

Multivariable predictive control considering time delay for load-frequency control in multi-area power systems

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In this paper, a multivariable model based predictive control (MPC) is proposed for the solution of load frequency control (LFC) in a multi-area interconnected power system. The proposed controller is designed to consider time delay, generation rate constraint and multivariable nature of the LFC system, simultaneously. A new formulation of the MPC is presented to compensate time delay. The generation rate constraint is considered by employing a constrained MPC and economic allocation of the generation is further guaranteed by an innovative modification in the predictive control objective function. The effectiveness of proposed scheme is verified through time-based simulations on the standard 39-bus test system and the responses are then compared with the proportional-integral controller. The evaluation of the results reveals that the proposed control scheme offers satisfactory performance with fast responses.

Key words: load frequency control, model predictive control, time delay, generation rate constraint.

1. Introduction

Undesirable frequency and scheduled tie-line power changes in multi-area power system is a direct result of the imbalance between generated power and system demand plus associated system losses. The main goals of load frequency control (LFC) are to track load demands, maintain frequency and tie-line scheduled power deviations in a desired range and ensure zero steady state error [1].

In control systems, it is well known that time delays can degrade a systems performance and even cause system instability. Considering this fact, because of the restructuring, expanding of physical setups, functionality and complexity of power systems in the new environment, communication delays are to become a significant challenge in LFC design and analysis [2]. In addition, due to the participation of many units in LFC task, the power system LFC is a multivariable control problem. Furthermore, generation rate constraints (GRC's) have major effects on the dynamic of power system LFC [3]. These features make load frequency control a time delayed, constrained, multivariable

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and multi-objective control problem. Many investigations in the area of power system LFC have been reported [2, 4, 5-8]. Nearly all such previous studies neglected time delay and some important features of the LFC. Two-degree-of-freedom Internal Model Control (IMC) method has been used in [4] for PID tuning of the LFC system. Application of Bacteria Foraging (BF) and craziness particle swarm optimizations (CPSO) to find PI gain controller have been proposed in [5] and [6], respectively. However, the effect of communication time delay has not been considered by these approaches. A linear matrix inequalities based approach has been proposed for the LFC system with time delay in [7]. Static output feedback (SOF) has been employed in [2] to design a robust decentralized controller for the LFC system with communication delays. However, these approaches have not taken the GRC into account. In [8], the authors have been presented a design of the optimal PI controller by employing a more realistic model considering time delay, the GRC and generator dead band simultaneously. However, the PI controllers have limited ability to deal with time delay and the GRC. Consequently, they are unable to provide acceptable dynamical performance. In addition, the PI controllers offer a number of difficulties in multi-objective and multivariable controller design. To overcome the disadvantages of the PI controllers, some efforts have been made to employ model predictive control. Model predictive control is a modern control theory which is known as a practical high performance technology. Main advantages of the MPC are full compensation of delayed systems, constraint handling ability, straightforward multivariable formulation and simple design approach for complex systems [9].

Design and simulation of decentralized MPC for multi-area power system has been presented in [10]. A distributed MPC framework for the LFC system has been proposed in [11]. In [12] and [13], the design and application of functional and robust multivariable MPC for the power system LFC have been performed. However, some of them have shortcomings in taking the GRC into account and a number of them simply assume that all subsystems are identical. Furthermore, none of the aforementioned MPC controllers considered communication time delay. This paper develops a novel MPC design for the LFC system employing all the advantages of this method. The proposed controller derives its optimal output by simultaneously considering time delay, generation rate constraints, economic allocation of generated power, minimizing output error and control effort. To achieve these goals, a multi-objective cost function is defined which consists three terms reflecting output error, control effort and economic allocation of the generation. Time delay is fully compensated by representing a new formulation which is the main contribution of this paper. Furthermore, constraint handling ability of the MPC is employed to effectively take the GRC into account. In addition, the proposed scheme is a multivariable approach which offers great advantages in comparison with single in-put single output (SISO) approaches. To evaluate the effectiveness of the proposed technique, time-based simulations are performed on the 39-bus test system and the results are then compared with the optimal PI controller.

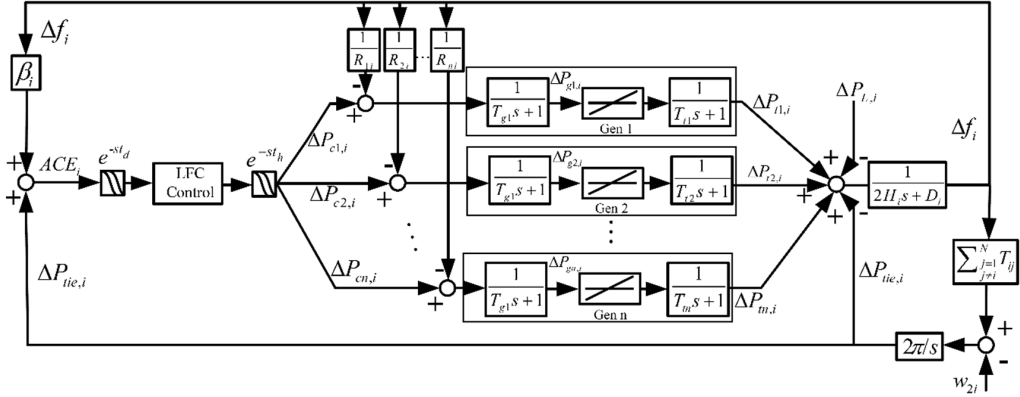


Figure 1: Dynamic model of i th control area.

2. Power system LFC dynamic

In a large power system, the overall power system structure can be considered as a collection of separated control areas which are connected through tie-lines. The block diagram representation of i th area with n generation units in an N -area power system is displayed in Fig. 1. Due to large scale of most power systems, the design and implementation of decentralized controllers are preferable. The state space model of each area is given in (1).

$$\begin{aligned} \dot{x}_i &= A_{c,i}x + B_{c,i}u_i + F_{c,i}w_i \\ y_i &= C_{c,i}x_i \end{aligned} \quad (1)$$

where x , u and w present state, input and disturbance vectors, respectively, y is designated as the control output or as the area control error (ACE). The state space variables of (1) are as follows:

$$x_i = [\Delta f_i, \Delta P_{tie,i}, \Delta P_{g1,i}, \dots, \Delta P_{gn,i}, \Delta P_{t1,i}, \dots, \Delta P_{tn,i}]^T$$

$$y_i = ACE_i = C_{c,i}x_i$$

$$u_i = [u_{i1}, \dots, u_{in}]^T = [\Delta P_{c,i1}, \dots, \Delta P_{c,in}]^T$$

$$w_i = [\Delta P_{L,i}, w_{2i}]$$

$$A_{c,i} = \begin{bmatrix} A_{i11} & A_{i12} & A_{i13} \\ A_{i21} & A_{i22} & A_{i23} \\ A_{i31} & A_{i32} & A_{i33} \end{bmatrix}, \quad B_{c,i} = \begin{bmatrix} B_{i1} \\ B_{i2} \\ B_{i3} \end{bmatrix}, \quad F_{c,i} = \begin{bmatrix} F_{i1} \\ F_{i2} \\ F_{i3} \end{bmatrix}$$

