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FATIGUE LIFE OF METALLIC MATERIAL ESTIMATED ACCORDING TO SELECTED MODELS AND LOAD CONDITIONS

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The authors present results of a fatigue test for specimens made of the aluminium alloy 2017A-T4 and alloy steels S355J2WP and S355J2G3 subjected to constant-amplitude proportional combined bending with torsion including mean stress values and for the S355J2WP alloy steel under uniaxial constant-amplitude and random loading with both zero and non-zero mean stress values. The test results were compared with the results of calculations according to the models proposed by Goodman, Gerber and Morrow as well as the stress-strain parameter. In the case of calculations based on the stresses, the multiaxial stress state was reduced to a uniaxial one using the Huber-Mises relationship. As for the method based on strain energy density, the multiaxial stress state was reduced to the uniaxial one with use of the stress-strain parameter. The plane in which the stress-strain parameter of shear loadings reaches its maximum value is assumed to be the critical plane.

Key words: uniaxial fatigue, multiaxial fatigue, mean stress

1. Introduction

Currently, according to the parameters used in the fatigue criterion, multiaxial fatigue damage models which take into consideration the influence of the mean stress can be mainly classified into three categories, namely the stress or strain-based approaches (Gerber, 1874; Goodman, 1899; Morrow, 1968; Lazzarin and Susmel, 2003; Kluger and Łagoda, 2004) and the energy critical plane criteria approach (Smith *et al.*, 1970; Glinka *et al.*, 1995; Palin-Luc and Lasserre, 1998; Fatemi and Socie, 1988; Papadopoulos, 1998; Łagoda, 2001a,b; Kardas *et al.*, 2008; Sonsino *et al.*, 2004; Macha *et al.*, 2006). The known stress, strain or energy models refer to a specified loading. The stress models are applied under high numbers of cycles, the strain models can be used in the case of low numbers of cycles, and the energy models refer to both high and low numbers of cycles (Łagoda, 2001a,b; Kardas *et al.*, 2008). Moreover, it is not easy to choose an adequate method for calculations.

The methods based on stresses or strains are simple and their application does not require much time, but they do not guarantee adequate accuracy of the results. In such models, for computing fatigue life only, the stress or strain amplitude and its mean value are directly taken into consideration (Gerber, 1874; Goodman, 1899; Morrow, 1968; Lazzarin and Susmel, 2003).

On the other hand, the methods based on strain energy density in the critical plane need more time and work, but they give much better results in comparison to the methods based on the stresses (Sonsino *et al.*, 2004). The approaches to fatigue life prediction using the concept of the critical plane have been found very effective because the critical plane concept is based on physical observations that cracks initiate and grow on favourable planes. The value of energy dissipated in the material during one cycle of loading or during all the cycles up to the failure are usually calculated from a history of the changes in cyclic strain and stress together with the number of cycles. This energy is connected with the area of the hysteresis loop (σ-ε) in the case plastic strains occur in the material. It is assumed that this area is proportional to the strain energy dissipated in the material during one cycle of loading. The total energy is the sum of areas of the hysteresis loop.

In this paper, the authors present the stress-strain parameter ($W_{\sigma\varepsilon}$) based on Lagoda-Macha's strain energy density parameter (Kluger and Łagoda, 2007; Łagoda and Ogonowski, 2005) and the models proposed by Goodman (Goodman, 1899; Lazzarin and Susmel, 2003), Gerber (Gerber, 1874; Lazzarin and Susmel, 2003) and Morrow (1968) for estimation of fatigue life of structure elements and machine sub-assemblies under combined bending with torsion, including the mean stress and strain values.

The authors present results of calculation and experimentation for the S355J2WP alloy steel under uniaxial constant-amplitude and random loading with both zero and non-zero mean stress values as well.

For the registered stress histories in the uniaxial loading for the S355J2WP alloy steel, elastic-plastic strains were calculated with the incremental kinematic model of material hardening formulated by Mroz (1967) and Garud (1982).

The paper also contains experimental verification of the considered model based on the obtained fatigue test results and show which model is the best for all kinds of loading.

2. Experimental data

Specimens made of the aluminium alloy 2017A-T4 (Kardas *et al.*, 2008) and the alloy steels S355J2WP (Łagoda *et al.*, 2001) and S355J2G3 (Gasiak and Pawliczek, 2001) were tested. Static and fatigue properties of the tested materials are given in Table 1.

Table 1. Static and fatigue properties of the analyzed materials

Material (EN)	ε'_f	c	σ'_f [MPa]	B	K' [MPa]	n'	E [GPa]	$\sigma_{0.2}$ [MPa]	σ_{UTS} [MPa]	ν
2017A-T4	1.879	−0.988	643	−0.065	617	0.066	72	395	545	0.32
S355J2WP	0.114	−0.420	1012	−0.105	853	0.156	215	414	556	0.29
S355J2G3	2.822	−0.491	1190	−0.143	869	0.287	213	394	611	0.31

For the uniaxial cyclic loading of the S355J2WP alloy steel (Sonsino *et al.*, 2004), the tests were performed for five different stress amplitudes and three levels of the mean loading, $\sigma_m = 75\,\text{MPa}$, $150\,\text{MPa}$ and $225\,\text{MPa}$. The tests were realized under force controlled. Under random loading, the tests were performed for 14 different values of the root mean square of stress σ_{RMS} and mean values σ_m (zero, compressive and tensile). The observation time for random loading was $T_0 = 649\,\text{s}$.

