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# STRONG SEQUENCES AND THEIR CONSEQUENCES IN SOCIAL CHOICE

**Abstract**. One of the most famous theorems in social choice theory – Arrow impossibility theorem – was published in 1951. Since Arrowian paper most researchers tried to find different versions of this theorem not only for finite but also for infinite sets of alternative and individuals, where one can treat this situation as anticipation for future social behaviour. The aim of this paper is to find some results concerning social voting for infinite sets using one of the combinatorial methods of set theory – strong sequences method. This method was introduced by Efimov in 1965 for proving well-known theorems in dyadic spaces, (i.e. continuous images of the Cantor cube).

## 1. Introduction

Since 1951 when K. Arrow published his impossibility theorem in social choice theory [1] a lot of researchers tried to find different versions of this theorem because Arrow's result does not hold for the case of infinite number of voters. The infinite case of Arrow's theorem has been mentioned for the first time by Fishburn in [4]. He proved the existence of introduced by Arrow "social welfare function" using a special kind of probability measure. Later in 70's of the last century

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Kirman and Sondermann [7] and Hansson [5] cast a new light on a structure of an Arrowian social welfare function with an infinite population only for the set of natural numbers, revealing the structure of decisive coalitions for this function as ultrafiltres. A related result was proved by Mas-Colell and Sonnenschein in [10] by using the additional condition of "positive responsiveness" which permits the reinstatement. Ferejohn and Page in [3] formulated the problem of intertemporal choice from an axiomatic perspective. The authors considered the problem of aggregating the preference orderings $R_i$, (preference relation of $i$-individual of an infinite sequence of future generations into an intertemporal social preference $R$). The aim of this paper is to show some results concerning infinite number of voters and alternatives. For this purpose one of the combinatorial methods – strong sequences method – was used.

## 2. Social choice results

We start with presenting selected results concerning Arrowian theorem and its versions. In the whole paper we will consider two sets: $X$ – a set of alternatives, $N$ – a set of individuals. Both such sets can be finite (but have at least two elements) or infinite. If they are finite we can identify them with finite subsets of a set of natural numbers, if they are infinite we can identify them with well-ordered sets of cardinality $\lambda$ or $2^\lambda$, for $\lambda$ – infinite cardinal. On a set $X$ we consider a relation $R_n$ which denotes the relation of preference of $n-$individual, where $n \in N$. Using $R_n$ we define the relation of indifference $I_n$ and the relation of strictly preference $P_n$ as follows: for all $x, y \in X$ and $n \in N$ $(x, y) \in I_n \Leftrightarrow (x, y) \in R_n \wedge (y, x) \in R_n$ and $(x, y) \in P_n \Leftrightarrow (x, y) \in R_n \wedge \neg(x, y) \in I_n$. A relation $R_n$ is *quasi-transitive* if $P_n$ is transitive. A relation $R_n$ is *acyclic* if there is no finite sequence $x_1, x_2, \ldots, x_k \in X$ such that $(x_1, x_2) \in P_n \wedge (x_2, x_3) \in P_n \wedge \ldots \wedge (x_k, x_1) \in P_n$. $R_n$ is *triple acyclic* if $k = 3$. Acyclity is weaker than quasi-transitivity, which is weaker than transitivity. See example below.

**Example 1.** Let $X = \{x, y, z\}$ and let $n \in N$. If $(x, y) \in P_n, (y, z) \in P_n$ and $(x, z) \in I_n$, then $R_n$ is acyclic but not quasi-transitive. If $(x, y) \in I_n, (y, z) \in I_n$ and $(x, z) \in P_n$, then $I_n$ is acyclic but not transitive.

Let $\mathcal{R}^N = \{(R_n)_{n \in N} \colon R_n$ is a preference relation of $n$-individual$\}$. Let $f$ be a function defined on $\mathcal{R}^N$ by the rule $f((R_n)_{n \in N}) = R$, where $R \subset X \times X$ is a social preference relation for a profile $(R_n)_{n \in N}$. We say that $f$ generates a relation $R$. This function is called *a collective choice rule*, (CCR). Let $A \subset X$ be a non-empty set and $x \in A$. Consider $C(x,R) = \{y \in A\colon (y,x) \in R\}$ and $C(A,R) = \bigcap_{x \in A} C(x,R)$. A CCR is a *social decision function* (SDF) if $R$ is a complete relation such that the set $C(A,R)$ is non-empty for all non-empty sets $A \subset X$. If $R$ is quasi-transitive then SDF is called SDF-Q. A SDF is a *social welfare function* (SWF) if $R$ is transitive. Arrow restricted his considerations to SWF. Accept the following conditions. Let $(R_n)_{n \in N} \in \mathcal{R}^N$:

*Condition U (Unrestricted Domain).* The domain of a function $f$ is $\mathcal{R}^N$.