$$\begin{aligned}
A_{i11} &= \begin{bmatrix} \frac{-D_i}{2H_i} & \frac{-1}{2H_i} \\ -2\pi \sum_{j=1, j \neq i}^N T_{ij} & 0 \end{bmatrix}, \quad A_{i12} = \begin{bmatrix} \frac{1}{2H_i} & \cdots & \frac{1}{2H_i} \\ 0 & \cdots & 0 \end{bmatrix} \\
A_{i22} &= \text{diag} \left[\frac{-1}{T_{g1,i}}, \frac{-1}{T_{g1,i}}, \dots, \frac{-1}{T_{gn,i}} \right] \\
A_{i33} &= -A_{i32} = \text{diag} \left[\frac{-1}{T_{t1,i}}, \frac{-1}{T_{t1,i}}, \dots, \frac{-1}{T_{tn,i}} \right] \\
A_{i21} &= \begin{bmatrix} \frac{-1}{(T_{g1,i}R_{1i})} & 0 \\ \frac{-1}{(T_{gn,i}R_{ni})} & 0 \end{bmatrix}, \quad A_{i12} = A_{i31}^T = 0_{2 \times n}, \quad A_{i23} = 0_{n \times n} \\
B_{i1} &= 0_{2 \times n}, \quad B_{i2} = \text{diag} \left[\frac{1}{T_{g1,i}}, \frac{1}{T_{g1,i}}, \dots, \frac{1}{T_{gn,i}} \right], \quad B_{i3} = 0_{n \times n} \\
C_{c,i} &= [\beta_i \ 1 \ 0_{1 \times n} \ 0_{1 \times n}], \quad \beta_i = D_i + \frac{1}{R_i} \\
F_{i1} &= \begin{bmatrix} \frac{-1}{2H_i} & 0 \end{bmatrix}, \quad F_{i2} = 0_{n \times 1}, \quad F_{i3} = 0_{n \times 1}
\end{aligned}$$

According to Fig. 1, in each control area, the control input is obtained by ACE signal.

$$u_i = \Delta P_{ci} = f_i(ACE_i). \quad (2)$$

All the existing LFC designs for power system are based on SISO models, although simulations are usually performed on nonlinear MIMO models. In a SISO design, each unit input u_{ik} is related to the area input $u_{i,tot}$ via (3)

$$u_i = [u_{i1}, \dots, u_{in}]^T = [\alpha_{i1}, \dots, \alpha_{in}]^T u_{i,tot} \quad (3)$$

where α_{ik} is assigned as the distribution participation factor for generating unit k in area i and $\sum_{k=1}^n \alpha_{ik} = 1$.

Generally, the SISO controller design and its implementation are straightforward; however, it is clearly a compromise for multivariable systems and is preferable to address the LFC design directly by multivariable methods.

An important constraint in the power system LFC is a limitation on the variation rate of mechanical movement, which is known as generation rate constraint. In the block diagram of Fig. 1, the GRC is modeled on the governor valve position ΔP_g . In [14] and [10], the GRC is imposed on ΔP_c , which is a simplified assumption. Since the control input signal ΔP_c is different from the governor valve position signal ΔP_g , assuming identity of these two signals is not valid and causes major deviation in the calculation of the

optimal control input. The GRC has significant impact on the dynamic response of the power system LFC and the effective inclusion of this constraint in the control scheme will greatly improve the control performance.

To apply the MPC, a reliable and fast communication link, which is currently available in most power systems, is vital. Fortunately, modern power systems employ wide area measurement system (WAMS) [15] which utilize phasor measurement unit (PMU) as a basis for data collection system. PMU's measure phasors of voltage, current and frequency with a time stamp in the time interval down to 20msec.

In any LFC system, with or without WAMS, there are several signal processing and data exchanging operations which introduce communication delay. For the purpose of this research, the communication delays are classified into two categories [2]: one is the induced delay τ_h between control center and individual generating units and the other is time delay τ_d related to receiving measured signals from the system (Δf and ΔP_{tie}). Depending on the size of a system, type of signal processing and communication technology, total time delay in a power system can be varied. Typically, communication time delay in the WAMS data communication is in the order of 100ms which can reach 250ms at the worst case [16]. In traditional communication systems, the time delay can increase to several seconds [1].

3. Control strategy

3.1. Model predictive control

The MPC has been extensively adopted in industry as an effective means to deal with multivariable constrained control problems. The MPC uses an explicit model of the system to predict the future trajectory of system states and outputs. This prediction capability allows computing optimal input sequence to minimize the output error over a finite horizon subjected to system constraints. To reduce the effect of future disturbance and model mismatch, only the first control signal in the optimal sequence is fed to the plant and the entire calculation is repeated at subsequent control intervals. Discrete form of state space model is used in this paper to model the power system LFC dynamic. It is of the form:

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m u(k) \\ y_m(k) &= C_m x_m(k) \end{aligned} \quad (4)$$

Where the state variable x_m have dimensionality of $n1$ and plant has p input and q output. To obtain Δu as the manipulated variable for system output, the original state space model (4) is augmented as follows. By assuming $\Delta x_m(k) = x_m(k) - x_m(k-1)$, $\hat{y}(k) = C_m x_m(k)$ and choosing a new state variable vector $x = [\Delta x_m \ \hat{y}(k)]^T$, the following

equation will be obtained.

$$\begin{bmatrix} \Delta x_m(k+1) \\ \hat{y}(k+1) \end{bmatrix} = \begin{bmatrix} A_m & 0_1 \\ C_m A_m & I_{p \times p} \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ \hat{y}(k) \end{bmatrix} + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k) \quad (5)$$

$$y(k) = [0_2 \ I] \begin{bmatrix} \Delta x_m(k) \\ \hat{y}(k) \end{bmatrix}$$

Where 0_1 , 0_m and 0_2 are zero matrices with sizes of $n_1 \times q$, $q \times n_1$ and $q \times 1$ respectively, and $I_{q \times q}$ is a unit matrix with dimension q . Equation (5) can be denoted by:

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k) \\ y(k) &= Cx(k) \end{aligned} \quad (6)$$

where k is designated as the sampling instant, x is assigned as the state vector and Δu represents the incremental input vector. The matrices A , B and C are the corresponding matrices given in the form of (5). The MPC formulation with embedded integral form has several advantages of full disturbance rejection, offset free tracking and simpler formulation in the problem with constraints on the variation rate of system variables [17].