As for the aluminium alloy 2017A-T4, two combinations of proportional constant-amplitude bending with torsion were considered (with constant mean value), where $\tau(t) = \sigma(t)$ and $\tau(t) = 0.5\sigma(t)$. In the case of steel alloy S355J2WP (with constant mean value) and S355J2G3 (with constant R ratio), only combined proportional constant-amplitude bending with torsion where $\tau(t) = \sigma(t)$ was taken into account. All tests were carried out under bending and torque controlled. In all multiaxial analyses, the authors used the elastic model (high cycle fatigue) in which the mean stress and amplitudes were counted as the nominal stress.

3. Models of fatigue life calculation

3.1. Energy model ($W_{\sigma\varepsilon}$)

3.1.1. Uniaxial constant-amplitude loading

The stress-strain parameter ($W_{\sigma\varepsilon}$) under the uniaxial stress state is the base of formulation of the energy model under complex loading states including the mean value. This parameter is defined as

$$W_{\sigma\varepsilon}(t) = \frac{1}{4}\{|\sigma(t)|[\varepsilon(t) - \varepsilon_m] + \sigma(t)|\varepsilon(t) - \varepsilon_m|\} \tag{3.1}$$

The absolute value of $\sigma(t)$ and $\varepsilon(t)-\varepsilon_m$ defines the tensile and compressive phases of loading. The application of the absolute value of $\sigma(t)$ and $\varepsilon(t)-\varepsilon_m$ in calculations brings about a change of the history of the stress-strain parameter in time in a symmetric way while cyclic stresses and strains change in relation to the mean values. Figure 1 shows the histories of the stress-strain parameter $W_{\sigma\varepsilon}(t)$ with and without the absolute value of $\sigma(t)$ and $\varepsilon(t) - \varepsilon_m$ $W^*(t)$. From the graphs, it appears that application of the absolute value of $\sigma(t)$ and $\varepsilon(t)-\varepsilon_m$ reduces the mean value of W_m.

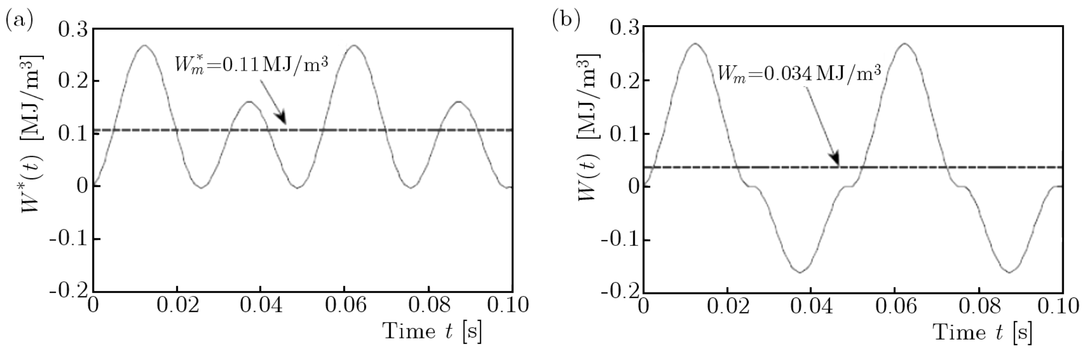

Fig. 1. Exemplary histories of the stress-strain parameter $W^*(t)$, $W(t)$ for aconstant-amplitude loading: (a) $W^*(t) = 0.5\sigma(t)[\varepsilon(t) - \varepsilon_m]$, (b) $W_{\sigma\varepsilon}(t) = 0.5\{|\sigma(t)|[\varepsilon(t) - \varepsilon_m] + \sigma(t)|\varepsilon(t) - \varepsilon_m|\}$

The stress-strain parameter was based on the multiaxial Łagoda-Macha model. Only in one case of loading (uniaxial tension-compression for small elastic-plastic strain and $\sigma_m > 0$), stress-strain parameter (3.1) is in a way similar to the Smith-Watson-Topper parameter (SWT) (Smith *et al.*, 1979) according to the following equation

$$W_{eq} = \frac{1}{2}P_{SWT} = \frac{1}{2}\sigma_{max}\varepsilon_a = \frac{1}{2}(\sigma_a + \sigma_m)\varepsilon_a \tag{3.2}$$

In other case of loadings (bending, torsion, combination bending and torsion) the stress-strain parameter works differently than SWT. The absolute value of $|\sigma(t)|$ and $|\varepsilon(t) - \varepsilon_m|$ changes history of the stress-strain parameter (see Fig. 1). If the elastic-plastic strain is higher, than difference between SWT and $W_{\sigma\varepsilon}$ is higher, too.

