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FRICTIONAL ASPECTS OF OPERATION FOR CENTRIFUGAL ACCELERATORS WITH RADIAL BLADES

TARCIOWE ASPEKTY PRACY ŁOPATEK PROMIENIOWYCH W AKCELERATORACH ODŚRODKOWYCH

Key words: centrifugal machine; blade of rotor, solid particle, friction, speed, energy loss.

Abstract: The work aims to determine the friction effect and change the velocity and kinetic energy during the movement of solid particles along the rotating machine's radial blade surface. It has been previously shown that the influence of the frictional forces of the particle against the rotor disc is negligible compared to the influence of the centrifugal force and the friction force of the particle against the rotor blade. Based on the analysis of the differential equation solution for particle motion along the surface of the blades, it was established that the total sliding velocity of a particle increase intensively in the initial period of motion and approaches asymptotically to the values described by a linear function, practically independent of the initial position of the particle. The obtained analytical expressions enable the determination of change in the relative and total velocity of the particle, the angle between the respective velocity vectors and its kinetic energy. Changes in the values of these parameters were also estimated for a wide range of variability of the friction coefficient.

Słowa kluczowe: maszyna odśrodkowa; łopatka wirnika, cząstka stała, tarcie, prędkość, straty energii.

Streszczenie: Celem pracy jest określenie wpływu tarcia cząstek stałych na ich prędkość i straty energii kinetycznej podczas ruchu po powierzchni łopatek promieniowych maszyn wirnikowych. Wykazano, że wpływ sił tarcia cząstki o tarczę wirnika jest pomijalnie mały w porównaniu z wpływem siły odśrodkowej i siły tarcia cząstki o łopatkę wirnika. Na podstawie analizy rozwiązania równania różniczkowego ruchu cząstek po powierzchni łopatek ustalono, że całkowita prędkość poślizgu cząstki intensywnie wzrasta w początkowym okresie ruchu i zbliża się asymptotycznie do wartości opisanych funkcją liniową, praktycznie niezależną od położenia początkowego cząstki. Uzyskane wyrażenia analityczne umożliwiają określenie: zmiany prędkości względnej i całkowitej cząstki, kąta pomiędzy odpowiednimi wektorami prędkości oraz jej energii kinetycznej. Dokonano również oszacowania zmiany wartości tych parametrów dla szerokiego zakresu zmienności współczynnika tarcia.

INTRODUCTION

The work process of many turbo machines is based on the movement of solid particles along the blades mounted on the rotors. Typical representatives of such machines are injection fans of pneumatic transport systems **[L. 1, 2, 3]**, equipment for feeding crushed fuel **[L. 4, 5]**, accelerators of centrifugal impact grinders **[L. 6–11]**, classifiers, separators **[L. 12]**, testing equipment and other devices for transporting dispersed materials, the main working body of which is a rotor with blades **[L. 13, 14]**. Despite the progress made in such a machine's calculation and design, many issues have not been sufficiently studied. In particular, there are no practical engineering methods for calculating the effect of particle friction on the disk

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and rotor blades on the movement of kinematic and energy parameters for displaced solid particles in the working area of the turbo machine. Using numerical methods **[L. 4, 6, 7]** makes it possible to determine the necessary parameters with a specific error but requires developing special software tools, which is quite time-consuming and not always convenient in engineering practice. In addition, the accuracy of calculations depends not only on the perfection of software tools but also largely on the accuracy of the initial data used in the calculations, primarily the friction coefficients of the particle on the blade, the values of which may not coincide under dynamic and static conditions of the wind, primarily due to the presence of vibration in high-speed machines. Graphoanalytic methods **[L. 1, 13]** are quite time-consuming. At the same time, information about the frictional aspects of the coarse particles sliding during their acceleration in the centrifugal grinders rotors is of considerable interest to the corresponding equipment designers. Firstly, friction affects the speed of movement, the kinetic energy of the particle on departure from the blade and the energy of impact interaction with the bump elements, which determines the grinding efficiency. Secondly, the ratio of radial relative and circumferential portable velocities depending on the friction of particles in the working area of the rotor, determines the angle of the particle's departure according to which the working surfaces of the bump elements should be installed to ensure a perpendicular impact, accompanied by minimal wear of the bump elements. Moreover, thirdly the friction-dependent sliding speed, along with the rotor angular velocity, determines the magnitude of the Coriolis force pressing the particle against the blade and causing erosive wear of the latter one.