The main goals of MPC are to minimize the predicted output error with minimal control effort by incorporating system constraints. Therefore, the optimization problem is stated as follows:

$$\begin{aligned} \sum_{j=1}^{N_p} \|y(k+j) - r_s(k+j)\|_2^2 Q_w + \sum_{j=1}^{N_c} \|\Delta u(k+j)\|_2^2 R_w \\ \text{s.t. } x_{\min} \leq x(\cdot) \leq x_{\max}, \quad \Delta x_{\min} \leq \Delta x(\cdot) \leq \Delta x_{\max} \end{aligned} \quad (7)$$

where $y(k+j)$ is assigned as the predicted output of the system at j th sample, $r_s(k+j)$ is designated as the future reference trajectory, N_p and N_c are prediction and control horizon, and R_w and Q_w are positive definite weighting matrices which compromise the output error and control effort. The first term in (7) reflects the future output error and second term reflects the consideration given to the control effort. The constraints of the MPC include constraints on the magnitude and change of system variables including the system inputs, states and outputs. Prediction expression for the state space model is introduced by (8) and its simplified expression is presented by (9).

$$\begin{bmatrix} y(k+1) \\ y(k+2) \\ y(k+3) \\ \vdots \\ y(k+N_p) \end{bmatrix} = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{N_p} \end{bmatrix} x(k) + \begin{bmatrix} CB & 0 & \dots & 0 \\ CAB & CB & \dots & 0 \\ CA^2B & CAB & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & \dots & CA^{N_p-N_c} \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \Delta u(k+N_c) \end{bmatrix} \quad (8)$$

$$Y = Fx(\cdot) + \phi\Delta U. \tag{9}$$

The statement (9) should be modified to predict future trajectory of the constrained variables in order to take system constraints into account. To accomplish this, the output matrix C should be replaced by a new matrix C_c which transforms constrained state variables into output variables. By applying this modification, equation (9) is expressed in the new form (10):

$$Y_C = F_C x(\cdot) + \phi_C \Delta U. \tag{10}$$

3.2. Economic allocation of generated power

As aforementioned, in areas with multiple units, the generated power should be allocated based on the economic criteria. In the previous LFC literatures, the total generating power required for each control area is calculated by employing a SISO control approach and the required power is then distributed among generation units by participation factors. This approach offers simplicity in practical implementation. However, it results in performance degradation due to the SISO approximation of a MIMO system.

In this study, a MIMO control approach is proposed for the LFC system. Considering major benefit of the MIMO control, this approach however cannot directly consider economic allocation of generating power. To alleviate this drawback, a heuristic modification which is first presented in [13], is applied in the cost function (7). This approach is then modified to be applicable in the new formulation of this study. Based on [13], an additional new term which reflects the consideration given to economic allocation should be added to the cost function. The new cost function is stated as follows:

$$J = \sum_{j=1}^{N_p} \|y(k+j) - r_s(k+j)\|_2^2 Q_w + \sum_{j=1}^{N_c} \|\Delta u(k+j)\|_2^2 R_w + \sum_{j=1}^{N_c} \|u(k+j)\|_2^2 R_\alpha \tag{11}$$

where $u(k+j)$ signifies of the system at j th sample and R_α is designated as the related weighting matrix. R_α is determined such that to ensure economic allocation of generating power. In order to derive R_α , (3) is rewritten in the new form (12).

$$\left[\begin{matrix} \frac{\alpha_{i1}}{\alpha_{i2}} u_{i2}, & \frac{\alpha_{i1}}{\alpha_{i3}} u_{i3}, & \dots, & \frac{\alpha_{i1}}{\alpha_{in}} u_{in} \end{matrix} \right] = [1, 1, \dots, 1]^T u_{i1} \tag{12}$$

By subtracting the left side from the right side of (12), summing square of each term and multiplying a tuning parameter $R_{\alpha 0}$, the following softened expression can be obtained:

$$R_{\alpha 0} \left[\left(\frac{\alpha_{i1}}{\alpha_{i2}} u_{i2} - u_{i1} \right)^2 + \left(\frac{\alpha_{i1}}{\alpha_{i3}} u_{i3} - u_{i1} \right)^2 + \dots + \left(\frac{\alpha_{i1}}{\alpha_{in}} u_{in} - u_{i1} \right)^2 \right] = u_i^T R_\alpha u_i \tag{13}$$

where

$$R_\alpha = R_{\alpha 0} \begin{bmatrix} n-1 & -\frac{\alpha_{i1}}{\alpha_{i2}} & -\frac{\alpha_{i1}}{\alpha_{i3}} & \dots & -\frac{\alpha_{i1}}{\alpha_{in}} \\ -\frac{\alpha_{i1}}{\alpha_{i2}} & \left(\frac{\alpha_{i1}}{\alpha_{i2}}\right)^2 & 0 & \dots & 0 \\ -\frac{\alpha_{i1}}{\alpha_{i3}} & 0 & \left(\frac{\alpha_{i1}}{\alpha_{i3}}\right)^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{\alpha_{i1}}{\alpha_{in}} & 0 & 0 & \dots & \left(\frac{\alpha_{i1}}{\alpha_{in}}\right)^2 \end{bmatrix}.$$

Based on the integral embedded form of MPC in (6), the unknown input to the state space model is $\Delta u(\cdot)$. Therefore, the input $U(\cdot)$ of (11) should be expressed as the incremental input $\Delta u(\cdot)$. To accomplish this, expression (14) with a compact form of $U = U_0 + G\Delta U$ can be used:

$$\begin{bmatrix} u(k+1) \\ u(k+2) \\ \vdots \\ u(k+N_C) \end{bmatrix} = \begin{bmatrix} u(k) \\ u(k) \\ \vdots \\ u(k) \end{bmatrix} + \begin{bmatrix} I & 0 & \dots & 0 \\ I & I & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ I & I & \dots & I \end{bmatrix} \begin{bmatrix} \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \Delta u(k+N_C) \end{bmatrix} \tag{14}$$

where I is an $n \times n$ identity matrix.

3.3. Delay compensation

The dynamic models of many engineering systems, including transportation of data, involve time delays. It is well known that time delays can degrade a systems' performance and even cause system instability. Therefore, effective incorporation of time delay in the LFC system increase control performance. Previous researches [7] and [2], conducted in this area, tried to reduce the impact of time delays and to guarantee system stability under maximum delay condition. These approaches, however, cannot completely eliminate the negative impact of time delay on the LFC performance.

A major advantage of the MPC is its ability in delay modeling which can fully compensate the effect of time delay. Due to the presence of the predictive mechanism in the MPC which provides the ability of prediction in time horizon larger than time delay, the control input can be computed such that to eliminate the effect of delay.

In the model shown in Fig. 1, the sending and receiving delays (τ_d, τ_h) are in series. Therefore, the system total time delay τ is the sum of these two delays.

$$\tau = \tau_d + \tau_h. \tag{15}$$

In a large scale power system, due to large number of sending and receiving signals and variable time of signal processing, there are some degrees of uncertainty in the amount

of time delay. In this paper, time delay τ_i s assumed to be constant and it is chosen to have its maximum expected value. To accomplish this and to include time delay, in the real time implementation, control signal should be kept to be applied after the time delay has elapsed.