*Condition P (Pareto Principle).* $(x,y) \in P_n \Rightarrow (x,y) \in P$ for all $x,y \in X$ and $n \in N$.

*Condition WP (Weak Pareto principle).* $(x,y) \in R_n \Rightarrow (x,y) \in R$ for all $x,y \in X$ and $n \in N$.

*Condition D (Non-Dictatorship).* There is no $n \in N$ such that for all $x,y \in X$ $(x,y) \in P_n \Rightarrow (x,y) \in P$.

*Condition WD (Weak Non-Dictatorship).* There is no $n \in N$ such that for all $x,y \in X$ $(x,y) \in P_n \Rightarrow (x,y) \in R$.

*Condition IIA (Independence of Irrelevant Alternatives).* For all $x,y \in X$ if $(R_n)_{n \in N}, (R'_n)_{n \in N} \in \mathcal{R}^N$ and $(x,y) \in R_n \Leftrightarrow (x,y) \in R'_n$ then $(x,y) \in R \Leftrightarrow (x,y) \in R'$.

A well-known Arrow's theorem says that

**Theorem 2 (Arrow's theorem).** *There is no SWF satysfying conditions U, P, IIA and D.*

The authors in [10] proved the similar results to Arrow's but for SDF-Q.

**Fact 1.** *There is no a SDF-Q satysfying U, P, IIA and WD.*

Moreover they introduced the following condition

*Condition $PR_K$ (Positive Responsivenes).* Let $(R_n)_{n \in N}, (R'_n)_{n \in N} \in \mathcal{R}^N$, $M \subset N$ of cardinality at least $K$ and $x,y \in X$. If $(x,y) \in R_n \Leftrightarrow (x,y) \in R'_n$ for all $n \in N \setminus M$ and $(y,x) \in P_n \wedge (x,y) \in I'_n$ or $(x,y) \in I_n \wedge (x,y) \in P'_n$ for all $n \in M$, then $(x,y) \in R \Rightarrow (x,y) \in P'$.

If $|K| = 1$ then we will write $PR$ instead of $PR_K$.

**Fact 2.** *There is no a SDF-Q satysfying U, P, IIA, D and PR.*

**Fact 3.** *If* $|N| > 3$ *then there is no SDF satisfying conditions U, P, IIA, PR and WD.*

**Fact 4.** *Any SDF-Q satisfying conditions U, P, IIA, and PR must be dictatorial if there are at least three alternatives.*

We show that $PR_K$ is not independent from IIA if condition WP holds.

**Lemma 3.** *Assume that conditions WP and* $PR_K$ *for* $|K| \geqslant 1$ *holds. Then the condition IIA holds.*

*Proof.* Let $(R_n)_{n\in N}, (R'_n)_{n\in N} \in \mathcal{R}^N$. We have $(x,y) \in R_n \Leftrightarrow (x,y) \in R'_n$ for all $x, y \in X$. By condition $PR_K$ we have $(x,y) \in R \Rightarrow (x,y) \in P'$. Obviously if $(x,y) \in P'$ then $(x,y) \in R'$ for all $x, y \in X$. Thus $(x,y) \in R \Rightarrow (x,y) \in R'$. Suppose now that $\neg((x,y) \in R' \Rightarrow (x,y) \in R)$. It means that there exist $x_0, y_0 \in X$ such that $(x_0,y_0) \in R' \wedge \neg(x_0,y_0) \in R$ (i.e. $(x_0,y_0) \in R' \wedge (y_0,x_0) \in P$). By condition WP there exists $m \in N$ such that $(y_0,x_0) \in P \Rightarrow (y_0,x_0) \in P_m$ and $(y_0,x_0) \in R' \Rightarrow (y_0,x_0) \in R'_m$. Thus $(x_0,y_0) \in R'_m \wedge (y_0,x_0) \in P_m$. Consider two cases

1) $m \in N \setminus M$. Then the previous statement is equivalent to $(x_0,y_0) \in R'_m \Rightarrow (x_0,y_0) \in R_m$ which contradicts to our assumption.