The work aims to create and test a method for calculating the friction effect on the speed regime and energy loss during the movement of coarse solid particles along the radial blades of turbomachines.

PROBLEM STATEMENT AND RESEARCH METHOD

The principal scheme of a centrifugal accelerator with a horizontal rotor – disk containing radially arranged blades is shown in **Fig. 1 [L. 15]**.

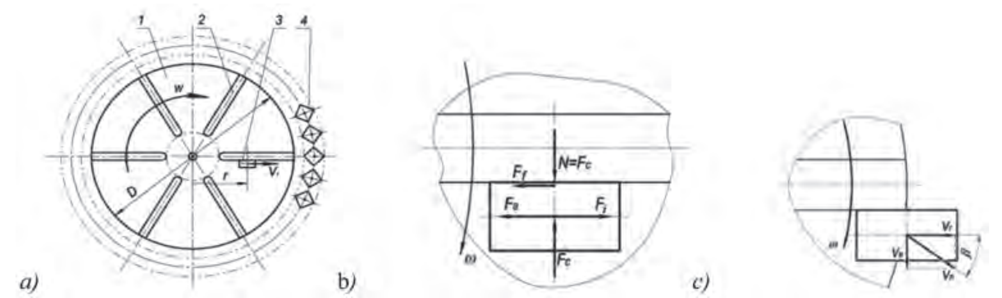

Fig. 1. Diagram for the structure of centrifugal accelerator with radially arranged blades and the distribution of forces acting on the particle and its velocities: a) diagram of the rotor with blades (1 – rotor, 2 – blade, 3 – particle, 4 – impact plate); b) diagram of forces acting on the particle; c) particle velocity diagram

Rys. 1. Schemat konstrukcji akceleratora odśrodkowego z promieniowo ułożonymi łopatkami oraz rozkład sił działających na cząstkę i jej prędkości: a) schemat wirnika z łopatkami (1 – wirnik, 2 – łopatka, 3 – cząstka, 4 – element, z którym zderza się cząstka po opuszczeniu łopatki); b) schemat sił działających na cząstkę; c) schemat prędkości cząstki

A particle of mass (m) after contact with the rotor blade is affected by gravity (G) and the disk reaction force balancing it; centrifugal force (F_i); friction force (F_f) of the particle on the rotor blade or the rotor disk (depends on the position of the particle along the height of the blade); aerodynamic drag force (F_a) pressing the particle against the disk blade; the Coriolis (F_C) inertia force as well as the reaction of the disk blade equal in magnitude and acting towards the Coriolis force. In this case, the centrifugal force (F_i) is directed along the radius and causes accelerated motion of the particle, and the friction force (F_f) of the particle on the rotor blade or the rotor disk and the aerodynamic drag force (F_a) is directed towards the movement. In some cases, for particles located in the angular space at the junction of the blade and the disk, the action of two friction forces simultaneously is possible – on the disk and the blade.

PERFORMING A COMPARATIVE ANALYSIS OF EACH OF THESE FORCES

The friction force of a particle on a disk can be determined from Coulomb's friction law as the product of the friction coefficient of the particle on the disk material (f_d) by the gravity of the particle (G), equal to the product of its mass (m) на by the acceleration from gravity (g):

$$F_{fd} = f_d G = f_d mg. \tag{1}$$

The centrifugal force (F_i) is equal to the product of the particle mass (m) by its centripetal acceleration and is directed from the centre of rotation.

$$F_i = m\omega^2 r, \tag{2}$$

where ω is the rotor angular velocity, r is the distance from the rotor centre to the particle.

Centrifugal accelerators are, as a rule, high-speed machines with an angular velocity of the rotor of about 150 s^{-1} for working fans of pneumatic transport systems **[L. 1, 4, 5]** and centrifugal crushers for coarse grinding to 380, …, 470 s^{-1} in impact centrifugal mills **[L. 6, 7, 9]**. The gravity forces of the particles and the friction forces due to them are negligibly small in comparison with the centrifugal force and the Coriolis force of inertia at such angular velocities. It is possible to show this in the following example. Let us determine the distance (r) at which the friction force of the particle on the disk does not exceed 5% of the centrifugal force. In mathematical notation, this condition has the form:

$$f_d mg \leq 0.05 m\omega^2 r. \tag{3}$$

It is possible to obtain from equation (3) after the transformations:

$$r \geq \frac{f_d g}{0.05\omega^2}. \tag{4}$$

As can be seen from (4), this distance does not depend on the mass of the particle but rather depends on the angular velocity (ω) of the rotor and the friction coefficient (f_d) of the particle on the rotor. Let us quantify the distance r using formula (4) using the example of quartz sand movement at the angular velocity of the rotor ω = 150 s^{-1}. We will also consider that turbo machines' operation is usually accompanied by vibration, which contributes to a decrease in the coefficient of friction compared with the data obtained during static tests. The rotor blades are usually made of steel, which is often coated with carbide coatings. The friction coefficient of mineral on steel disc (f_d) changes for such materials in the range f_d= 0.3–0.6 **[L. 6, 9, 14]**. The calculations performed show that this distance is:

$$r \geq \frac{(0,3, \ldots, 0,6) \cdot 9,8}{0,05 \cdot 150^2} = (0,006, \ldots, 0,0052)\ \text{m} = (2,6, \ldots, 5,2)\ \text{mm}.$$

Thus, in the case under consideration, at a distance more than 2.6…5.2 mm from the disk centre, the friction force effect for the particle on the disk surface, due to the gravity of the particle, is negligible in comparison with the centrifugal force. Moreover, with an increase in the angular velocity ω of the rotor, this value decreases further, which makes it possible to ignore this friction force for industrial and even laboratory rotary machines in technical calculations. In addition, previous studies have shown **[L. 16]** that the influence of aerodynamic drag forces on coarse particles can be neglected due to their smallness compared with the centrifugal force and the friction force from pressing the particle to the blade by the Coriolis force.

The Coriolis force (F_C) is equal to the product of the particle mass (m) by the Coriolis acceleration (twice the product of the rotor angular velocity (ω) by the velocity (V_r) of the particle sliding relative to the blade):

$$F_C = m \cdot 2f\omega r V_r = 2mf\omega\dot{r}, \tag{5}$$

where f is the friction coefficient of the particle on the rotor blade.

Taking into account the accepted assumptions about the smallness of the friction forces of the particle on the rotor disk as well as the smallness of the aerodynamic drag force, the differential equation for particle motion along the accelerator blade has the form:

$$m\ddot{r} = m\omega^2 r - m \cdot 2f\omega\dot{r} \tag{6}$$

or after the transformations:

$$\ddot{r} + 2f\omega\dot{r} - \omega^2 r = 0. \tag{7}$$

then perform the following replacement in order to analyse the velocity distribution along the disk radius:

$$\frac{dV_r}{dt} = \frac{dV_r}{dt}\frac{dr}{dt} = V_r\frac{dV_r}{dr}. \quad (8)$$

Then the differential equation (7) takes the form

$$V_r\frac{dV_r}{dr} + 2f\omega V_r = \omega^2 r. \quad (9)$$

It is difficult to obtain analytical solutions for equation (9). Usually, numerical **[L. 4, 5, 6, 9]** or graphic analytical **[L. 1, 13]** methods are used, which are either quite time-consuming or require the development of appropriate software tools.

The analysis shows that one of the partial solutions of the differential equation (9) is a linear function passing through the origin. At the same time, using the method of indefinite coefficients, it is possible to obtain an equation of the form:

$$V_{r0} = \left(\sqrt{1+f^2} - f\right)\omega r, \quad (10)$$

where V_{r0} is the particle's velocity that started moving near the rotor axis ($r = 0$).

Search for a general solution. Making a substitution:

$$V_r = u \cdot r. \quad (11)$$

where u is an auxiliary variable mathematical parameter.

Then:

$$V_r' = u'\,r + u, \quad (12)$$

$$V_r V_r' = ur(u'r+u) = uu'r^2 + u^2 r\,. \quad (13)$$

Substituting (11) and (12) into (9) gives:

$$uu'r^2 + u^2 r + 2f\omega ur = \omega^2 r. \quad (14)$$

The following equation can be obtained after reducing r and separating the variables,

$$\frac{udu}{u^2+2f\omega u-\omega^2} = -\frac{dr}{r}\,. \quad (15)$$

Integration (15) leads to the following analytical dependence (intermediate calculations and transformations are omitted):

$$Cr\sqrt{|u^2 + 2f\omega u - \omega^2|} = \left|\frac{u-\omega\left(\sqrt{1+f^2}-f\right)}{u+\omega\left(\sqrt{1+f^2}+f\right)}\right|^{\frac{f}{2\sqrt{1+f^2}}}. \quad (16)$$

where C is the constant of integration.