By assuming sampling time interval of Δt , the discrete time delay horizon N_t is $T_D = \Delta t$. To add the ability of delay compensation in the MPC formulation, the prediction expression (8) is rewritten in the detailed form as (16). It is assumed that the first N_t input samples are known and constant. Therefore, corresponding output samples are fixed. The next N_c input samples are unknown variables and should be optimized to provide the minimum output error corresponding to the output samples.

$$\begin{bmatrix} y(k+1) \\ \vdots \\ y(k+N_T) \\ \vdots \\ y(k+N_P) \end{bmatrix} = \begin{bmatrix} CA \\ \vdots \\ CA^{N_T} \\ \vdots \\ CA^{N_P} \end{bmatrix} x(k) + \tag{16}$$

$$+ \begin{bmatrix} CB & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ CA^{N_T-1}B & \dots & CB & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ CA^{N_P-1}B & \dots & CA^{N_P-N_T-1}B & \dots & CA^{N_P-N_C-N_T}B \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \vdots \\ \Delta u(k+N_T) \\ \vdots \\ \Delta u(k+N_T+N_C) \end{bmatrix}$$

Equation (16) can be presented by the simplified expression of (17):

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} x(\cdot) + \begin{bmatrix} \phi_1 & \phi_2 \\ \phi_3 & \phi_4 \end{bmatrix} \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix} \tag{17}$$

where $\Delta u_1, y_1, F_1, \phi_1$ and ϕ_2 relate to the first N_T fixed samples. ϕ_2 is a zero matrix with dimension of $N_T \times N_C$ which reflects the lack of any relationship between unknown input variable Δu_2 and fixed output y_1 . Consequently, $\Delta u_2, y_2, F_2, \phi_3$ and ϕ_4 are unknown input and output variables with their corresponding matrices. The extraction of the unknown variables from the prediction expression (17) leads to the following equation

$$y_2 = F_2x(\cdot) + \phi_3\Delta u_1 + \phi_4\Delta u_2 \tag{18}$$

where y_2 is the corresponding output which its error should be minimized over the prediction horizon. $F_2x(\cdot) + \phi_3\Delta u_1$ is a constant term and Δu_2 is unknown control vector which should be calculated by optimization process. Similarly, the prediction expression for the constrained variables can be obtained by (19).

$$y_{2C} = F_{2C}x(\cdot) + \phi_{3C}\Delta u_1 + \phi_{4C}\Delta u_2 \tag{19}$$

Predictive control is a feedback control policy. Therefore, there is a risk that the resulting closed loop may be unstable. In the MPC strategy, stability can usually be ensured by making the prediction horizon sufficiently long [9]. In the special case of time delayed system, prediction horizon must be larger than delay horizon N_T . Otherwise, system instability may occur and closed loop performance will be deteriorated.

3.4. Solving predictive control problem

To solve the predictive control problem, the final optimization problem should be converted to a standard quadratic programming (QP) problem. The main reason for this conversion is the presence of several powerful algorithms to solve this type of convex optimization problem [18]. In the literatures of quadratic programming, the decision variable is denoted by x . The objective function J and the inequality constraints are expressed as:

$$\begin{aligned} \min J(x) &= \frac{1}{2}x^T E x + x^T f \\ \text{s.t. } Mx &\leq \gamma \end{aligned} \quad (20)$$

where E , f , M and are compatible matrices and vectors in the quadratic programming problem. Without the loss of generality, E is assumed to be symmetric and positive definite.

To derive the standard QP parameters for the classical MPC formulation, equation (9) and (10) should be substituted in (7) which yield:

$$J = (R_s - Fx(k))^T Q_w (R_s - Fx(k)) - 2\Delta U^T \phi^T (R_s - Fx(k)) + \Delta U^T (\phi^T \phi + R_w) \Delta U$$

$$y_{2C} = F_{2C}x(\cdot) + \phi_{3C}\Delta u_1 + \phi_{4C}\Delta u_2$$

$$\text{s.t. } \begin{cases} \phi_C \Delta U + F_C x(\cdot) < \Delta P'_{g,\max} \\ -\phi_C \Delta U + F_C x(\cdot) < -\Delta P'_{g,\min} \end{cases} \quad (21)$$

Therefore, the QP parameters can be obtained as follows:

$$E = \phi^T \phi + R_w, \quad f = \phi^T F \quad (22)$$

$$M = \begin{bmatrix} \phi_C \\ -\phi_C \end{bmatrix}, \quad \gamma = \begin{bmatrix} \Delta P'_{g,\max} - F_C x(k) \\ -\Delta P'_{g,\min} + F_C x(k) \end{bmatrix}$$

Solving the objective function (20) with parameters introduced in (22) generates a constrained control approach without considering time delay and economic allocation. To enable economic allocation and delay compensation property in the MPC strategy, prediction variable in the objective function (11) should be replaced by the prediction statements (18) and (19) which results in:

$$\begin{aligned}
 J &= (R_s - F_2x(k) - \phi_3\Delta u_1 - \phi_4\Delta u_2)^T (R_s - F_2x(k) - \phi_3\Delta u_1 - \phi_4\Delta u_2) \\
 &\quad + \Delta u_2^T R_w \Delta u_2 + (u_0 + G\Delta u_2)^T R_\alpha (u_0 + G\Delta u_2) \\
 \text{s.t. } &\begin{cases} \phi_{4C}\Delta u_2 + (F_{2C}x(k) + \phi_{3C}\Delta u_1) < \Delta P'_{g,\max} \\ -\phi_{4C}\Delta u_2 - (F_{2C}x(k) + \phi_{3C}\Delta u_1) < -\Delta P'_{g,\min} \end{cases} \quad (23)
 \end{aligned}$$

with the standard QP parameters:

$$\begin{aligned}
 E &= \phi_{4C}^T \phi_{4C} + R_w + G^T R_\alpha G \\
 f &= \phi_4^T (R_s - F_2x(k) + \phi_3\Delta u_1) - G^T R_\alpha u_0
 \end{aligned} \quad (24)$$

$$M = \begin{bmatrix} \phi_{4C} \\ -\phi_{4C} \end{bmatrix}, \quad \gamma = \begin{bmatrix} \Delta P'_{g,\max} - F_{2C}x(k) - \phi_{3C}\Delta u_1 \\ -\Delta P'_{g,\min} + F_{2C}x(k) + \phi_{3C}\Delta u_1 \end{bmatrix}. \quad (25)$$

In this paper, active set method is employed to solve real time quadratic optimization problem [18].

4. Simulation results and discussion

To demonstrate the effectiveness of the proposed control design, some nonlinear simulations are performed in the Power System Toolbox (PST) [19] of MATLAB (R2009b) software. All the nonlinear simulations are executed on an Intel core i7 2.8 GHz computer with 2G RAM. The proposed multivariable MPC is applied to the IEEE 10 generators 39-bus test system. The system base frequency is 50 Hz and the system data are taken from [20]. The single line diagram of the 39-bus test system is illustrated in Fig. 2 and the system LFC specifications are presented in Appendix.

The power system is divided to three control areas. Area-2 contains four generation units while other areas contain three generation units. Each area further contains a separated MPC controller. The total system is simulated by a centralized nonlinear model and the MPC is applied on the decentralized way. In the simulations, the performance of the closed-loop system using the designed predictive controllers is compared with well-tuned conventional PI controllers. The parameters of PI controller are obtained with genetic algorithm based on the integral square error criteria (ISE) with respect to step load change in all areas.