The transformed amplitudes $W_{eq,aT}$ of the strain energy density parameter for tensile and compressive states were calculated from the following formula

$$W_{eq,aT} = \frac{(\sigma_a + \psi\sigma_m)\varepsilon_a}{2} = \begin{cases} \dfrac{(\sigma_a + \sigma_m)\varepsilon_a}{2} = \dfrac{W_a + W_m}{2} & \text{for} \quad \sigma_m \geqslant 0 \ \wedge \ \psi = 1 \\ \dfrac{\sigma_a\varepsilon_a}{2} = \dfrac{W_a}{2} & \text{for} \quad \sigma_m < 0 \ \wedge \ \psi = 0 \end{cases} \tag{3.3}$$

where $W_a = \sigma_a\varepsilon_a$ and $W_m = \sigma_m\varepsilon_a$.

The number of cycles to failure was calculated according to Eqs. (3.1), (3.6), $(N_f = N_{cal})$

$$W_{eq,aT} = \frac{1}{4}[|\sigma_a + \psi\sigma_m|\varepsilon_a + (\sigma_a + \psi\sigma_m)|\varepsilon_a|] = \frac{{\sigma'_f}^2}{2E}(2N_f)^{2b} + \frac{1}{2}\varepsilon'_f\sigma'_f(2N_f)^{b+c} \tag{3.4}$$

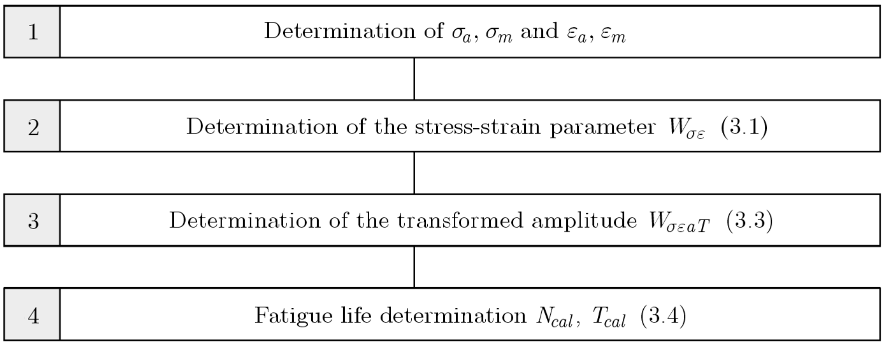

Fig. 2. Algorithm of fatigue life determination for a uniaxial constant-amplitude loading

3.1.2. Uniaxial random loading

Figure 3 shows the algorithm for the determination of the fatigue life of S355J2WP steel according to the stress-strain parameter.

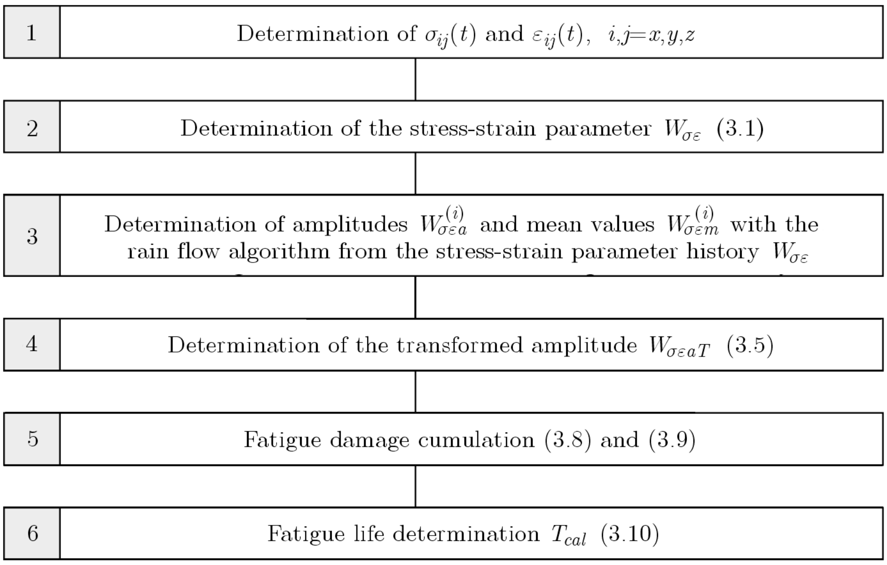

Fig. 3. Algorithm of fatigue life determination for a uniaxial random loading

The rain flow algorithm (Downing and Socie, 1982, [1]) was used for determination of amplitudes $W_a^{(i)}$ and mean values $W_m^{(i)}$ of cycles and half-cycles.

