	0.689	0.062
$\overline{\|\Delta d\|}$ [mm]			-	0.13	-	0.06		0.05
R^2 [%]			97.1	-	97.2	-		99.2

Fig. 8 shows the comparison of the errors obtained by frother type between the two mentioned models and the proposed model. It can be seen that using the parameter by type of frother reduces the errors in the proposed model. Therefore, the arithmetical mean of the parameters per type of frother was used for the model proposed, as shown in Table 4, Fig. 5 and Fig. 6, resulting in an about 99.2% determination coefficient, increasing fit significantly, as compared with the models studied. So, it is possible to state that the drift flux model better fitting experimental data is the one proposed in this study.

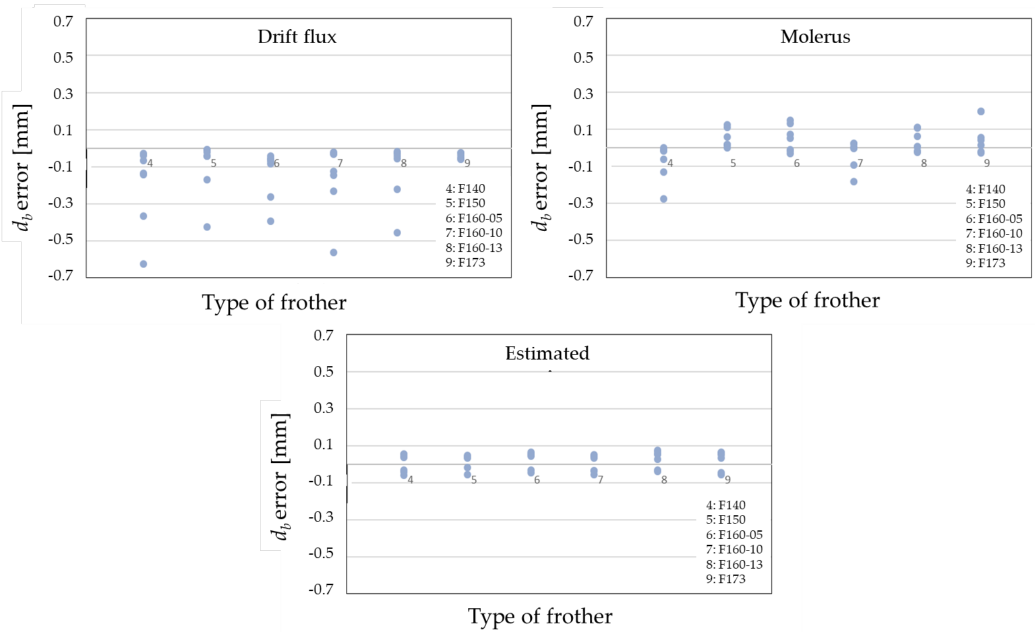

Fig. 8. Error by frother type

4. Conclusions and future work

This study deals with drift flux adjustment modelling, based on reagents in a flotation process, by adding a parameter per type of frother to the expression relating the rising velocity of bubbles in a swarm with the bubble diameter in countercurrent flotation columns. A drift flux adjustment model was developed, according to Dowfroth 250C (DOW), triethoxybutane (TEB), and methylisobutylcarbinol (MIBC) frothers, corresponding to polyglycol, alkoxy, and aliphatic alcohols, respectively, by adding parameter (C_L) for each type of frother in the drift flux model proposed.

The adjusted model was validated with operational data from lab tests for the different concentrations and types of frothers provided by Yianatos et al. (1987), obtaining a 0.01 [mm] diameter difference as the absolute mean and a 99% determination coefficient. Thus, the model estimated fits better in diameter estimation, according to the adjustments analyzed throughout the modelling. Lab tests using other types of frothers are recommended to obtain new operational data and a new parameter fitting and validating the models published. In addition, other tests should be made, using a variety of frothers of the same nature as the ones analyzed to validate the parameter proposed. Finally, further analysis should be conducted to determine if the parameter proposed is related to the physical and/or chemical characteristics of the type of frother, such as structures and types of links to find the physical explanation relating these characteristics.

The authors of this article consider the followings topics for future research:

- Perform additional tests to verify if the (C_L) parameter can be used under the same conditions using the same frothers.
- Study if the (C_L) factor has any relationship with the type of frother based on its chemical composition or hydrophile-lipophile balance (HLB).

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Appendix 1: Nomenclature

C_L	Adimensional parameter by frother type
d_b	Bubble diameter, cm
d_c	Column diameter, cm
g	Gravity acceleration, cm/s^2
J_g	Superficial gas rate, cm/s
J_l	Superficial liquid rate, cm/s
m	Factor according to the Reynolds
r_o	Characteristic dimension of the particles
Re_b	Reynolds number of bubbles
Re_s	Reynolds for a bubble in a swarm
U_{bs}	Bubble swarm velocity, cm/s
U_t	Terminal velocity of a single bubble cm/s
β	Dimensionless bubble size
δ	Pores
ε_g	Holdup
ρ_b	Bubble density, g/cm^3
ρ_l	Liquid density, g/cm^3
ζ	Packing parameter
μ_l	Liquid viscosity, g/cm s

Appendix 2: Data Yianatos et al.