For the purpose of this study, all generators are participated in LFC task. Each control area is supposed to be a MIMO system. The input and output of each control area are $\Delta U = [\Delta P_{c1,i}, \dots, \Delta P_{cn,i}]$ and $Y = [ACE_i]$. The signal $ACE_{ref} = 0$ is further implemented as the reference trajectory. Other control parameters are set as follows:

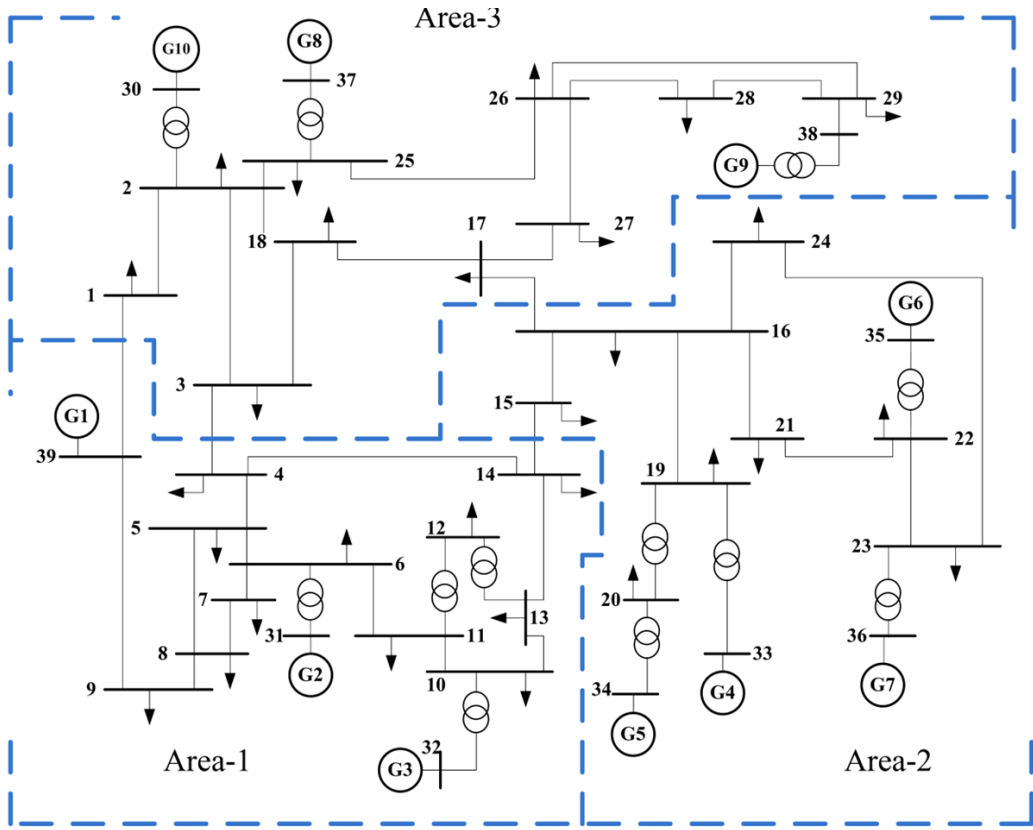


Figure 2: Single line diagram of 39-bus power system.

Number of prediction horizon: $N_p = 80$,

Number of control horizon: $N_C = 5$,

Output weighting matrix: $Q_w = 1.0 \times I_{N_p \times N_p}$,

Input weighting matrix: $R_w = 2.0 \times I_{N_C \times N_C}$,

Economic allocation weighting coefficient: $R_{\alpha 0} = 0.10$.

Choosing larger R_w yields slower control response and selecting larger $R_{\alpha 0}$ provides faster tendency of generating power to their economic values. The discretization time interval is selected as 50ms and the Zero-Order Hold method is employed to discrete the continuous system of (1)[21]. The typical generation rate constraint (GRC) of each thermal unit is 10%/min. Therefore, the GRC of the each generator is assumed to be 0.17%/sec.

$$\Delta P'_{g,\max} = 0.1 \frac{pu}{min} (0.0017 \frac{pu}{sec}), \quad -\Delta P'_{g,\max} \leq \Delta P'_g \leq \Delta P'_{g,\max}$$

In all simulations, a 0.12 pu step load increase (sufficiently significant to cause large overshoot in the frequency deviation) is applied to the control areas. Three scenarios with different time delays and load changes are investigated and the results are then analyzed. For further evaluation and deep assessment of the designed MPC controller, the detailed results with quantitative comparisons are presented. The effect of delay compensation, multivariable design and constraint handling ability are further analyzed and discussed in detail.

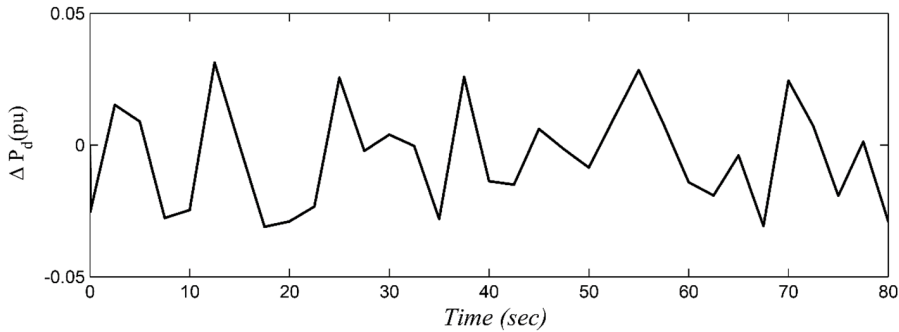


Figure 3: Random load variations.

4.1. Scenario 1: response to random load changes

In this scenario, a bounded random load changes, as displayed in Fig. 3, are applied to the control area-1 and area-3. The ΔP_{d1} and ΔP_{d3} are selected as $\Delta P_d/2$ and time delay is chosen to be 0.3 sec. The frequency deviation (Δf), area control error (ACE) and total generation power (ΔP_T) of control area-1 and area-3 are presented in Fig. 4 and Fig. 5.

As shown in the results, all controllers change the generated power to match the load fluctuations. However, the performance of MPC is more effective compared to the PI controller. The figures show that the MPC is faster than the PI controller and provides ACE and frequency deviation less than the PI controller. Since the MPC takes the GRC and time delay into account, it provides more satisfactory results compared to the conventional PI controller.

4.2. Scenario 2: 0.3 sec time delay

In this scenario, the system performance is tested with 0.3 sec time delay following a 0.12 pu step load increase ($\Delta P_{L1} = 0.00$ pu, $\Delta P_{L2} = 0.00$ pu, $\Delta P_{L3} = 0.12$ pu). The closed loop system response, including frequency deviation Δf and ACE of control areas are displayed in Fig. 6 and Fig. 7.