It is possible to transform the resulting formula by the square of both sides:

$$C^2 r^2|u^2 + 2f\omega u - \omega^2| = \left|\frac{u-\omega\left(\sqrt{1+f^2}-f\right)}{u+\omega\left(\sqrt{1+f^2}+f\right)}\right|^{\frac{f}{\sqrt{1+f^2}}}. \quad (17)$$

When returning to the original variable V_r using equation (11), the formula for the relationship between the sliding velocity (V_r) of a particle and its coordinate (r) takes the following form after substitution u and transformations:

$$C^2|V_r^2 + 2f\omega V_r r - \omega^2 r^2| = \left|\frac{V_r-\left(\sqrt{1+f^2}-f\right)\omega r}{V_r+\left(\sqrt{1+f^2}+f\right)\omega r}\right|^{\frac{f}{\sqrt{1+f^2}}}. \quad (18)$$

It is possible to find the constant integration C by using the following boundary conditions $r = r_0$, $V_r = 0$, then:

$$C^2 = \frac{1}{\omega^2 r_0^2}\left|\frac{\sqrt{1+f^2}-f}{\sqrt{1+f^2}+f}\right|^{\frac{f}{\sqrt{1+f^2}}}. \quad (19)$$

Substituting this value into formula (18) gives:

$$\frac{\left(\sqrt{1+f^2}-f\right)^{\frac{2f}{\sqrt{1+f^2}}}}{\omega^2 r_0^2}|V_r^2 + 2f\omega rV_r - \omega^2 r^2| = \left|\frac{V_r-\left(\sqrt{1+f^2}-f\right)\omega r}{V_r+\left(\sqrt{1+f^2}+f\right)\omega r}\right|^{\frac{f}{\sqrt{1+f^2}}}. \quad (20)$$

This equation relates the velocity of the particle (V_r) with the coordinate (r) of its location in an implicit form. The dependence is difficult to analyse and inconvenient for practical use. At the same time, it makes it possible to obtain some interesting and important results for engineering applications. First, let us consider the condition for the graph to pass through the origin: particle movement beginning from a point adjacent to the origin ($r = r_0 = 0$). However, division by zero is possible only in the case when the division of 0 by 0 is ensured. Thus, the expression in brackets on the left side of the formula, which is under the sign of the absolute value (positive value), must be equal to zero:

$$V_r^2 + 2f\omega r V_r - \omega^2 r^2 = 0. \tag{21}$$

The following functions are the roots of the quadratic equation (21):

$$V_{r1} = \left(\sqrt{1+f^2} - f\right)\omega r. \tag{22a}$$

$$V_{r2} = -\left(\sqrt{1+f^2} + f\right)\omega r. \tag{22b}$$

Obviously, only the positive velocity has a physical meaning that corresponds to the analytical dependence (22a), which coincides with the formula (10) previously obtained by the selection method.

When performing a qualitative analysis of the differential equation (9) for other particles that started sliding relative to the blade at a point with a distance of $r_{0,}$ it is possible to determine the formula from (9) the gradient of the particle sliding velocity along the rotor radius:

$$\frac{\mathrm{d}V_r}{\mathrm{d}r} = \frac{\omega^2 r - 2f\omega r V_r}{V_r} \tag{23}$$

At the initial moment of the sliding, the particle velocity must be equal to zero. Since the numerator in the fraction (23) is not equal to zero, infinity is obtained from dividing any number by zero. It follows from this that the tangent to the velocity change graph at the initial motion moment must be perpendicular to the radius. On the other hand, at very high values of the sliding velocity (V_r), one should expect a linear dependence of velocity on the radius to ensure the fulfilment of the dimension rule.

When obtaining the analytical solution of the differential equation (7) for a more detailed analysis of the change in the particle sliding velocity along the radius of the disc, the equation is linear and describes the particle coordinate at an arbitrary moment of time. It is not difficult to make sure that the solution (7) under initial conditions ($t = 0$; $r = r_0$; $V_r = 0$) is the function:

$$r = \frac{r_0\left(\sqrt{1+f^2}+f\right)}{2\sqrt{1+f^2}} e^{\left(\sqrt{1+f^2}-f\right)\omega t}(1+\delta_r), \tag{24}$$

where:

$$\delta_r = \left(\sqrt{1+f^2} - f\right)^2 e^{-2\sqrt{1+f^2}\omega t} \tag{25}$$