The transformed amplitudes of the strain-stress parameter including the mean load values could be calculated from the distinguished amplitudes of cycles of the parameter W_a. The corresponding mean values W_m are determined from the history according to

$$W_{aT} = \begin{cases} W_a + W_m & \text{for} \quad W_m \geqslant 0 \\ W_a & \text{for} \quad W_m < 0 \end{cases} \tag{3.5}$$

where W_m is determined from the rain flow algorithm.

For cyclic loadings, (3.5) can be expressed by means of the stress criteria. For elastic materials under a high number of cycles to failure, the transformed amplitude of the strain energy density parameter in the stress approach can be written as

$$\sigma_{aT} = \sqrt{(\sigma_m + \sigma_a)\sigma_a} \tag{3.6}$$

This notation conforms with the models based on the Smith-Watson-Topper parameter (P_{SWT}) (Smith *et al.*, 1970). The amplitudes of elastic-plastic strains are obtained from the Ramberg-Osgood relationship, and in the stress approach can be expressed as

$$\frac{\sigma_{aT}}{2}\Big[\frac{\sigma_{aT}}{E} + \Big(\frac{\sigma_{aT}}{K'}\Big)^{\frac{1}{n'}}\Big] = \frac{(\sigma_a + \sigma_m)\sigma_a}{2E} + \frac{\sigma_a + \sigma_m}{2}\Big(\frac{\sigma_a}{K'}\Big)^{\frac{1}{n'}} \tag{3.7}$$

As a result, it is possible to determine the requested value σ_{aT} numerically.

Differences between the elastic-plastic strain and the elastic strain depend on the loading level.

For determination of the damage degree, the Palmgren (1924) and Miner (1945) hypothesis was used

$$S(T_0) = \sum_{i=1}^{n} \frac{1}{N_f^{(i)}} \tag{3.8}$$

where $N_f^{(i)}$ is determined from Eq. (3.9) for the transformed amplitudes $W_{aT}^{(i)}$

$$W_{aT}^{(i)} = \frac{{\sigma'_f}^2}{2E}\big(2N_f^{(i)}\big)^{2b} + \frac{1}{2}\varepsilon'_f\sigma'_f\big(2N_f^{(i)}\big)^{b+c} \tag{3.9}$$

and fatigue life determination was realized according to the following relationship

$$T_{cal} = \frac{T_0}{S(T_0)} \tag{3.10}$$

where T_0 is the observation time.

3.1.3. *Multiaxial constant-amplitude loading*

In the presented stress-strain parameter approach (3.1), the mean stress value strongly influences the fatigue life and the mean strain value does not influence the fatigue process.

Under multiaxial loading, the material effort is determined by the maximum value of the linear combination of the stress-strain parameters $W_\eta(t)$ (normal loadings) and $W_{\eta s}(t)$ (shear loadings). It leads to the equation for the equivalent value $W_{\sigma\varepsilon,eq}$ in the form

$$W_{\sigma\varepsilon,eq}(t) = \beta W_{\eta s}(t) + \kappa W_\eta(t) \tag{3.11}$$

where β is the constant value for selection of a particular form (Łagoda and Ogonowski, 2005), see $(3.14)_2$, κ – material constant obtained from uniaxial fatigue tests (Łagoda and Ogonowski, 2005), see $(3.14)_1$

$$\begin{aligned} W_\eta(t) &= \frac{1}{4}\{|\sigma_\eta(t) + \sigma_{\eta m}|[\varepsilon_\eta(t) - \varepsilon_{\eta m}] + [\sigma_\eta(t) + \sigma_{\eta m}]|\varepsilon_\eta(t) - \varepsilon_{\eta m}|\} \\ W_{\eta s}(t) &= \frac{1}{4}\{|\tau_{\eta s}(t)|[\varepsilon_{\eta s}(t) - \varepsilon_{\eta sm}] + \tau_{\eta s}(t)|\varepsilon_{\eta s}(t) - \varepsilon_{\eta sm}|\} \end{aligned} \tag{3.12}$$

and σ_η – normal stress to the critical plane (Fig. 4), $\sigma_{\eta m}$ – mean value of the normal stress in the critical plane (Fig. 4), ε_η – normal strain in the critical plane (Fig. 4), $\tau_{\eta s}, \varepsilon_{\eta s} = 0.5\gamma_{\eta s}$ – shear

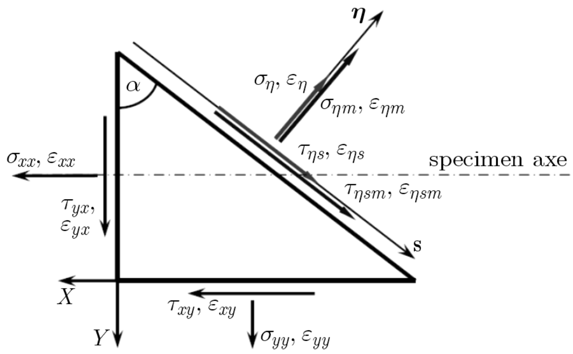

Fig. 4. Orientation of particular stress and strain on the critical plane (Łagoda and Ogonowski, 2005; Macha *et al.*, 2006)

stress and a half of the shear strain in the critical plane (Fig. 4), respectively, $\varepsilon_{\eta m}, \varepsilon_{\eta sm} = 0.5\gamma_{\eta sm}$ – mean normal strain and a half of the shear strain in the critical plane (Fig. 4), respectively.