As demonstrated in Fig. 6 and Fig. 7, both the MPC and the PI controller make frequency and ACE deviations move towards zero. The PI controller offers overshoot and settling time of 0.0930 Hz and 63.1 sec for Δf_1 . The results reveal that the MPC controller is superior to the PI controller. The MPC offers satisfactory performance and

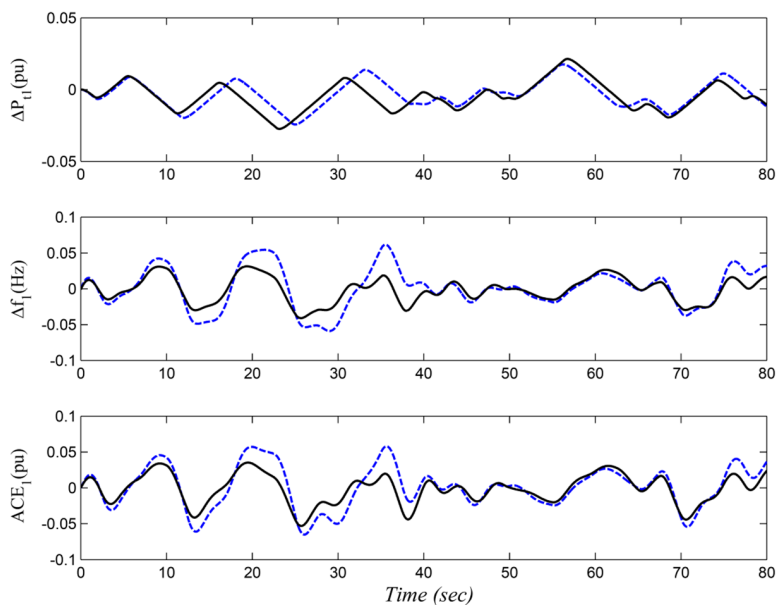


Figure 4: System response to random load variations for area-1. Solid (MPC), dashed (PI).

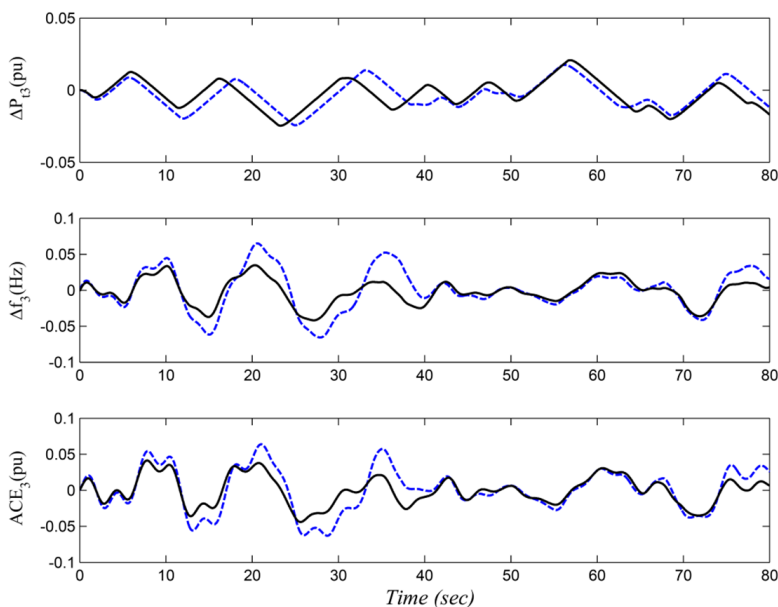


Figure 5: System response to random load variations for area-3. Solid (MPC), dashed (PI).

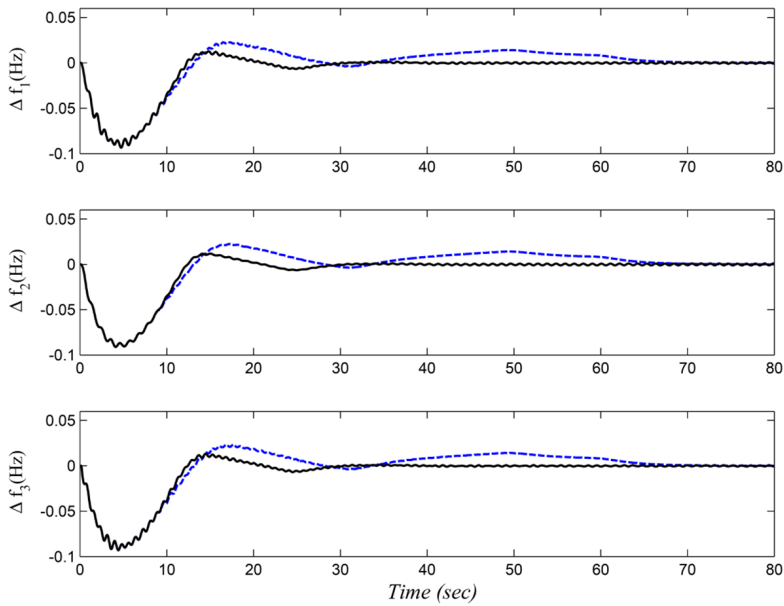


Figure 6: Frequency oscillations of the control area's in response to step load change for area-1. Solid (MPC), dashed (PI).

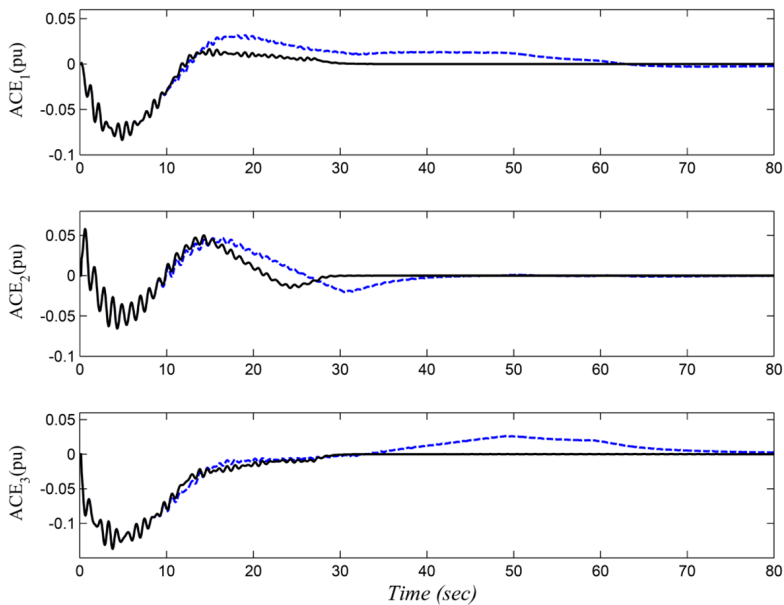


Figure 7: ACE oscillations of the control area's in response to step load change for area-1. Solid (MPC), dashed (PI).

provides frequency Δf_1 overshoot and settling time of 0.0930 Hz, 27.3 sec. Compared to the PI controller, the settling time is decreased and the closed loop system performance is considerably improved. In comparison with the PI controller, the main reasons for superiority of the MPC are multivariable design and constraint handling ability. The multivariable design provides sufficient flexibility to keep constrained variables close to their margins. In addition to multivariable design and constraint handling ability, the MPC compensates the time delay which yields to more improvement in closed loop performance.

4.3. Scenario 3: 3 sec time delay

For scenario 3, the system performance is examined with 3 sec time delay. The purpose of this scenario is to investigate the effect of long time delay on the closed loop performance of LFC. The system responses are depicted in Fig. 8 and Fig. 9.