Jg	Eg	Jl	dc	ρ_l	ρ_b	μ_l	Frother	ppm	d_{32} measured
1.000	9.50	0.91	3.81	1.000	0.0010	0.010	1	5	1.200
1.000	12.90	0.85	3.81	1.000	0.0010	0.010	1	10	0.860
1.000	15.80	0.82	3.81	1.000	0.0010	0.010	1	15	0.770
1.000	15.50	0.85	3.81	1.000	0.0010	0.010	1	20	0.690
1.000	16.20	0.77	3.81	1.000	0.0010	0.010	1	25	0.730
2.100	15.70	0.30	10.31	1.000	0.0010	0.010	1	10	1.510
1.500	14.00	0.30	10.31	1.000	0.0010	0.010	1	15	1.130
0.500	12.30	1.00	5.71	1.000	0.0010	0.010	1	15	0.620
0.800	17.00	1.00	5.71	1.000	0.0010	0.010	1	15	0.670
1.000	20.00	1.00	5.71	1.000	0.0010	0.010	1	15	0.700
1.200	23.40	1.00	5.71	1.000	0.0010	0.010	1	15	0.740
1.500	28.00	1.00	5.71	1.000	0.0010	0.010	1	15	0.810
1.800	32.00	1.00	5.71	1.000	0.0010	0.010	1	15	0.880
1.000	11.20	0.96	3.81	1.000	0.0010	0.010	2	5	0.970
1.000	13.20	0.88	3.81	1.000	0.0010	0.010	2	10	0.850
1.000	14.40	0.91	3.81	1.000	0.0010	0.010	2	15	0.850
1.000	17.70	0.87	3.81	1.000	0.0010	0.010	2	20	0.820
1.000	21.50	0.83	3.81	1.000	0.0010	0.010	2	25	0.710
1.000	13.20	0.90	3.81	1.000	0.0010	0.010	3	20	0.780
1.000	13.30	0.90	3.81	1.000	0.0010	0.010	3	30	0.750
1.000	13.60	0.91	3.81	1.000	0.0010	0.010	3	45	0.800
1.000	15.30	0.91	3.81	1.000	0.0010	0.010	3	60	0.730
1.000	18.00	0.96	3.81	1.000	0.0010	0.010	3	75	0.670

Appendix 3: Gomez and Maldonado

Jg	Eg	Jl	dc	ρ_l	ρ_b	μ_l	Frother	ppm	d_{32} measured
0.605	4.17	0.00	10.00	0.999	0.0012	0.012	4	2	2.063
0.601	4.96	0.00	10.00	0.999	0.0012	0.012	4	5	1.581
0.604	6.82	0.00	10.00	0.999	0.0012	0.012	4	10	1.045
0.602	8.71	0.00	10.00	0.999	0.0012	0.012	4	15	0.884
0.601	10.46	0.00	10.00	0.999	0.0012	0.012	4	30	0.721
0.604	13.21	0.00	10.00	0.999	0.0012	0.012	4	60	0.607
0.604	13.84	0.00	10.00	0.999	0.0012	0.012	4	100	0.584
0.606	3.82	0.00	10.00	0.998	0.0012	0.011	5	2	1.993
0.601	4.44	0.00	10.00	0.999	0.0012	0.012	5	5	1.527
0.602	6.05	0.00	10.00	0.999	0.0012	0.012	5	10	1.055
0.601	7.42	0.00	10.00	0.999	0.0012	0.012	5	15	0.894
0.606	11.90	0.00	10.00	0.999	0.0012	0.012	5	30	0.621
0.610	14.26	0.00	10.00	0.998	0.0012	0.011	5	60	0.547
0.612	15.13	0.00	10.00	0.998	0.0012	0.011	5	100	0.527
0.609	3.69	0.00	10.00	0.998	0.0012	0.011	6	2	2.012
0.604	3.85	0.00	10.00	0.999	0.0012	0.012	6	5	1.821
0.609	5.29	0.00	10.00	0.998	0.0012	0.011	6	10	1.212
0.607	7.18	0.00	10.00	0.998	0.0012	0.011	6	15	0.922
0.611	10.66	0.00	10.00	0.998	0.0012	0.011	6	30	0.703
0.606	13.25	0.00	10.00	0.999	0.0012	0.012	6	60	0.607
0.611	14.91	0.00	10.00	0.998	0.0012	0.011	6	100	0.579
0.585	3.75	0.00	10.00	1.000	0.0013	0.014	7	2	2.231
0.581	4.49	0.00	10.00	1.000	0.0013	0.014	7	5	1.637
0.605	6.23	0.00	10.00	0.999	0.0012	0.012	7	10	1.101
0.600	10.40	0.00	10.00	0.999	0.0012	0.012	7	15	0.807
0.607	12.28	0.00	10.00	0.999	0.0012	0.012	7	30	0.620
0.606	13.86	0.00	10.00	0.999	0.0012	0.012	7	60	0.582
0.602	14.69	0.00	10.00	0.999	0.0012	0.012	7	100	0.566
0.600	3.72	0.00	10.00	0.999	0.0012	0.012	8	2	2.079
0.601	4.24	0.00	10.00	0.999	0.0012	0.012	8	5	1.637
0.600	6.05	0.00	10.00	0.999	0.0012	0.012	8	10	1.054
0.600	7.87	0.00	10.00	0.999	0.0012	0.012	8	15	0.841
0.602	12.08	0.00	10.00	0.999	0.0012	0.012	8	30	0.655
0.610	13.70	0.00	10.00	0.998	0.0012	0.011	8	60	0.569
0.610	14.93	0.00	10.00	0.998	0.0012	0.011	8	100	0.541
0.598	3.90	0.00	10.00	0.999	0.0012	0.012	9	2	1.744
0.603	4.85	0.00	10.00	0.999	0.0012	0.012	9	5	1.292
0.610	7.84	0.00	10.00	0.998	0.0012	0.011	9	10	0.837
0.604	9.16	0.00	10.00	0.999	0.0012	0.012	9	15	0.741
0.607	11.84	0.00	10.00	0.999	0.0012	0.012	9	30	0.625
0.607	12.78	0.00	10.00	0.999	0.0012	0.012	9	60	0.630
0.607	13.14	0.00	10.00	0.999	0.0012	0.012	9	100	0.627

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