A formula for determining the radial velocity (V_r) was obtained by differentiating (24):

$$V_r = \frac{\omega r_0}{2\sqrt{1+f^2}} e^{\left(\sqrt{1+f^2}-f\right)\omega t}(1-\delta_V), \tag{26}$$

where:

$$\delta_V = e^{-2\sqrt{1+f^2}\omega t} \tag{27}$$

Linear speed (V_e) for particle rotation together with the rotor is equal:

$$V_e = \omega r = \frac{\omega r_0\left(\sqrt{1+f^2}+f\right)}{2\sqrt{1+f^2}} e^{\left(\sqrt{1+f^2}-f\right)\omega t}(1+\delta_r). \tag{28}$$

The absolute (total) velocity (V_a) is then:

$$V_a = \sqrt{V_e^2 + V_r^2} = \frac{\omega r_0}{2\sqrt{1+f^2}} e^{\left(\sqrt{1+f^2}-f\right)\omega t} \sqrt{\left(\sqrt{1+f^2}+f\right)^2(1+\delta_r)^2 + (1-\delta_V)^2}. \tag{29}$$

The tangent of the angle (β) between the relative radial (V_r) and the total (V_a) velocities of the particle (**Fig. 1c**) will be found from the following relation:

$$\mathrm{tg}\beta = \frac{V_e}{V_r} = \frac{\left(\sqrt{1+f^2}+f\right)(1+\delta_r)}{1-\delta_V}. \quad (30)$$

The above formulas allow us to analyse the change in displacements and velocities at any time and, accordingly, to determine the change in the velocity of the particle as it moves along the rotor blade. They also allow us to find the change in the corresponding kinematic parameters as a function of the particle's coordinate (r) on the blade's radius and the friction coefficient (f).

RESEARCH RESULTS

It was considered for analysis that centrifugal accelerators are widely used in various branches of engineering and technology for coarse and fine grinding of raw mineral materials. They are used, for example, in the mining and building materials industry in the manufacture of high-quality cube-shaped crushed stone from granite or other rocks, in the processing of crushing waste of these rocks, in the mechanical activation of concrete components, in the grinding of apatite concentrate, and asbestos, dolomite, lime and cement stone and other components of building mixtures, quartz sand and cullet in glass production **[L. 6, 8, 9]**. The friction coefficient of minerals on steel (in the manufacture of rotor blades of hardened steel) is for quartz sand, dolomite, apatite concentrate and cement clinker approximately 0.3, …, 0.34 **[L. 6, 17]**; for glass, granite, limestone, coke, and cryolite is one about 0.5, …, 0.6 **[L. 17]**. The friction coefficient differs from the above values in manufacturing the blades with special wear-resistant coating, which should be taken into account when performing tribo technological calculation of centrifugal machinery.

The examples in **Tables 1** and **2** are the results of calculations of the particle's dimensionless (relative) displacements (r/r_0), as well as the parameters δ_r and δ_V for a rotor with angular velocity $\omega = 150$ s^{-1} when a coefficient of friction is $f = 0.3$ and $f = 0.5$. Smaller values of the friction coefficient ($f = 0.3$) correspond to the sliding of quartz sand-type particles on the hardened steel blade surface **[L. 6]**, and larger values ($f = 0.5$) correspond to the glass or granite particles sliding off on a hardened steel blade's surface. An angular velocity of 150 s^{-1} corresponds to the operation of centrifugal accelerators, for example, in the grinding complex KI-1.0 while crushing mineral raw materials.

Table 1. The calculated values of the relative displacement (r/r_0), as well as the parameters δr and δ_V of the particle sliding along the centrifugal accelerator blade at $\omega = 150$ s^{-1}

Tabela 1. Obliczone wartości przemieszczenia względnego (r/r_0), oraz parametrów δ_r i δ_V cząstki ślizgającej się wzdłuż łopatki akceleratora odśrodkowego przy $\omega = 150$ s^{-1}

Time, ms	The calculated value of the parameter at the coefficient of friction					
	$f = 0.3$			$f = 0.5$		
	δ_r	δ_v	r/r_0	δ_r	δ_v	r/r_0
5	0.116	0.209	1.255	0.071	0.187	1.232
10	0.024	**0.044**	**2.012**	0.013	0.035	1.853
15	0.005	**0.009**	**3.451**	0.002	0.007	2.914
20	0.001	$2{*}10^{-3}$	6.005	$4.66{*}10^{-4}$	0.001	4.623
25	$2.2{*}10^{-4}$	$3.98{*}10^{-4}$	10.483	$8.71{*}10^{-5}$	$2.28{*}10^{-4}$	7.346
30	$4.6{*}10^{-5}$	$8.3{*}10^{-5}$	18.313	$1.63{*}10^{-5}$	$4.27{*}10^{-5}$	11.677

For higher values of the rotor, angular velocity similar data are presented in **Tables 3** and **4**.

