

## TWO-PHASE FLOW IN A CATHETERIZED ARTERY WITH ATHEROSCLEROSIS

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We consider the problem of blood flow in a catheterized artery and in the presence of atherosclerosis, which is chosen based on the available experimental data. The atherosclerosis is a condition where an artery wall thickens as a result of fatty materials such as cholesterol. The use of catheter is of immense importance as a standard tool for diagnosis and treatment in a patient whose artery is affected adversely by the presence of atherosclerosis within the artery. The blood flow in the arterial tube is represented by a two-phase model composing a suspension of erythrocytes (red cells) in plasma. The coupled differential equations for both fluid (plasma) and particles (red cells) are solved theoretically subjected to reasonable modeling and approximations. The important quantities such as plasma speed, velocity of red cells, blood pressure force, impedance (blood flow resistance) and the wall shear stress are computed for different values of the catheter size, axial location of atherosclerosis and the hematocrit due to the red cells-plasma combination of the blood flow system.

*Key words:* arterial flow, blood flow, impedance, atherosclerosis, pressure, shear stress

### 1. Introduction

Diseases in the blood vessels and in the heart, such as heart attack and stroke, are the major causes of mortality worldwide. The underlying cause for these events is the formation of lesions, known as atherosclerosis. These lesions and plaques can grow and occlude the artery and hence prevent blood supply to the distal bed. Plaques with calcium in them can also rupture and initiate the formation of blood clots (thrombus). The clots can form as emboli and occlude the smaller vessels that can also result in interruption of blood supply to the distal bed. Plaques formed in coronary arteries can lead to heart attacks and clots in the cerebral circulation can result in the stroke. There are a number of risk factors for the presence of atherosclerotic lesions. The common sites for the formation and development of atherosclerosis include the coronary arteries, the branching of the subclavian and common carotids in the aortic arch, the bifurcation of the common carotid to internal and external carotids especially in the carotid sinus region distal to the bifurcation, the renal arterial branching in the descending aorta and in the iliofemoral bifurcations of the descending aorta. The common feature in the location for the development of the lesion is the presence of curvature, branching, and bifurcation present in these sites. The fluid dynamics at these sites can be anticipated to be vastly different from other segments of the arteries that are relatively straight and devoid of any branching segments. Hence, several investigators have attempted to link the fluid dynamically induced stress with the formation of atherosclerotic lesion in the human circulation. By assuming the artery to be circularly cylindrical in shape, Mishra and Panda (2005) studied the flow of blood in stenosed artery for the Casson type fluid. Young and Tsai (1973) discussed some characteristics of flow of blood in stented arteries. The blood vessels carry blood from the heart to all the organs and tissues of the body including brain, kidneys, gut, muscles, and the heart itself. Venkateswarlu and Rao (2004) studied an assumed oscillatory form of the blood flow through an indented tube in the

presence of steady single stenosis with a very simple shape. They used the so-called Einstein model for the viscosity of the blood but for a variable volume flow rate and the prescribed value for the magnitude of the pressure gradient. Srivastava *et al.* (2010) studied arterial blood flow through an overlapping stenosis (Mishra and Panda, 2005) by using a Casson type fluid flow. They calculated the impedance and shear stress for different stenosis height. Riahi *et al.* (2011) investigated arterial blood flow in the presence of an overlapping stenosis using the variable viscosity model due to Einstein for the blood flow. All the investigations described above were for the cases where no catheter was inserted into the artery, although there have also been a number of studies of the blood flow systems in catheterized arteries (Kanai *et al.*, 1970; Back, 1994; Back *et al.*, 1996; Srivastava and Rastogi, 2010).

In all the studies that have been carried out so far and investigated arterial blood flow system, and some of which were listed in the previous paragraph, the shape of stenosis was based on the assumed analytical description due to some forms of certain functions. In the present study, we apply for the first time an experimentally based form (Back *et al.*, 1984) for the atherosclerosis shape in the artery (Fig. 1), where the blood is represented by a two-phase macroscopic model. In Fig. 1, we provide a dimensional shape function  $R(z)$  versus the dimensional value of the axial variable of an artery based on the actual dimensional data determined from the experimentally values of the cross-sectional area of the artery of a human (Back *et al.*, 1984). Here, as in more realistic cases, the blood flow is composed of a suspension of erythrocytes (red cells) in plasma.