In this case, the critical plane is determined by the maximum value of the shear stress-strain parameter $\max\{W_{\eta s}(t)\}$.

For high cycle fatigue (HCF), stress amplitudes are calculated directly from loading history (for example for bending as nominal stress amplitude). The mean value of stress is determined as a global expected non-zero value from the load history (Łagoda, 2001a,b). To determine the strain amplitude and its mean value in HCF were plastic strains are almost zero, we can calculate it from the stress history. In the case the strain history was registered, the stress history could be derived from calculation in the reverse direction.

The equivalent value of the shear loading according to Eq. (3.11) for the critical plane defined by the maximum stress-strain parameter takes the form

$$\begin{aligned} W_{\sigma\varepsilon,eq}(t) &= \frac{1}{4}\beta\{|\tau_{\eta s}(t)|[\varepsilon_{\eta s}(t) - \varepsilon_{\eta sm}] + \tau_{\eta s}(t)|\varepsilon_{\eta s}(t) - \varepsilon_{\eta sm}|\} \\ &+ \frac{1}{4}\kappa\{|\sigma_\eta(t) + \sigma_{\eta m}|[\varepsilon_\eta(t) - \varepsilon_{\eta m}] + [\sigma_\eta(t) + \sigma_{\eta m}]|\varepsilon_\eta(t) - \varepsilon_{\eta m}|\} \end{aligned} \tag{3.13}$$

The analysis of the stress and strain states for pure torsion and alternating bending under constant-amplitude loadings offers grounds for determination of weighted coefficients for the particular components in the combination. The weights can be written as (Łagoda and Ogonowski, 2005)

$$\kappa(N_f) = \frac{4 - k(N_f)}{1 - \nu} \qquad \beta(N_f) = \frac{k(N_f)}{1 + \nu} \qquad k(N_f) = \Big(\frac{\sigma_a(N_f)}{\tau_a(N_f)}\Big)^2 \tag{3.14}$$

Depending on the constant k and the assumed life Nf determined from Eq. $(3.14)_3$, it is possible to derive special forms of the criteria presented and verified in Łagoda (2001a,b), Karolczuk *et al.* (2002).

In a general case, the coefficient $k(N_f)$ (Łagoda and Ogonowski, 2005) is given by the relation between the amplitude of the normal stress σ_a and amplitude of the shear stress τ_a for a given number of cycles. The values $\sigma_a(N_f)$ and $\tau_a(N_f)$ are calculated from the fatigue curves S-N for simple loadings: tension (bending), shearing (torsion). If there are no distinct irregularities in the S-N curves (σ_a-N_f, τ_a-N_f), for the purpos of simplification we can assume that $k(N_f) = \text{const}$, e.g. 10^5 or 10^6 cycles, or usually for the fatigue limit level.

The plane where the stress-strain parameter of shear loading $W_{\eta s}$ assumes its maximum value as the critical plane for the aluminum alloy 2017A-T4 and both steels S355J2WP and S355J2G3. The proposed algorithm includes the effect of the mean values of stress and strain during fatigue life calculations.

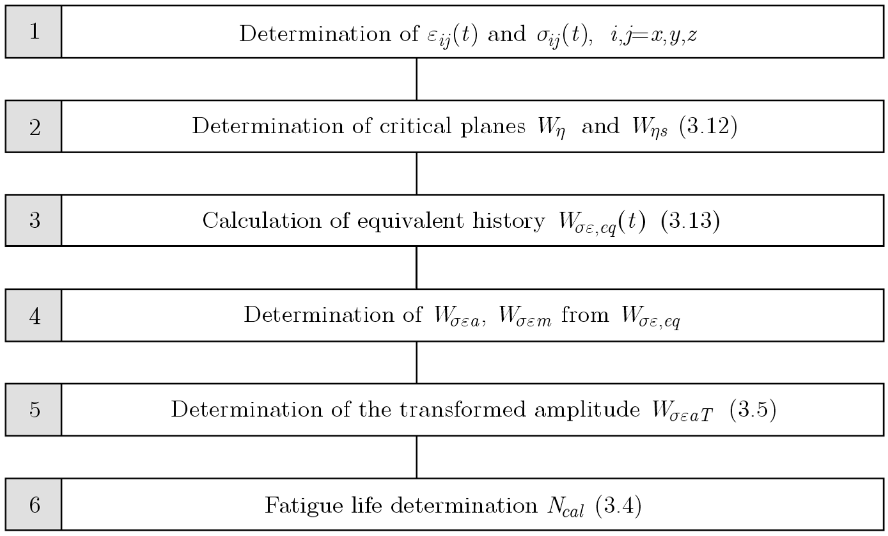

Fig. 5. Algorithm of fatigue life determination for multiaxial constant-amplitude loading

Table 2. Computational data for considered materials under multiaxial loading

Material	k	β	κ
S355J2WP	3.63	2.77	0.53
S355J2G3	2.14	1.63	2.70
2017A-T4	2.79	2.11	1.78

Table 2 contains values of the coefficients k, β and κ from Eqs. (3.14). In this case, parallel fatigue characteristics were assumed for bending and torsion.