An angular velocity of 150 s^{-1} corresponds to the operation of centrifugal accelerators, for example, in grinding complex KI-0.4 during fine grinding of mineral raw materials **[L. 6, 8]**. It was taken into account in the calculations that the linear velocity of the particles should increase with an increase in the rotor angular velocity. Therefore, the time range t is assumed to be smaller in comparison with the range for **Tables 1** and **2**.

Table 2. Calculated values of relative particle sliding velocities along the centrifugal accelerator rotor blade and the angle (*β*) between them at $\omega = 150\ s^{-1}$

Tabela 2. Obliczone wartości prędkości względnych poślizgu cząstek wzdłuż łopaty wirnika akceleratora odśrodkowego i kąta (*β*) między nimi przy $\omega = 150\ s^{-1}$

Time, ms	The calculated value of the parameter at the coefficient of friction					
	$f = 0.3$			$f = 0.5$		
	$V_r/\omega r$	$V_a/\omega r$	tgβ	$V_r/\omega r$	$V_a/\omega r$	tgβ
5	0.528	1.261	1.895	0.469	1.183	2.132
10	0.695	1.247	1.439	0.589	1.176	1.699
15	0.734	1.246	1.363	0.612	1.176	1.633
20	0.742	1.246	1.348	0.617	1.176	1.621
25	0.744	1.246	1.345	0.618	1.176	1.619
30	0.744	1.246	1.344	0.618	1.176	1.618

Table 3. Calculated values of the relative displacement (*r/r_0*), as well as the parameters δ_r and δ_V of the particle sliding along the centrifugal accelerator blade at $\omega = 450\ s^{-1}$

Tabela 3. Obliczone wartości przemieszczenia względnego (r/r_0) oraz parametrów δ_r i δ_V cząstki ślizgającej się wzdłuż łopatki akceleratora odśrodkowego przy $\omega = 450\ s^{-1}$

Time, ms	The calculated value of the parameter at the coefficient of friction					
	$f = 0.3$			$f = 0.5$		
	δ_r	δ_v	r/r_0	δ_r	δ_v	r/r_0
2	0.085	0.153	1.364	0.051	0.134	1.326
4	0.013	**0.023**	**2.488**	0.007	0.018	2.216
6	0.002	**0.004**	**4.808**	0.001	0.002	3.842
8	$3.01*10^{-4}$	0.001	9.377	$1.22*10^{-4}$	$3.19*10^{-4}$	6.696
10	$4.60*10^{-5}$	$8.3*10^{-5}$	18.313	$1.63*10^{-5}$	$4.27*10^{-5}$	11.677
12	$7.02*10^{-6}$	$1.27*10^{-5}$	35.774	$2.18*10^{-6}$	$5.70*10^{-6}$	20.366
14	$1.07*10^{-6}$	$1.9*10^{-6}$	69.884	$2.91*10^{-7}$	$7.62*10^{-7}$	35.520

Table 4. Calculated values of relative particle sliding velocities along the rotor blade of the centrifugal accelerator and the angle (*β*) between them at $\omega = 450\ s^{-1}$

Tabela 4. Obliczone wartości względnych prędkości poślizgu cząstek wzdłuż łopatki wirnika akceleratora odśrodkowego oraz kąta (*β*) między nimi przy $\omega = 450\ s^{-1}$

Time, ms	The calculated value of the parameter at the coefficient of friction					
	$f = 0.3$			$f = 0.5$		
	$V_r/\omega r$	$V_a/\omega r$	tgβ	$V_r/\omega r$	$V_a/\omega r$	tgβ
2	0.581	1.254	1.720	0.509	1.180	1.963
4	0.717	1.247	1.394	0.603	1.176	1.659
6	0.740	1.246	1.351	0.616	1.176	1.623
8	0.743	1.246	1.345	0.618	1.176	1.619
10	0.744	1.246	1.344	0.618	1.176	1.618
12	0.744	1.246	1.344	0.618	1.176	1.618
14	0.744	1.246	1.344	0.618	1.176	1.618