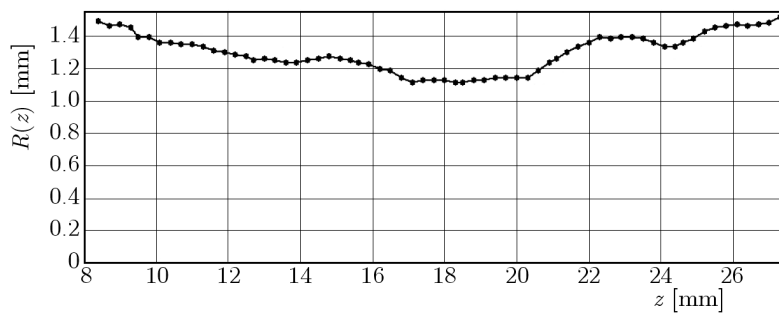


Fig. 1. Experimentally collected data for the atherosclerosis (Back *et al.*, 1984)

## 2. Formulation and analysis

We consider the problem of axisymmetric flow of blood in a catheterized artery in the form of a circular cylindrical annulus tube with the outer radius  $R_0$  (radius of the artery) and the inner radius  $r_1$  (radius of the catheter) and in the presence of an atherosclerosis whose shape (Fig. 1) is determined from the experimentally collected data (Back *et al.*, 1984). The artery length is assumed to be sufficiently large in comparison to its radius so that the end effects can be neglected.

The two-phase flow system in a catheterized artery is based on the original governing equations for the mass conservation and momentum (Batchelor, 1970) for both fluid plasma and the suspended particles (red cells) as their steady axisymmetric form in the cylindrical coordinate system with the axial direction along the co-axial direction of the catheterized artery are given by (Srivastava and Rastogi, 2010)

$$\begin{aligned}
 (1 - C)\rho_f \left( u_f \frac{\partial u_f}{\partial z} + v_f \frac{\partial u_f}{\partial r} \right) &= -(1 - C) \frac{\partial P}{\partial z} + (1 - C)\mu_s \nabla^2 u_f + CS'(u_p - u_f) \\
 (1 - C)\rho_f \left( u_f \frac{\partial v_f}{\partial z} + v_f \frac{\partial v_f}{\partial r} \right) &= -(1 - C) \frac{\partial P}{\partial r} + (1 - C)\mu_s \nabla^2 v_f + CS'(v_p - v_f) \\
 \frac{1}{r} \frac{\partial}{\partial r} [(1 - C)v_f] + \frac{\partial}{\partial z} [(1 - C)u_f] &= 0
 \end{aligned} \tag{2.1}$$

and

$$\begin{aligned}
C\rho_p\left(u_p\frac{\partial u_p}{\partial z} + v_p\frac{\partial u_p}{\partial r}\right) &= -\frac{\partial P}{\partial z} + CS'(u_f - u_p) \\
C\rho_p\left(u_p\frac{\partial v_p}{\partial z} + v_p\frac{\partial v_p}{\partial r}\right) &= -\frac{\partial P}{\partial r} + CS'(v_f - v_p) \\
\frac{1}{r}\frac{\partial}{\partial r}(Cr v_p) + \frac{\partial}{\partial z}(C u_p) &= 0
\end{aligned}
\tag{2.2}$$