2) $m \in M$. It also makes contradiction because $(x_0,y_0) \in R'_m \not\Rightarrow (x_0,y_0) \in I'_m$.

□

Another results related to the topic of Arrow's theorem concerns the existence of "a veto" situation.

We say that an individual $n \in N$ *has a veto* for a pair $(x,y) \in X \times X$ if $(x,y) \in P_n \Rightarrow (x,y) \in R$.

We have the following facts (see [11]):

**Fact 5.** *For any SDF satisfying conditions U, P, IIA and PR if there are at least three alternatives and at least four individuals then someone has a veto.*

**Fact 6.** *For any CCR generating only reflexive, complete and triple acyclic R satisfying conditions U, P, IIA and PR if there are at least four alternatives, then someone has a veto.*

Other results related to Arrow's theorem concern so called decisive sets.

A set $S \subset N$ *is a decisive set (a D-set) for a pair* $(x, y) \in X \times X$ if and only if $(x, y) \in P_n \Rightarrow (x, y) \in P$ for all $n \in S$. A set $S \subset N$ *is D-set* if and only if it is a D-set for each pair $(x, y) \in X \times X$. A set $S \subset N$ *is a weak decisive set (WD-set) for a pair* $(x, y) \in X \times X$ if and only if $(x, y) \in P_n \Rightarrow (x, y) \in R$ for all $n \in S$. A set $S \subset N$ *is a WD-set* if and only if it is a WD-set for each pair $(x, y) \in X \times X$.

It is obvious that if condition D holds then condition WD holds. The inverse theorem holds only under some assumptions.

**Lemma 4.** *Let a CCR satisfies conditions U, WP, $PR_K$. If $S \subset N$ with $|S| \leqslant |K|$ is a WD-set then $S$ is D-set.*

*Proof.* Suppose that $S$ has property WD and does not have property D for some $x, y \in X$. We have $(x, y) \in P_n \wedge (y, x) \in R$ for some $n \in N$. By condition WP $(y, x) \in P \Rightarrow (y, x) \in P_m$ for some $m \in N$. But if $(y, x) \in I$ we have $(y, x) \in I_{m_0}$ for some $m_0 \in S$. Thus $(y, x) \in R \Rightarrow (y, x) \in R_m$ for some $m \in N$. Divide the set $N$ as follows: $N_0 = \{n \in N \colon (x, y) \in P_n\}$ and $M_0 = \{n \in N \colon (y, x) \in R_n\}$. Both sets are non-empty. Indeed. $N_0 \neq \emptyset$ because $N_0 = S$. If $M_0 = \emptyset$ then $N_0 = N = S$. Thus by condition WP we would obtain $(x, y) \in P$. Contradiction to our assumption. Divide the set $M_0$ as follows: $M_0^P = \{n \in M_0 \colon (y, x) \in P_n\}$ and $M_0^I = \{n \in M_0 \colon (y, x) \in I_n\}$. If $M_0^I = \emptyset$ then $(x, y) \in P_n \wedge (y, x) \in P_m$ for all $n \in S$ and $m \notin S$. By condition WP for the set $N \setminus S$ we have $(y, x) \in P$. It contradicts to the assumption that $S$ has property WD. Let $M_0^I \neq \emptyset$. Let $(R_n)_{n \in N}, (R'_n)_{n \in N} \in \mathcal{R}^N$ be such that $R_n = R'_n$ for $n \in M_0^P = M_0 \setminus M_0^I$ and $(y, x) \in P'_n$ for $n \in M_0^I$. By condition $PR_K$ we have $(y, x) \in R \Rightarrow (y, x) \in P'$. Since $(y, x) \in R_n \Leftrightarrow (y, x) \in R'_n$ for all $n \in M_0^P$ and $(y, x) \in I_n \Leftrightarrow (y, x) \in P'_n$ for all $n \in M_0^I$ we have $(y, x) \in R_n \Leftrightarrow (y, x) \in R'_n$ for all $n \in M_0$. By condition $PR_K$ and the result above we obtain $(y, x) \in P \Leftrightarrow (y, x) \in P'$. Finally we obtain $(x, y) \in P_n \wedge (y, x) \in P_m \Rightarrow (y, x) \in P$ for all $n \in S$ and $m \notin S$ which contradicts to the assumption that $S$ has property WD. If we were not able to define such $(R'_n)_{n \in N}$ we would have to divide the set $M_0^I$ for sets $M_1^P = \{n \in M_0^I \colon (y, x) \in P''_n\}$ and $M_1^I = M_0^I \setminus M_1^P$ for some $(R''_n)_{n \in N}$ such that $R_n = R''_n$ for $n \in M_0^P \cup M_1^P$ and repeat our consideration for $M_1^P$. The construction we can repeat as long as we obtain our claim. $\square$