3.2. The Goodamn, Gerber and Morrow models

The stress models are often applied by engineers. In this paper, they are used for the purpos of comparison of the existing models. These models describe the boundary between the mean value of cycle, its amplitude and material constants, the calculations are quite simple and they do not require much time. The applied models were formulated by:

— Goodman

$$\frac{\sigma_a}{\sigma_{aT}} + \frac{\sigma_m}{R_m} = 1 \tag{3.15}$$

— Gerber

$$\frac{\sigma_a}{\sigma_{aT}} + \Big(\frac{\sigma_m}{R_m}\Big)^2 = 1 \tag{3.16}$$

— Morrow

$$\frac{\sigma_a}{\sigma_{aT}} + \frac{\sigma_m}{\sigma'_f} = 1 \tag{3.17}$$

where σ_{aT} is the normalized stress amplitude.

The familiar and widely used Huber-Mises criterion was applied for reduction of the complex stress state, namely bending combined with torsion.

A number of cycles to failure was calculated from

$$N_f = 10^{A-m\log\sigma_{aTH-M}} \tag{3.18}$$

where m is the coefficient of the S-N curve slope (for bending), A – constant of the fatigue curve regression (for bending), σ_{aTH-M} – transformed amplitudes related to the stress mean value under the uniaxial stress state.

The relationship above was obtained from conversion of the known S-N fatigue characteristic S-N [2]

$$\log N_f = A - m \log \sigma_a \tag{3.19}$$

on the assumption that $\sigma_a = \sigma_{aTH-M}$.

In the stress models, just as in the case of the energy model, after reduction from the complex stress state to the uniaxial one, the obtained results were compared with the simple state, namely bending in the considered case.

In the case of random loading, for fatigue life determination, the cycle counting method was adopted. In this method, schematization of the random stress history was undertaken by means of the rain flow algorithm, damages were accumulated according to Palmgren (1924) and Miner (1945) hypothesis including the influence of amplitudes less than the fatigue limit σ_{af}

$$S_{PM}(T_o) = \sum_{i=1}^{k} \frac{n_i}{N_0\Big(\frac{\sigma_{af}}{\sigma_{ai}}\Big)^m} \tag{3.20}$$

The damage degree $S(T_o)$ at observation time T_o of stress history was calculated by summation of damages from successive amplitudes of cycles and half-cycles σ_{ai} according to relationship (3.10).

4. Verification of the models

A statistical analysis understood as a measure of usability was performed for all the considered models and materials. The analysis included determination of the mean scatter expressed by (Macha *et al.*, 2006)

$$\overline{T}_N = 10^{\overline{E}} \tag{4.1}$$

where

$$E_i = \log \frac{N_{exp_i}}{N_{cal_i}} \qquad \overline{E} = \frac{1}{n} \sum_{i=1}^{n} E_i \tag{4.2}$$

and the scatter coefficient expressed as

$$T_N = 10^{\frac{t_{n-1}\alpha}{2s}} \tag{4.3}$$

where s is the standard deviation defined according to

$$s = \sqrt{s^2} \qquad s = \frac{1}{n-1} \sum_{i=1}^{n} (E_i - \overline{E})^2 \tag{4.4}$$

The required confidence level for each desired quantity, for most applications, at the level of 95% is retained as the default value, the significance level is usually assumed as $\alpha = 5\%$ ([20], Sutherland and Veers, 2000). Thus, the mean value should be included in the interval $-t_{(n-1),\alpha/2}(s) \leqslant \overline{E} \leqslant t_{(n-1),\alpha/2}(s)$, where $t_{(n-1),\alpha/2}$ is the constant from t-Student's distribution.

The constant $t_{(n-1),\alpha/2}$ from t-Student's distribution is determined for a half of the significance level $\alpha/2$ because of the boundary section of the normal distribution.

Figures 6-8 show a comparison between the calculated and experimental fatigue lives for all the considered materials. The solid line means ideal conformity of the results. The dashed lines represents a scatter band with a coefficient of 3, i.e. $N_{f\,exp}/N_{f\,cal} = 3$ (1/3). In the case of stress methods, the large scatter of the results is attrributable to small material constants which were used in the calculations (σ_{TS}, σ'_f). In the case of the energy model, more material constants and strain history were applied in the calculations. This data offered better results of calculation as we can see in Table 3, however, it has a big influence on the calculation time (longer calculation time as in the stress models).