Here  $\nabla^2 \equiv [(1/r)(\partial/\partial r)(r\partial/\partial r) + \partial^2/\partial z^2]$  is the Laplacian operator,  $r$  ( $r_1 \leq r \leq R_0$ ) and  $z$  are the cylindrical coordinates with the axial variable  $z$  along the tube axis and the radial variable  $r$  along the direction perpendicular to the tube axis, subscripts  $f$  and  $p$  refer to fluid (plasma) and particle (erythrocyte) quantities, respectively,  $u$  and  $v$  are the axial and radial velocity components, respectively,  $\rho$  is density,  $P$  is pressure,  $C$  is the volume fraction density of the particles, refers here as the hematocrit % in the blood, and the expressions for the viscosity of suspension  $\mu_s$  and the drag coefficient of interaction  $S'$  have been chosen to be (Srivastava, 1996; Srivastava and Srivastava, 2009)

$$\begin{aligned}
\mu_s &= \frac{\mu_0}{1 - mC} \quad m = 0.07 \exp\left[2.49C + \frac{1107}{T} \exp(-1.69C)\right] \\
S' &= 4.5 \frac{\mu_0}{a_0^2} \frac{4 + 3\sqrt{8C - 3C^2} + 3C}{(2 - 3C)^2}
\end{aligned}
\tag{2.3}$$

where  $\mu_0$  is the plasma viscosity,  $a_0$  is the radius of a red cell and  $T$  is absolute temperature measured in Kelvin. Based on the reasonable suggestion by Charm and Kurland (1974), the expression for plasma viscosity given by (2.3)<sub>1,2</sub> is accurate up to 60% hematocrit ( $C = 0.6$ ), and expression (2.3)<sub>1,2</sub> was derived first by Tam (1969) representing classical Stokes drag valid for a small particle Reynolds number.

We consider governing equations (2.1)-(2.3) for the blood flow in the axisymmetric form and use the cylindrical coordinate system with  $r$  as the radial variable,  $z$  as the axial variable and with the  $z$ -axis along the axis of cylindrical artery tube, where a catheter in the form of a tube with a small radius but along the axis of artery is placed in the artery. The inside boundary of the artery is partially structured along a distance  $L_0$  due to the presence of atherosclerosis (Fig. 2). In Fig. 2, where the flow system and the geometry is shown in the cylindrical annulus, the catheterized arterial tube is given over a distance  $L = 2d + L_0$  in the axial direction,  $\delta$  is the maximum height of atherosclerosis into the lumen, which appears at a particular location in the axial direction. In particular, we refer to a location corresponding to a value very close to the maximum height  $\delta$  of the atherosclerosis as the critical height such as the location at a distance  $z = d + L_0/2$  from the origin of the coordinate system.

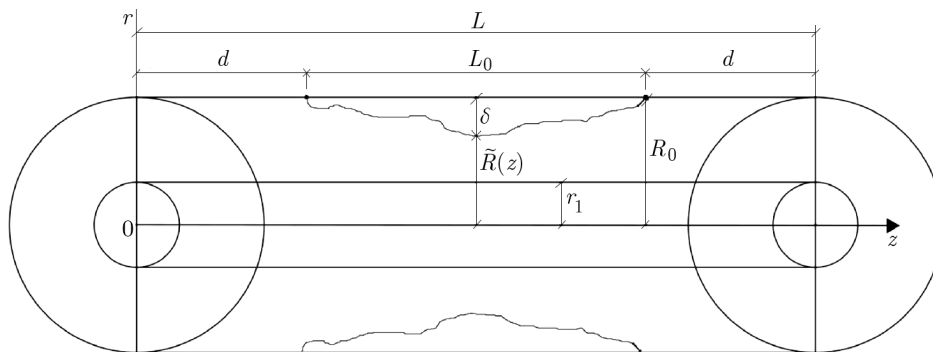


Fig. 2. Flow geometry in a catheterized artery with atherosclerosis

We now first non-dimensionalize governing equations (2.1)-(2.3) using  $U$ ,  $L_0$ ,  $R_0$ ,  $\delta$  and  $\mu_0 U L_0 / \delta^2$  as scales for the velocity, axial length, radial length, rate of radial change and pressure, respectively, where  $U$  is the maximum velocity for the unidirectional flow in a cylindrical annulus (White, 1991). Next, we simplify the dimensionless forms of governing equations (2.1)-(2.3) under the reasonable conditions for mild atherosclerosis with  $\delta/R_0 \ll 1$ , unidirectional flow assumption (White, 1991) where the axial velocity component dominates over the radial velocity component, and subjected the assumptions that the inertial terms in governing momentum equations (2.1)<sub>1,2</sub>, (2.2)<sub>1,2</sub> are small and  $R_e(\delta/L_0) \ll 1$ . Under these conditions and assumptions (Srivastava and Rastogi, 2010), the pressure is only a function of  $z$  and (2.1)-(2.3) lead to simpler equations. These simpler equations in non-dimensional forms are given below using the same symbols for the variables as their dimensional ones for simplicity of notations