By previous lemma we immediately obtain the following

**Corollary 5.** *Let CCR fulfills conditions U, WP and IIA. If $S \subset N$ is a WD-set then $S$ is a D-set.*

A set $S \subset N$ *is an almost decisive set (AD-set) for a pair* $(x, y) \in X \times X$ if and only if $(x, y) \in P_n \wedge (y, x) \in P_m \Rightarrow (x, y) \in P$ for all $n \in S$ and $m \notin S$. A set $S \subset N$ *is an AD-set* iff it is AD for each pair $(x, y) \in X \times X$.

**Lemma 6.** *Let SWF satisfies condition WP. If the set $S \subset N$ is a AD-set then is a WD-set.*

*Proof.* Let $(x, y) \in X \times X$ be an element such that $(x, y) \in P_n$ for all $n \in S$. By our assumptions $(y, x) \in P_m$ for any $m \in N \setminus S$. Hence $(x, y) \in R_m$ for any $m \in N \setminus S$. By condition WP we have that $(x, y) \in R$ for any $n \in N$. $\square$

In the previous lemma if we assume that a function is CCR only we have the following counterexample.

**Example 7.** Suppose $N = \{1, 2\}$. Consider the CCR defined as follows $xPy$ if $[(xP_1y \wedge xP_2y) \vee (xP_1y \wedge yP_2z)]$; $yPx$ if $(xP_1y \wedge xI_2y)$; $xIy$ otherwise. Notice that $\{1\}$ is almost decisive but not weakly decisive. WP is satisfied.

**Lemma 8.** *Assume that a SWF satisfies conditions U, WP and $PR_K$ for $|K| \geqslant 1$. Then there can be only one D-set.*

*Proof.* By previous lemma it is enough to show that there is unique AD-set. Suppose that there are two sets, i.e. $S_1, S_2 \subset N$ such that $S_1 \neq S_2$ which are AD. It means that for a pair $(x, y) \in X \times X$ $(x, y) \in P_n \wedge (y, x) \in P_m$ for all $n \in S_1$ and $m \in S_2 \setminus S_1$. Since $S_1$ is an AD-set, thus the previous statement follows that $(x, y) \in P$ but $S_2$ is also an AD-set, thus the previous statement follows that $(y, x) \in P$. Contradiction. $\square$

According to previous lemma and Lemma 3 we have

**Lemma 9.** *For any SWF satisfying conditions U, WP and IIA there can be only one D-set.*

# 3. Strong sequences in social choices

We start with presenting some historical notes on the strong sequences method. This method was introduced by B.A. Efimov in [2] as a useful method for proving well-known theorems in dyadic spaces. Let us remind Efimov's main result. Let $T$ be an infinite set. Let $D^T = \{p\colon p\colon T \to \{0,1\}\}$ denotes the Cantor cube. For $s \subset T$, $i\colon s \to \{0,1\}$ let $H^i_s = \{p \in D^T : p|s = i\}$. Efimov defined strong sequences in the subbase of the Cantor cube $\{H^i_{\{\alpha\}} \colon \alpha \in T, i\colon \{\alpha\} \to \{0,1\}\}$ as follows.