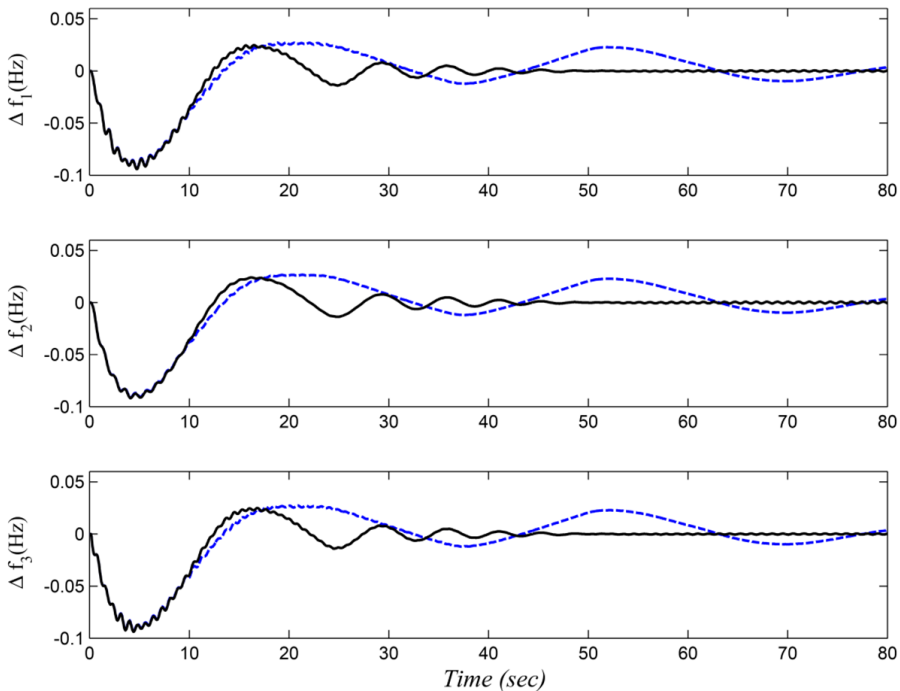


Figure 8: Frequency oscillations of the control area's in response to step load change (Scenario-3). Solid (MPC), dashed (PI).

The obtained results reveal that the MPC provides settling time of 37.3 sec whereas the PI controller offers settling time of 99.6 sec. This means that in the presence of long time delay, the MPC preserves its satisfactory performance, however, the performance of the PI controller deteriorates drastically. Unlike to the PI controller, the MPC worked well and properly returned frequency and ACE oscillations back to zero. This is due to

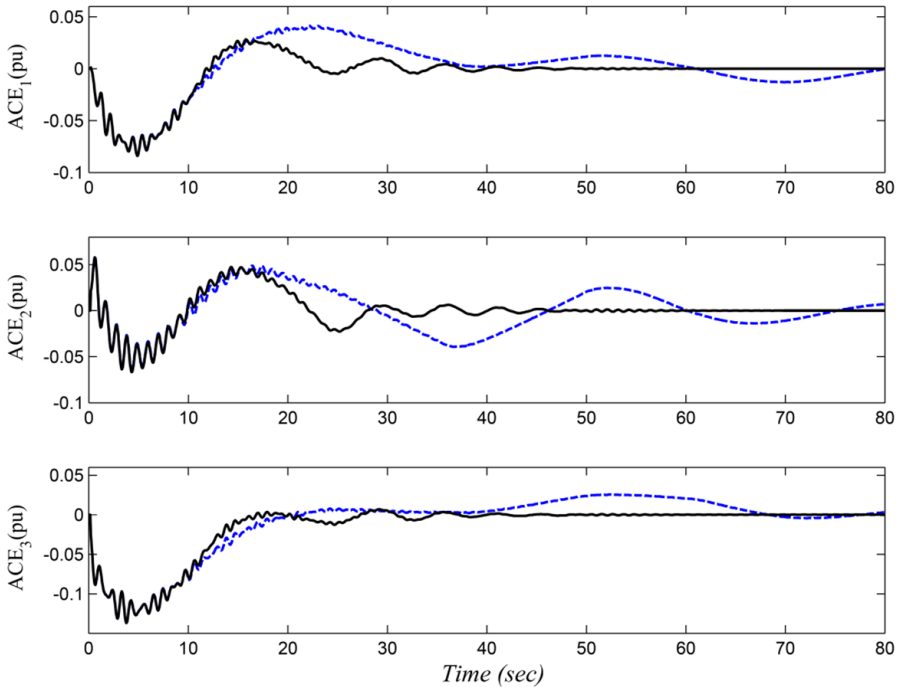


Figure 9: ACE oscillations of the first control area's in response to step load change (Scenario-3). Solid (MPC), dashed (PI).

the delay compensation ability of the MPC which can reduce the harmful effect of time delay on the closed loop performance.

4.4. Discussion on the performance of MPC controller

To better understand and analyze the behavior of the MPC controller, the detailed results related to economic allocation of generated power, constraint handling and delay compensation are presented.

The cost function and settling time of controllers are presented in Tab. 1 in order to investigate the effect of time delay on the closed loop performance. The related objective function is the quadratic cost function given in (7).

Tab. 1 indicates that at complete delay compensation, the MPC provides more satisfactory responses compared to the PI controller. When time delay is small, the impact of multivariable design and constraint handling is dominant. In the large time delays, the effect of delay compensation is dominant and is the main reason for superiority of the MPC. The results further reveal the high sensitivity of the MPC with respect to incomplete delay compensation. In the case of partial delay compensation, the performance of the MPC can be worse compared to the PI controller. Therefore, in the application of the MPC, the value of delay compensation should not be considered less than its true value.

Table 1: Performance of different controllers with respect to changes in time delay.

Controller	Time delay (sec)	Delay Compensation (%)	T_s (sec)	Cost function
PI	0.0	—	63.0	1.361
	0.5	—	63.4	1.373
	1.0	—	67.3	1.412
	1.5	—	71.4	1.497
	2.0	—	80.2	1.550
	2.5	—	88.3	1.611
	3.0	—	99.6	1.671
MPC	0.0	100%	26.8	0.934
	0.5	100%	29.1	0.942
	1.0	100%	30.5	0.978
	1.5	100%	31.7	1.010
	2.0	100%	33.0	1.083
	2.5	100%	35.1	1.126
	3.0	100%	37.3	1.675
MPC	3.0	75%	49.4	1.976
	3.0	50%	64.2	2.231
	3.0	25%	85.9	2.502
	3.0	0%	—	2.766

The governor valve position of the PI and the MPC controllers are displayed in Fig. 10. The results reveal that if the PI controller demands a control action which exceeds system constraints; then it will be replaced with a control action at the constraint. The MPC, however, has the knowledge of the constraints within the optimization. Therefore, it can be far more intelligent than the PI controller and compute the control actions in such a way that the constrained variables are maintained close to their margins. Inability of the PI controller to take the GRC into account yields to larger violation of constrained variable from their margin. This leads to extra unnecessary control actions, poor damping of system oscillations and slower response.