$$(1-C) \frac{\partial P}{\partial z} = \frac{1-C}{1-mC} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_f}{\partial r} \right) + CS\beta^2(u_p - u_f) \quad (2.4)$$

$$\frac{dP}{dz} = S\beta^2(u_f - u_p) \quad S = 4.5 \frac{4 + 3\sqrt{8C - 3C^2} + 3C}{(2 - 3C)^2}$$

where  $\beta = \delta/a_0$ . Equations (2.4) are subjected to the following no slip boundary conditions

$$u_f = 0 \quad \text{on} \quad r = r_1 \quad \text{and} \quad u_f = 0 \quad \text{on} \quad r = R(z) \quad (2.5)$$

Using (2.4)<sub>2,3</sub> for  $(u_f - u_p)$  in (2.4)<sub>1</sub> and integrating twice with respect to  $r$  and making use of the boundary conditions given in (2.5), we find

$$u_f = -\frac{1-mC}{4(1-C)} \frac{dp}{dz} \left[ R^2 - r^2 + \frac{(R^2 - r_1^2) \ln \frac{r}{R}}{\ln \frac{R}{r_1}} \right] \quad (2.6)$$

The expression for the axial velocity for the red cells is then found from (2.4)<sub>2,3</sub> in terms of the axial velocity for the plasma in the form

$$u_p = u_f - \frac{1}{S\beta^2} \frac{dp}{dz} \quad (2.7)$$

Since both expressions for the axial velocity of plasma and red cells given by (2.6) and (2.7) are in terms of the unknown pressure gradient ( $dP/dz$ ), we obtain an expression for the pressure gradient by assuming a prescribed volume flow rate in the annulus given by

$$Q = 2\pi \int_{r_1}^R r[(1-C)u_f + Cu_p] dr \quad (2.8)$$

Using (2.6) and (2.7) in (2.8) and solving for the pressure gradient, we find

$$\frac{dP}{dz} = -\frac{Q}{\frac{\pi}{16}(R^2 - r_1^2)} \left[ \frac{16c}{S\beta^2} + 2 \frac{R^2 - r_1^2}{1-C} (1-mC) \left( \frac{R^2 + r_1^2}{R^2 - r_1^2} - \frac{1}{\ln \frac{R}{r_1}} \right) \right] \quad (2.9)$$

The flow resistance, referred to as the impedance  $\lambda$ , is given by

$$\lambda = \frac{\Delta P}{Q} \quad (2.10)$$

where  $\Delta p$  is the pressure drop across the length  $1 + 2b$  given by

$$\Delta P = P(0) - P(1 + 2b) = \int_0^{1+2b} \frac{dP}{dz} dz \quad (2.11)$$

and  $b = d/L_0$ .

From (2.6), we find the wall shear stress  $\tau_w$  to be

$$\tau_w = -\left. \frac{\partial u_f}{\partial r} \right|_{r=R(z)} = -\frac{R(1-mC)}{2(1-C)} \frac{dP}{dz} - \frac{1-mC}{4R(1-C)} \frac{dP}{dz} \frac{r_1^2 - R^2}{\ln \frac{R}{r_1}} \quad (2.12)$$

and the shear stress force  $F$  over the surface of the artery from  $z = 0$  to  $z = 1 + 2b$  is then given by

$$F = 2\pi \int_0^{1+2b} \tau_w dz \quad (2.13)$$

The value of the wall shear stress at a different location where the atherosclerosis is located can then be found from (2.12) for a specific  $z$  value.