DISCUSSION OF RESEARCH RESULTS

The data given in the tables show the change in relative kinematic parameters, such as dimensionless displacements (r/r_0), dimensionless relative ($V_r/\omega r$) and dimensionless total ($V_a/\omega r$) velocity as functions on the time of particle motion along the blades. It can be seen from the data obtained (highlighted in bold) that even at relatively small distances from the point r_0 of the start of movement along the blade, the parameters δ_r and δ_V, included in the equations (26), (28), (29) and (30), becomes negligible in comparison with unity. So, at ω = 150 s^{-1} and f = 0.3 already at a distance of $r = 2.01r_0$ (t = 10 ms) the parameter δ_V becomes equal to δ_V = 0.04, and at a distance of $r = 3.45r_0$ it will be only δ_V = 0,01, which is negligible in comparison with unity. At higher rotor speeds (ω = 450 s^{-1} and f = 0.3) at a distance of $r = 2.488r_0$ (t = 4 ms), the parameter δ_V = 0.023, and at $r = 4.808r_0$ the total δ_V = 0.0036, which is also negligible in comparison with unity.

It can also be seen from the tables that the ratio of radial and total (absolute) particle velocities to the product of (ωr) very quickly approaches some constants. From a physical point of view, this result can be regarded as an asymptotic approximation of the sliding velocities to some linear dependencies (**Fig. 2**) **[L. 15]**, which is certainly true for the middle and end parts of the blade.

Fig. 2. Scheme of particle velocity change along the rotor blade at different values of the initial particle position (r_{01}, r_{02}, r_{03} – the distance from the rotor centre to the starting points of the particle movement); 1 – asymptote of particle velocity

Rys. 2. Schemat zmiany prędkości cząstki wzdłuż łopatki wirnika przy różnych wartościach początkowego położenia cząstki (r_{01}, r_{02}, r_{03} – odległości od środka wirnika do punktów początkowych ruchu cząstki); 1 – asymptota prędkości cząstek

In general, it follows from the performed analysis that for the middle and end parts of the accelerating blade, the second term in parentheses (parameters δ_r and δ_V) in formulas (25) – (30) can be neglected. Then, taking $\delta_r = 0$ and $\delta_V = 0$ and dividing the right and left parts of formulas (25) by (24), we obtain an analytical dependence to describe the relative velocity distribution for the particle moving along the rotor radius in this part of the length that is most important for the operation of centrifugal accelerators

$$\frac{V_r}{r} = \frac{\omega}{\sqrt{1+f^2}+f}. \quad (31)$$

Let us get the absolute velocity as an analogy for dividing (29) by (24)

$$\frac{V_a}{r} = \omega\sqrt{1+\left(\sqrt{1+f^2}-f\right)^2}. \quad (32)$$

The analysis shows at the same time that the formula (31) completely coincides with the previously obtained formula (10). To do this, multiply the numerator and denominator in (31) by the multiplier conjugate to the denominator and multiply the values in the denominator

$$V_r = \frac{r\omega}{\sqrt{1+f^2}+f}\cdot\frac{\sqrt{1+f^2}-f}{\sqrt{1+f^2}-f} = \left(\sqrt{1+f^2}-f\right)\omega r. \quad (33)$$

It can be seen from (31) and (32) that on the main part of the length of the rotor blade, the sliding speed practically does not depend on the initial position (coordinates r_0) and the mass of the particle. Moreover, relative (V_r) and absolute (total) (V_a) velocities of the particle motion are linearly dependent on the rotor angular velocity and non-linearly on the friction coefficient of the particle material on the rotor blade. With an increase in the coefficient of friction, the relative (31) and absolute (total) (32) particle velocities decrease.

It can be written as the angle (β) between the relative (V_r) vector and the total (V_a) particle velocities taking in (30) $\delta_r = 0$ and $\delta_V = 0$

$$\mathrm{tg}\beta \approx \sqrt{1+f^2}+f. \quad (34)$$

As can be seen from formula (34), the departure angle for the particle (β) does not depend on (r) and (ω) but depends on the friction coefficient (f) of the particle material on the rotor blade.