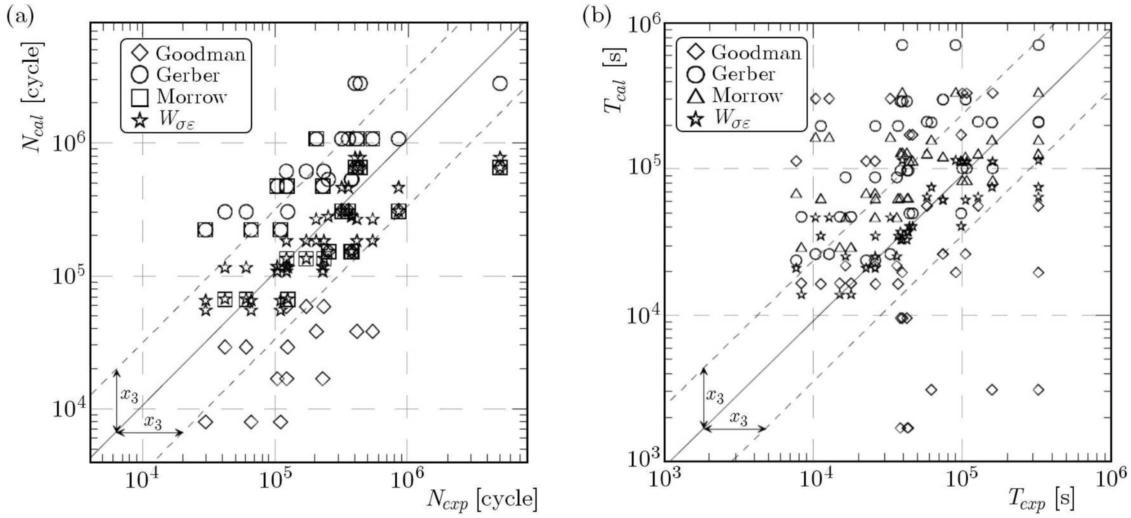

Fig. 6. Comparison of the calculated and experimental fatigue lives for specimens made of alloy steel S355J2WP under tension-compression cyclic loading (a) and random loading (b)

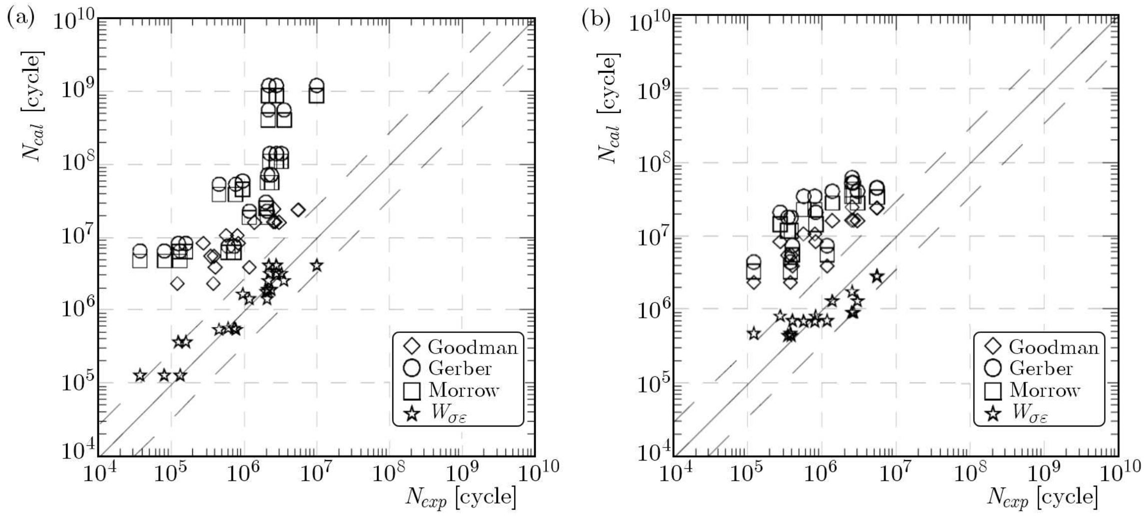

Fig. 7. Comparison of the calculated and experimental fatigue lives for specimens made of aluminium alloy 2017A-T4 under combined bending with torsion; (a) $\tau(t) = 0.5\sigma(t)$, (b) $\tau(t) = \sigma(t)$

From Table 3 it appears that the models for fatigue life estimation based only on stresses have a higher mean scatter. The perfect conformity of the results is the case for $\overline{T} = 1$. The majority of the results are found in the safe region as calculation results are greater than the experimental, but when $\overline{T}$ is too big, the mechanical values are no longer familiar with them. T_N coefficient