### 3. Results and discussion

We carried out numerical calculations of various expressions obtained in the previous section for several different values of the hematocrit parameter  $C$ , the radius of the catheter and over a range of values of the axial and possibly radial variables. For all the calculations, we set the volume flow rate  $Q = 1$ ,  $b = 0.5$  and  $\beta = \delta/0.004$ , where  $\delta = 1 - \min R$ .

Figures 3-5 present results for  $dP/dz$  (axial rate change of the blood pressure in the catheterized artery). Figure 3 presents the pressure gradient versus the axial variable for the catheter radius = 0.2 and for several values of the hematocrit parameter. It can be seen from this figure that the blood pressure gradient is negative implying that the blood pressure force is in the direction of the positive  $z$ -axis. The blood pressure force does not vary with respect to the axial variable at the axial locations outside the atherosclerosis zone. However, the magnitude of the blood pressure force increases with the atherosclerosis effect in the atherosclerosis zone. The magnitude of the pressure force also increases with the hematocrit effect. These results are physically and bio-medically reasonable since higher percentage of the blood cells amount in the plasma as well as more severity of the atherosclerosis can intensify the blood pressure force in the artery.

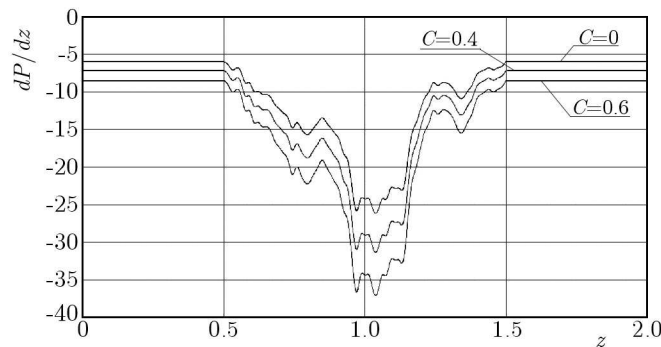


Fig. 3. Axial rate of change of blood pressure versus axial variable for catheter radius = 0.2 and several values of hematocrit parameters

In Figs. 4 and 5, similar results to those in Fig. 3 are presented but for higher values of the catheter radius. It can be seen that the magnitude of the blood pressure force in the artery increases with the catheter radius, which again makes sense physically since higher catheter radius implies smaller annulus gap leading to a higher value of such a force.