The most important parameter for centrifugal-impact devices (grinding equipment) is the kinetic energy of the particle, which can be determined as the product of the mass (m) of the particle by half the square of the full velocity calculated by the formula (35)

$$E = \frac{mV_a^2}{2} = \frac{m(\omega r)^2}{2}\left[1 + \left(\sqrt{1+f^2} - f\right)^2\right]. \quad (35)$$

Formula (36) can also be represented as

$$E = E_i\eta = \frac{E_i}{2}\left[1 + \left(\sqrt{1+f^2} - f\right)^2\right], \quad (36)$$

where E_i is the kinetic energy of the particle in an ideal mechanism in which there is no friction of the particle on the blade.

η – is a tribomechanical coefficient that takes into account particle acceleration efficiency and kinetic energy loss for the particle due to friction against the disk blade.

The value of kinetic energy for an ideal mechanism can be found by the formula (36), taking the coefficient of friction equal to zero in it.

The coefficient (η), which is, in this case, the main part of the efficiency of the rotor, is equal.

$$\eta = \frac{1}{2}\left[1 + \left(\sqrt{1+f^2} - f\right)^2\right]. \quad (37)$$

Formula (37) actually describes a decrease in the rotor accelerator efficiency due to the energy dissipation under the friction of the particle on the rotor blade.

The coefficient (λ) of the particle energy loss due to friction against the rotor blade can be calculated based on the formula (38)

$$\lambda = 1 - \eta = f\left(\sqrt{1+f^2} - f\right). \quad (38)$$

The effects of the friction coefficient (f) on the calculated values of the particle departure angle (β) also, the tribomechanical coefficient (η) and the energy loss coefficient (λ) are given in **Table 5**.

Table 5. The effects of the coefficient of friction (*f*) on the calculated values of the angle (*β*) of the departure of particles, the tribomechanical coefficient (*η*) and the energy loss coefficient (*λ*)

Tabela 5. Wpływ współczynnika tarcia (*f*) na obliczone wartości kąta (*β*) odejścia cząstek, współczynnik trybomechaniczny (*η*) i współczynnik strat energii (*λ*)

Parameter	Coefficient of friction				
	0.2	0.3	0.4	0.5	0.6
tgβ	1.220	1.344	1.477	1.618	1.766
β, grad	50°39′	53°21′	55°54′	58°17′	60°29′
η	0.836	0.777	0.729	0.691	0.661
λ	0.164	0.223	0.271	0.309	0.334

It can be seen from the table that with increasing the friction coefficient of the particle on the blade of the disk, for example, due to the wear of the blade or the use of materials (coatings) with an increased value of the friction coefficient (f), the tribomechanical coefficient (η) of the rotary accelerator efficiency decreases and the coefficient (λ) of frictional energy losses increases. This must be taken into account when designing and operating such equipment. The angle (β) of the particle departure varies in a relatively narrow range. In particular, with a friction coefficient f = 0.3, this angle is 53°21′. With a change in the coefficient of friction from f = 0.2 to f = 0.6, the angle also changes in a narrow range from β = 50°39′ to 60°29′, which does not always require resetting (turning) the bump elements during their operation to reduce wear. The calculated values obtained are in good agreement with the available experimental data given in **[L. 9, p. 47]**. Tests on centrifugal accelerators TSUK-2 and TSUK-3M equipped with special rotors have shown that the angle of departure of quartz sand practically does not depend on the radius of the rotor and is, on average equal to β = 55°. This is an important result for designers and operators of equipment such as shock-centrifugal grinders.

CONCLUSIONS

Based on the analysis of the solution of the differential equation for the motion of the coarse particles along the radial blades of turbo machines, it is found that the sliding velocity of a particle asymptotically approaches to linear function that practically does not depend on the initial position of the particle. However, it linearly depends on the rotor's angular velocity and non-linearly on the friction coefficient of the particle on the blade. Analytical expressions are obtained that allow us to determine the change in relative and total velocity, the angle between the corresponding velocity vectors, and the kinetic energy of the particle as it moves along the radius of the blade in the working space of the rotor. It is shown that with an increase in the friction coefficient of the particle on the disk blade, for example, due to the wear of the blade or the use of materials (coatings) with an increased value of the friction coefficient, the efficiency of the rotary accelerator decreases. In this case, the angle of the particle departure changes in a relatively narrow range, which does not always require resetting (turning) the bump elements during their operation to reduce wear. The result of research indicates that when designing and operating centrifugal accelerators, it is necessary to consider not only the issues of increasing the wear resistance of the rotor blades but also the energy loss due to friction when particles slide along the blade.

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