The output power of units of area-1, controlled by the MPC, is shown in Fig. 11. Initially, all units provide approximately equal amount of power to respond to load increase. After a few seconds and near the steady state, their output powers change slowly to comply with the economic power sharing. In this time span, the first and the second terms of the objective function (11) tend toward zero which dominates the weight of the

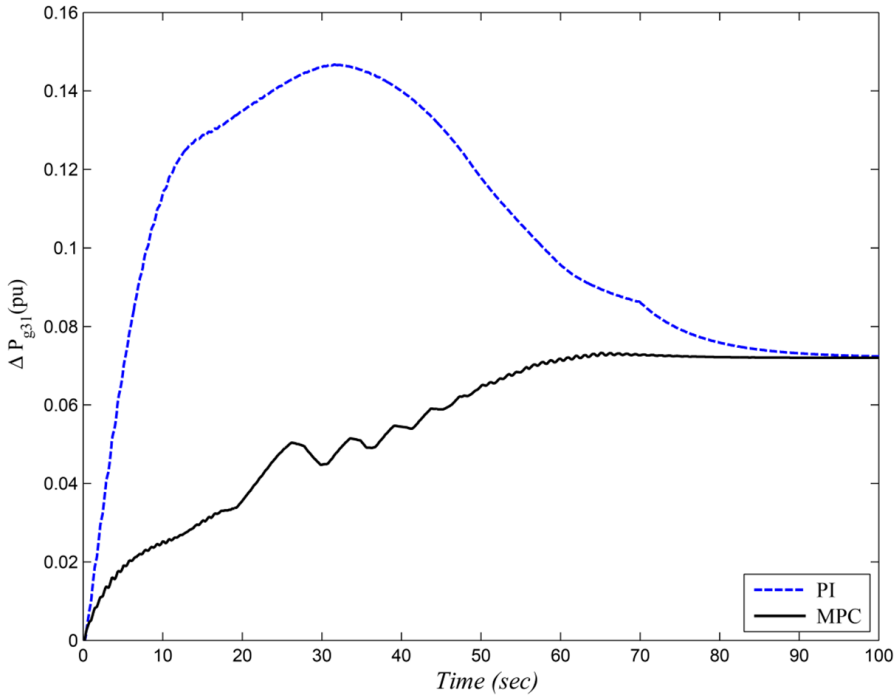


Figure 10: Governor valve position $\Delta P_{g3,1}$.

third term. Since this term is added to ensure the economic allocation, the control inputs are forced to fulfill the economic sharing.

In the optimization problem (24), each local decentralized controller should solve 9 (3×3) unknown variables in each time interval. By employing active set method, the average required time to solve the QP problem is 2.3 msec which is insignificant relative to other sources of delay. This time delay can be ignored or considered as another source of delay to be taken by the MPC into account.

Generally, the design and implementation of MPC is more complex than the PI. However, in order to respond new challenges of the LFC in the future, a compromise between control performance and ease of implementation should be performed. The authors believe that expected future advancement in digital technology and power system control and monitoring systems, easily can fulfill the practical requirement for the real-time applications of the MPC based LFC in power system. Therefore, it will be logical to employ MPC to utilize its valuable control advantages.

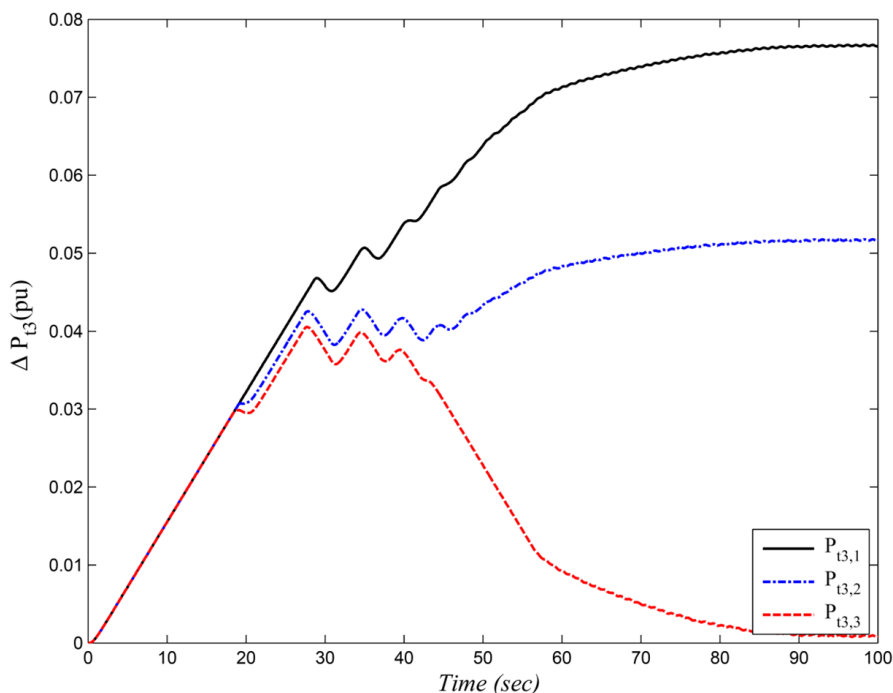


Figure 11: Generating powers of area-3 (ΔP_t , $\alpha_{31} = 0.6$, $\alpha_{32} = 0.4$, $\alpha_{33} = 0.0$).

5. Conclusion

A novel MPC design considering time delay has been proposed for the power system LFC. Generation rate constraint, MIMO nature of the LFC and economic allocation of generating are further incorporated in the presented approach. Taking time delay into account, the quadratic cost function of predictive controller is reformulated and the optimal control trajectory is calculated by solving this minimization problem.

The simulation results demonstrate that the proposed control strategy provided satisfactory closed loop performance in the presence of time delay. Compared to the PI controller, it offered smoother response with less oscillation and settling time. Considering a more complete model of the system, offering great flexibility and providing superior performance are the main advantages of the proposed controller.

Appendix

The LFC specifications of 39-bus test system are as follows:

Area-1: $K_{i1} = -0.1672$, $K_{p1} = -1.17e - 3$, $\alpha_{11} = 0.50$, $\alpha_{12} = 0.25$, $\alpha_{13} = 0.25$,
 Area-2: $K_{i2} = -0.1558$, $K_{p2} = -2.23e - 3$, $\alpha_{21} = 0.50$, $\alpha_{22} = 0.50$, $\alpha_{23} = 0.00$
 $\alpha_{24} = 0.00$,
 Area-3: $K_{i3} = -0.1777$, $K_{p3} = -7.82e - 4$, $\alpha_{31} = 0.60$, $\alpha_{32} = 0.40$, $\alpha_{33} = 0.00$.

List of symbols

ACE_i	area control error
i	area number
R_i	droop characteristic
D_i	area load frequency characteristic
β_i	frequency bias
T_{ij}	synchronizing coefficient between area i and j
$T_{g,i}$	governor time constant
$T_{t,i}$	turbine time constant
$M_i (2H_i)$	area equivalent inertia
Δf_i	change in area frequency (Hz)
$\Delta P_{c,i}$	change in governor load set point
$\Delta P_{g,i}$	change in governor valve position
$\Delta P_{t,i}$	change in turbine power
$\Delta P_{tie,i}$	tie-line power deviation
$\Delta P_{L,i}$	power demand deviation

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