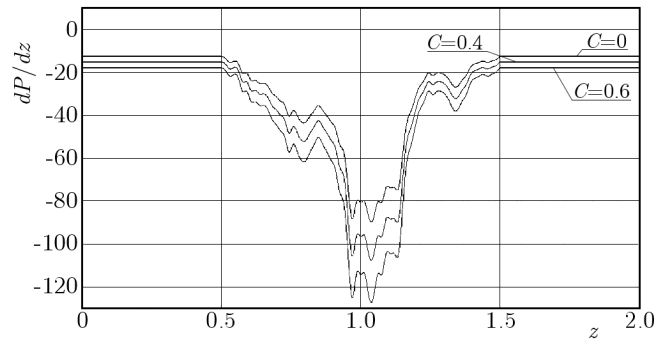


Fig. 4. The same as in Fig. 3 but for catheter radius = 0.4

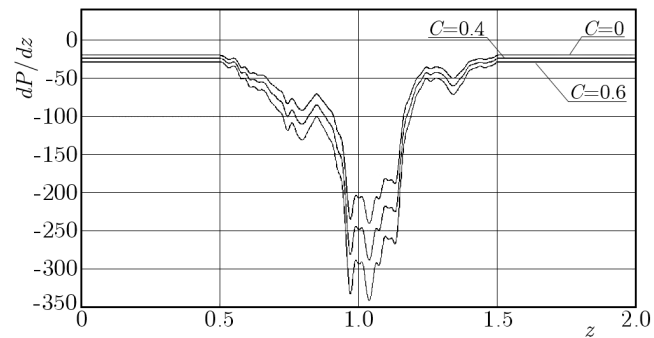


Fig. 5. The same as in Fig. 3 but for the catheter radius = 0.5

Figure 6 presents the axial velocity of the blood plasma versus the axial variable for the hematocrit parameter = 0.4, the radial variable = 0.6 and for two different values of the catheter radius. It can be seen from this figure that the plasma velocity is positive, which makes sense since the blood pressure force is in the positive direction of the axis of the catheterized artery system. The plasma velocity is constant outside the atherosclerosis zone, while it is variable and its magnitude increases with the atherosclerosis effect. This result is reasonable since higher atherosclerosis effect increases the strength of the pressure force leading to higher blood plasma speed. The plasma velocity also increases with the catheter radius, which is reasonable since higher catheter radius decreases the annulus gap leading to higher plasma speed. Our additional generated data for the plasma velocity versus different values of the hematocrit percentage indicated insignificant changes of the plasma speed with respect to the hematocrit effect.

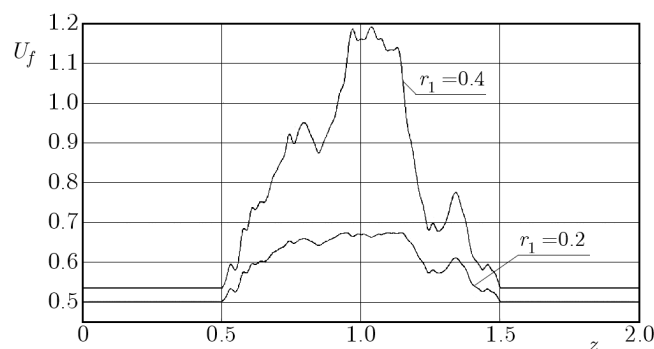


Fig. 6. Axial velocity of plasma versus the axial variable for  $C = 0.4$ ,  $r = 0.6$  and  $r_1 = 0.4$

Figure 7 present the values of the plasma velocity versus the radial variable for the catheter radius = 0.2, the axial value of  $z = 1$  and the value of 0.4 for the hematocrit parameter. It can be seen from this figure that the plasma velocity satisfied its zero no-slip conditions at the two

boundaries of the catheterized artery system, while it has a maximum value at some location in the annulus away from the boundary.

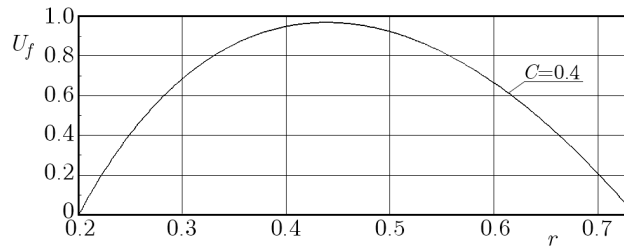


Fig. 7. Axial velocity of plasma versus  $r$  ( $r_1 \leq r \leq R$ ) for  $r_1 = 0.2$ ,  $z = 1$  and  $C = 0.4$

Figure 8 presents the axial velocity of the red cells versus the axial variable for a fixed radial location ( $r = 0.6$ ), for the value of 0.4 for the hematocrit parameter and for two values 0.2 and 0.4 of the catheter radius. As can be seen from this figure, the red cells velocity is higher for higher value of the catheter radius. The speed of the red cells is constant outside the atherosclerosis zone, but it varies significantly in the atherosclerosis zone and increases significantly at the locations where the atherosclerosis is more severe.

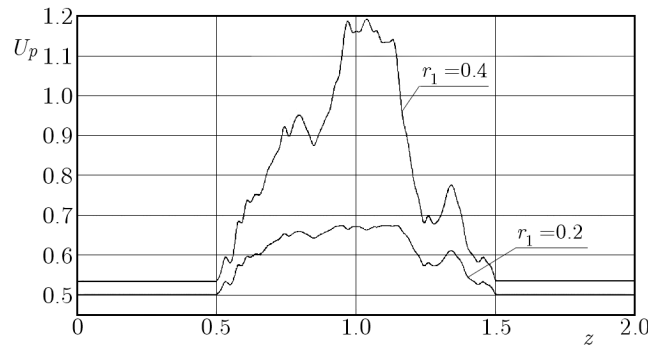


Fig. 8. Axial velocity of red cells versus the axial variable for  $r = 0.6$ ,  $C = 0.4$  and two different  $r_1$  values 0.2 and 0.4.