A pair $(H^i_s, H^i_v)$ where $|s| < \omega$ is called *a connected pair* if $H^i_s \cap H^i_v \neq \emptyset$. A sequence $(H^{i_\alpha}_{s_\alpha}, H^{i_\alpha}_{v_\alpha})$ consisting of connected pairs is called *a strong sequence* if $H^{i_\alpha}_{s_\alpha} \cap H^{i_\beta}_{v_\beta} = \emptyset$ whenever $\alpha > \beta$.

**Theorem 10 (Efimov's theorem).** *Let $\kappa$ be a regular, uncountable cardinal number. In the space $D^T$ a strong sequence $(H^{i_\alpha}_{s_\alpha}, H^{i_\alpha}_{v_\alpha}), \alpha < \kappa$ such that $|s_\alpha| < \omega$ and $|v_\alpha| < \kappa$ for each $\alpha < \kappa$ does not exists.*

In the paper [13] the method of strong sequences was introduced as follows:

Let $X$ be a set, and let $\mathcal{B} \subset P(X)$ be a family of non-empty subsets of $X$ closed under finite intersections. We say that a family $\mathcal{C} \subset \mathcal{B}$ is *a centered family* if and only if $\bigcap \mathcal{F} \neq \emptyset$ for each finite subfamily $\mathcal{F} \subset \mathcal{C}$. Let $S, H \subset \mathcal{B}$ with $|S| < \omega$. A pair $(S, H)$ is called *connected* if $S \cup H$ is centered. A sequence $(S_\phi, H_\phi)$; $\phi < \alpha$ consisting of connected pairs is called *a strong sequence* if for all $\lambda$, in the range $\phi < \lambda < \alpha$, a family $S_\lambda \cup H_\phi$ is not centered.

**Fact 7 ([13]).** *If for $\mathcal{B} \subset P(X)$ there exists a strong sequence $(S_\phi, H_\phi)_{\phi<(\kappa^\lambda)^+}$ such that $|H_\phi| \leqslant \kappa$ for each $\phi < (\kappa^\lambda)^+$, then the family $\mathcal{B}$ contains a subfamily of cardinality $\lambda^+$ consisting of pairwise disjoint sets.*

This method was used for proving the following theorems: Kurepa theorem [8], Marczewski theorem [9] on cellurality of dyadic spaces, Shanin theorem [12] on a calibre of dyadic spaces, Erdös-Rado theorem (see [13]).

For the purpose of this paper the method of strong sequences will be introduced on a set with relation $r \subset Y \times Y$. We do not assume anything about this relation as far as possible.

Let $(Y, r)$ be a set with relation $r$ on $Y$. A set $A \subset Y$ *has a bound* if there exists $b \in A$ such that for all $a \in A$ we have $(a, b) \in r$. Elements $a, b \in Y$ are *compatible* if they have a bound. A set $A \subset Y$ is *$\omega-$directed* if every subset of $A$ of cardinality less than $\omega$ has a bound.

**Definition 11.** *Let $(Y, r)$ be a set with a relation $r$. A sequence $(H_\phi)_{\phi<\alpha}$, where $H_\phi \subset Y$, is called a $\kappa$-strong sequence if:*

$1^o$ *$H_\phi$ is $\omega$-directed for all $\phi < \alpha$;*

$2^o$ *$H_\psi \cup H_\phi$ is not $\omega$-directed for all $\phi < \psi < \alpha$.*

Using arguments like in [6], Theorem 3.5 we conclude that the following theorem is true

**Theorem 12.** *Let $\lambda$ be a regular cardinal number such that $2^\lambda$ is also regular. If for $(Y, r)$ there exists a strong sequence $(H_\phi)_{\phi<2^\lambda}$ such that $|H_\phi| < 2^\lambda$ for each $\phi < 2^\lambda$ then there exists a strong sequence $(T_\phi)_{\phi<\lambda}$ such that $T_\phi \subset H_\phi$ and $|T_\phi| < \omega$ for each $\phi < \lambda$.*

The main result of this paper is the following theorem.