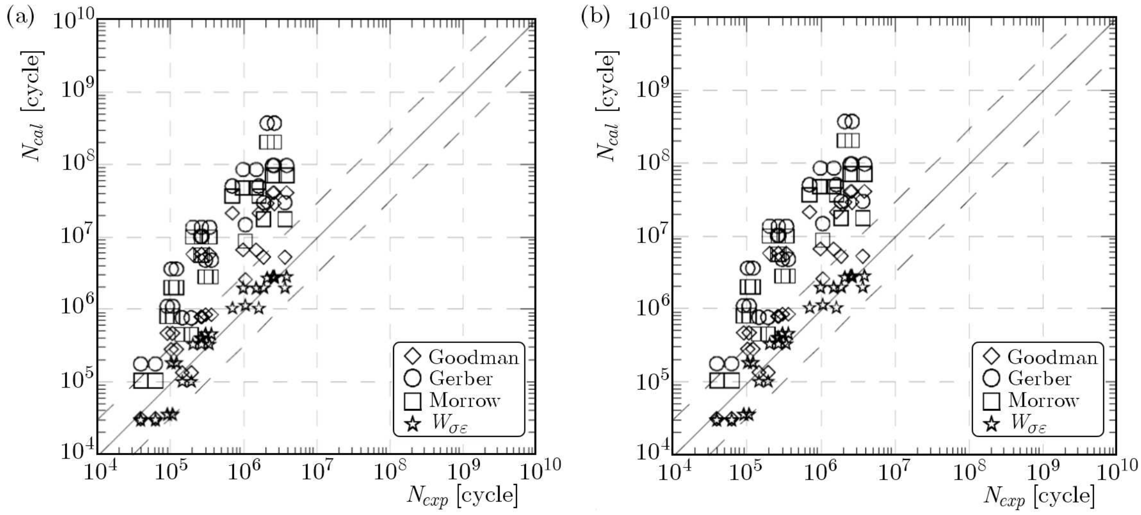

Fig. 8. Comparison of the calculated and experimental fatigue lives for specimens made of alloy steel S355J2WP under combined bending with torsion; (a) $\tau(t) = \sigma(t)$, (b) $\tau(t) = \sigma(t)$

Table 3. Statistic analysis of the considered models

	Uniaxial loading				Multiaxial loading									
	S355J2WP				S355J2G3		S355J2WP		2017A-T4					
	cylic		random						$\tau = \sigma$		$\tau = 0.5\sigma$			
	$\overline{T}$	T_N	$\overline{T}$	T_N	$\overline{T}$	T_N	$\overline{T}$	T_N	$\overline{T}$	T_N	$\overline{T}$	T_N	$\overline{\overline{T}}$	$\overline{T}_N$
Goodman	3.31	6.44	1.51	47.59	2.63	4.43	4.65	10.69	8.96	3.28	20.37	5.85	9.15	6.06
Gerber	3.22	2.93	2.53	6.11	7.20	3.90	24.36	8.71	21.75	4.11	50.31	9.99	25.91	6.68
Morrow	1.25	2.65	2.12	5.31	3.74	3.80	15.22	8.91	15.31	3.84	38.70	9.14	18.24	6.42
$W_{\sigma\varepsilon}$	1.02	2.94	1.04	3.75	1.04	4.79	1.02	2.65	1.15	3.52	1.04	3.28	1.06	3.56

describes the range of the scatter band. If T_N assumes a higher value, the calculation method is worse.

Table 4 show comparisons the advantages and faults of the considered models.

Table 4. Usability analysis of the considered models

	Goodman	Gerber	Morrow	$W_{\sigma\varepsilon}$
Usable in multiaxial loading in base form	no	no	no	yes
Usable in uniaxial loading in base form	yes	yes	yes	yes
Easy to use	yes	yes	yes	no
Time counting in cyclic loading	short	short	short	short
Time counting in random loading	short	short	short	long
Scatter band in multiaxial cyclic loading	medium	poor	poor	good
Scatter band in uniaxial cyclic loading	medium	good	good	good
Scatter band in uniaxial random loading	poor	medium	medium	good

5. Conclusions

The undertaken verification of the energy model offered satisfactory results regarding the comparison of the calculated and experimental data for aluminum alloy 2017A-T4 for combined bending with torsion including mean values. The calculated results are included in the scatter band for tests of cyclic bending with the zero mean value.

The application of $W_{\sigma\varepsilon}$ including the influence of the stress mean value for fatigue life determination of steels S355J2WP and S355J2G3 makes it possible to obtain results close to those obtained from tests.

The energy model gives satisfactory results because, in this research, a low mean scatter $\overline{T}$ and a small scatter band T_N has been noted. The Goodman model seems to be the most appropriate for the considered materials, and the Gerber model seems to be the worst one.

Models based only on stress history should only be used in engineering calculations, in the design process of machines which do not bear heavy loads. Concurrently, when we have to do with structures or parts of machines that are crucial for human life, designers should use new methods based on the stress and strain history together.

In the case of the considered materials, the stress models are characterized by a very high mean scatter, leading to overestimation of the calculated lives. Thus, their application to machine designing seems to be unacceptable.

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