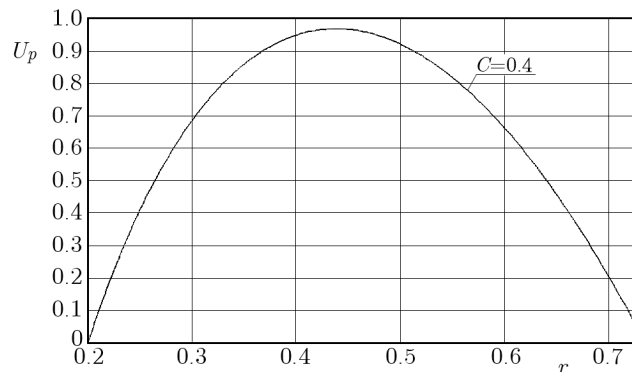


Fig. 9. Axial velocity of the red cells versus the radial variable for  $z = 1$ ,  $C = 0.4$  and  $r_1 = 0.2$

Figure 9 presents the speed of the red cells versus the radial variable for  $z = 1$ ,  $C = 0.4$  and  $r_1 = 0.2$ . It can be seen from this figure that the highest speed of cells is near mid-region of the annulus, the rate of increase of the speed with respect to the radial variable is higher near the catheter, while the rate of decrease of the speed is smaller near the artery wall. Our additional generated data for the variation of the velocity of the red cells with respect to the hematocrit parameter indicated that the magnitude of such a velocity increases with  $C$ , even though such increase is not very significant.

Figure 10 presents the impedance versus the hematocrit parameter for values of 0.2, 0.4 and 0.6 of the catheter radius. It can be seen from this figure that the impedance increases with respect to the hematocrit parameter especially noticeably for  $C < 0.2$  and  $C > 0.4$ . Thus, the flow resistance increases with increase in the hematocrit character of the blood especially for a relatively small percentage of the red cells or a large percentage of such cells in the plasma. The flow resistance also is higher if the catheter radius is larger.

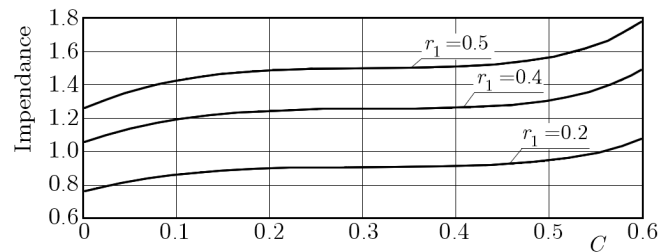


Fig. 10. Impedance versus  $C$  for  $r_1 = 0.2, 0.4$  and  $0.6$

Figure 11 presents the wall shear stress versus the axial variable for the value of 0.4 for the hematocrit parameter and for two values 0.2 and 0.4 of the other catheter radius. It can be seen from this figure that the shear stress increases with the catheter radius. The shear stress is constant outside the atherosclerosis zone, while it varies with respect to the axial variable and increases significantly in the atherosclerosis zone, and such an increase intensifies if the atherosclerosis becomes more severe. Although the shear stress was found to increase with the hematocrit parameter, such an increase was found to be weak.