**Theorem 13.** *Let $\lambda$ be a regular cardinal number such that $2^\lambda$ is also regular. Let $N, X$ be sets such that $|N| \geqslant 2^\lambda$ and $|X| \geqslant 2^\lambda$. If a SWF satisfies conditions U, WP and $PR_K$ for $|K| \geqslant 1$, then either there exists a unique D-set $S \subset N$ with $|S| = 2^\lambda$ or there exists $T \subset N$ with $|T| = \lambda$ such that each $t \in T$ has a veto.*

*Proof.* If there exists a D-set of cardinality $2^\lambda$ we have our claim. The uniqueness of such a D-set follows from Lemma 8. Suppose that for eaxh $X' \subset X$ with $|X'| < 2^\lambda$ a $D$-set has cardinality less than $2^\lambda$. Let $(x_0, y_0) \in X \times X$. Let $S_0 \subset N$ be a maximal D-set for the pair $(x_0, y_0)$. Let $X_0 \times X_0$ be a maximal set for which $S_0$ is a D-set. Let $S_0$ be the first element of a strong sequence. Let $(x_1, y_1) \in (X \times X) \setminus (X_0 \times X_0)$. Let $S_1 \subset N$ be a maximal D-set for the pair $(x_1, y_1)$. Let $X_1 \times X_1$ be a maximal set for which $S_1$ is a D-set. Let $S_1$ be the second element of a strong sequence.

Assume that the following sequences have been defined for $\beta < \alpha < 2^\lambda$:

1) $\{(x_\beta, y_\beta) \in (X \times X) \setminus \bigcup_{\gamma<\beta}(X_\gamma \times X_\beta) \colon \beta < \alpha\}$;

2) $\{S_\beta \colon \beta < \alpha\}$, where $S_\beta$ is a maximal D-set for $(x_\beta, y_\beta)$;

3) $X_\beta \times X_\beta \subset (X \times X) \setminus \bigcup_{\gamma<\beta}(X_\gamma \times X_\gamma)$, where $X_\beta \times X_\beta$ is a maximal set for which $S_\beta$ is a D-set.

Since $|X_\beta| < 2^\lambda$ for $\beta < \alpha$ there exists $(x_\alpha, y_\alpha) \in X \times X \setminus \bigcup_{\beta<\alpha}(X_\beta \times X_\beta)$. Let $S_\alpha \subset N$ be a maximal D-set for the pair $(x_\alpha, y_\alpha)$. Let $X_\alpha \times X_\alpha \subset (X \times X) \setminus \bigcup_{\beta<\alpha}(X_\beta \times X_\beta)$ be a maximal set for which $S_\alpha$ is a D-set. The claim $S_\beta \cup S_\alpha$ is not $\omega$- directed for $\bigcup_{\alpha<2^\lambda}(X_\alpha \times X_\alpha)$ follows from the above construction because on each step we choose maximal sets of required property. Thus $\{S_\alpha \colon \alpha < 2^\lambda\}$ forms the strong sequence. By Theorem 12 there exists a strong sequence $\{T_\alpha \colon \alpha < \lambda\}$ such that $T_\alpha \subset S_\alpha$ and $|T_\alpha| < \omega$ for any $\alpha < \lambda$. For each $\alpha < \lambda$ let $t_\alpha \in T_\alpha$ be a bound. By our construction $\{t_\alpha \colon \alpha < \lambda\}$ is not a D-set for any $(x, y) \in \bigcup_{\alpha<\lambda}(X_\alpha \times X_\alpha)$. □

By Theorem 13 and Lemma 3 we have

**Corollary 14.** *Let $\lambda$ be a regular cardinal number such $2^\lambda$ is also regular. Let $N, X$ be sets such that $|N| \geqslant 2^\lambda$ and $|X| \geqslant 2^\lambda$. If a SWF satisfies conditions U, WP and IIA, then either there exists a unique D-set $S \subset N$ with $|S| = 2^\lambda$ or there exists $T \subset N$ with $|T| = \lambda$ such that each $t \in T$ has a veto.*

## 4. Interpretation of the main result

In this section we present the social consequences of Theorem 13 (Corollary 14, respectively). If in the theorem we assume that $\lambda = \aleph_0$ then we have $|N| = |X| = \mathfrak{c}$, i.e. $N$ is equinumerous with each interval of the real line. Thus we can partition all individuals into sets number by reals with respect to for example height or weight or other condition. Then under given assumptions in the Theorem 13 (Corollary 14, respectively) we have two possibilities: we have that individuals from each set of the partition which form a D-set which is unique or we have countable number of individuals belonging to different sets of such a partition such that each such an individual has a veto.

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