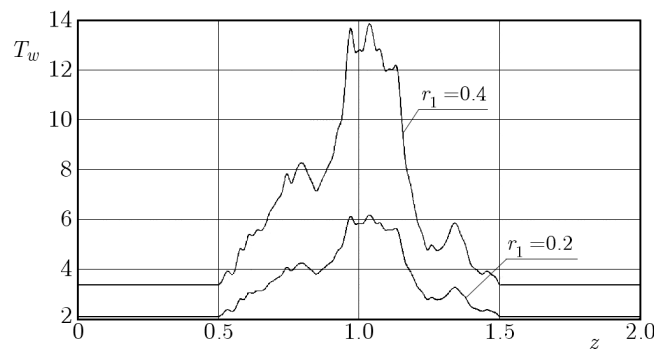


Fig. 11. Wall shear stress versus  $z$  for  $C = 0.4$  and  $r_1 = 0.2, 0.4$

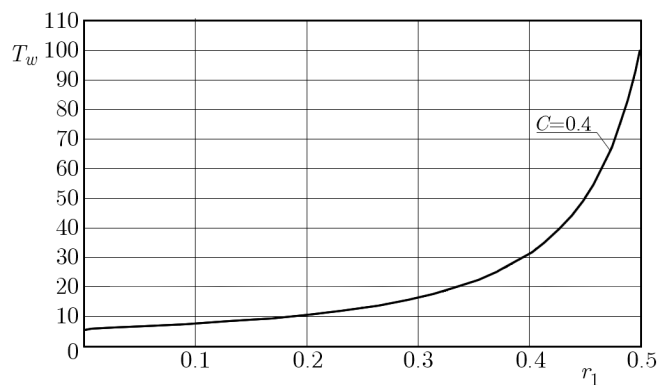


Fig. 12. Wall shear stress versus catheter radius for  $z = 1$  and  $C = 0.4$

Figure 12 presents the shear stress versus the radius of the catheter for  $z = 1$  and  $C = 0.4$ . It can be seen from this figure that the shear stress increases with the radius of the catheter, and its rate of increase with respect to the catheter radius also increases with the catheter radius.



#### 4. Conclusion

We investigated the catheterized arterial blood flow in the presence of the atherosclerosis effect. Our formulation was based on a two-phase blood fluid flow model of combination of plasma and red cells. We calculated steady cases for important quantities, such as the pressure gradient force, plasma velocity, red cells velocity, impedance and wall shear stress in the presence of the steady atherosclerosis effect, which is taken into account from the available experimental results for a human. These quantities were evaluated at several values of the hematocrit parameter  $C$ , the axial variable  $z$  or the radial variable  $r$  and the catheter radius. We found, in particular, that the pressure gradient force, plasma velocity, red cells velocity and wall stress are significant in atherosclerosis zone, and they become stronger if the effect of the atherosclerosis is stronger. These conditions could be relevant to the corresponding conditions for certain patients, which can require higher care and attention.

The extension of the present paper to the cases of unsteady arterial flow in the presence of the pulse frequency, whose calculated results turn out to depend on the results of the present paper, will be presented in a forthcoming paper. Another important extension of the present study can be for more medically realistic finite systems with cases conforming to more medically generated data in order to identify the components of arterial blood flow diseases, which can improve health conditions of the corresponding patients.

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### **Dwufazowy przepływ krwi w tętnicy ze zmianami miażdżycowymi po wykonanym zabiegu koronarografii**

#### Streszczenie

W pracy omówiono problem przepływu krwi w tętnicy po zabiegu koronarografii w obecności zmian miażdżycowych, opierając się na osiągalnych danych z badań klinicznych. Miażdżycą nazywamy stan, w którym ściana tętnicy pogrubia się do wewnątrz wskutek odkładania się tłuszczu, głównie cholesterolu. W standardowej metodzie leczenia miażdżycy stosuje się zabieg koronarografii polegający na wprowadzeniu cewnika do upośledzonej tętnicy. W pracy opisano przepływ krwi w przekroju tętnicy za pomocą dwufazowego modelu odzwierciedlającego zawiesinę czerwonych ciałek krwi w osoczu. Sprężone, różniczkowe równania przepływu płynu (osocza) i ruchu cząstek (czerwonych ciałek) rozwiązano analitycznie w stopniu akceptowalnie przybliżonym. Tak istotne wielkości, jak prędkość przepływu osocza, prędkość czerwonych ciałek, ciśnienie krwi, impedancja (opory przepływu) oraz naprężenia ścinające w ścianie tętnicy obliczono dla różnych rozmiarów cewnika, osiowego rozkładu złogów miażdżycowych oraz hematokrytu wywołanego dwufazową kombinacją czerwone ciała-osocze w badanym układzie krwionośnym.

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