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APPLICATION OF DIRECT AND INVERSE KINEMATICS AND DYNAMICS IN MOTION PLANNING OF MANIPULATOR LINKS

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For the synthesis of manipulators and robots, an accurate analysis of movements of the individual links is essential. This paper deals with motion planning of the effector of a multi-linked manipulator. An important issue in this area is the orientation and position of links and kinematic pairs in space. In particular, attention should be paid to the position of their endpoint as well as other significant points. Trajectory planning allows the manipulator to perform complex tasks, such as picking and placing objects or following a particular path in space. Overall, trajectory planning of a multibody manipulator involves a combination of direct and inverse kinematics calculations, as well as control theory and optimization techniques. It is an important process enabling manipulators to perform complex tasks such as assembly, handling and inspection. In the design of robot kinematic structures, simulation programs are currently used for their kinematic and dynamic analysis. The proposed manipulator was first solved by inverse kinematics problem in MATLAB. Subsequently, the trajectories of the end-effector were determined in MATLAB by a direct kinematics problem. In Simulink, using the SimMechanics library, the inverse problem of dynamics was used to determine the trajectories of the moments.

Key words: kinematics analysis, manipulator, inverse kinematics problem, motion planning, inverse dynamics problem.

1. Introduction

With technological advances, industries are gradually entering the era of full automation. Industrial robots are used instead of humans to perform repetitive work that is highly risky, too complex, too difficult and takes a long time. Most of the manufacturing and industrial processes are now performed by modern industrial robots, which provide different types of object manipulation and make individual production processes easier and faster. The basic function of a handling device is to hold an object and precisely transfer it to a specified position. Mobile robots for remote handling are used in the production process. With gradual development, robotic manipulators with two or more arms have been designed that are capable of simultaneously imitating the movements of a human hand.

For the synthesis of manipulators and robots, an accurate analysis of the movements of individual links is essential. In the analysis of the motions of individual links, the start and end positions of the manipulator are important, which are used as a basis for determining or planning the trajectory of a single link. For kinematic and dynamic analysis of motions, the matrix method is often used, on the basis of which simulation programs are also built. Simulation programs are used in the kinematic and dynamic analysis of robots and allow a reliable analysis of robotic devices as a whole and also their single links. The individual computer simulation programs contain predefined basic bodies and kinematic pairs from which the user can choose while building a simulation model of a multi-link system.

Compared to the analytical approach, this procedure saves time considerably. Trajectory planning allows the manipulator to perform complex tasks. Overall, trajectory planning of a multibody manipulator

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involves a combination of direct and inverse kinematics calculations as well as control theory and optimization techniques. It is an important process enabling manipulators to perform complex tasks such as assembly, handling and inspection (Hroncova *et al.* [1], Lewis *et al.* [2], Angeles [3]).
Direct kinematics and inverse kinematics are used to control the manipulator. The proposed manipulator was first solved by inverse kinematics problem in MATLAB, where the rotation angles of both robot arms were determined. Subsequently, the trajectories of the end effector were determined in MATLAB by the direct kinematics problem. In Simulink, using the SimMechanics library, the inverse dynamics problem was used to determine the moment trajectories.

2. Trajectory planning

The movements performed by robots and manipulators should be as smooth as possible. Sudden changes of movements, i.e. sudden changes of position, speed, acceleration, should be avoided. Sudden changes in movements can also occur when the manipulator comes into contact with another object, which should be avoided. It is therefore necessary to plan or program in advance the movements that a given manipulator can perform with respect to surrounding objects. In practice, however, it is still impossible to avoid unforeseen situations that can lead to system failure. Unpredictable situations should therefore be taken into account when designing a robotic system. Two trajectory planning methods can be used for motion planning in a known environment

- pick-and-place and
- continuous pathways.

The task of the robotic manipulator is to move an object from a given initial position to a specified final position. These positions are determined with respect to the manipulator's base space and its orientation. When moving from one position to another, it is not the path that is important, but that there is no collision with other objects and at the same time that the movement is smooth. (Lewis *et al.* [2], Kuryło *et al.* [4], Sapietová *et al.* [5], Božek *et al.* [6]).

Although the initial and final positions are prescribed in the basic Cartesian coordinate system, the motions of individual robot links are realized in their local space. Therefore, individual local spaces need to be transformed into the base space. Planning movements in the base space ensures that the robot does not collide with other objects in its environment. Therefore, it is necessary to render the motion of all moving links of the robot, each of which has its own geometry. However, for general three-dimensional motions and arbitrary geometries, this approach is impractical due to the computational requirements. A more pragmatic approach could consist of two steps, namely

- planning a preliminary trajectory in local space, without taking into account obstacles to movement and
- visual verification of the occurrence of collisions, based on a plotted animation of the motion of the manipulator in the presence of obstacles.

To determine the trajectories of the manipulator's moving links, it is necessary to know or choose the initial and final positions of the manipulator Let the vector of joint variables in the robot's initial and final configurations be denoted by θ_0 and θ_F. We assume that time is calculated from the initial position when $t = 0$. If the motion takes place at time t, then at the final position let the time be labeled $t = t_F$. In this case, the path of the end effector is not important.

The position vector of the endpoint (effector) with respect to the frame is denoted by $\boldsymbol{p}$, that is, at the initial position $\boldsymbol{p}(t = 0) = \boldsymbol{p}(0) = \boldsymbol{p}_0$ and at the final position $\boldsymbol{p}(t = t_F) = \boldsymbol{p}(t_F) = \boldsymbol{p}_F$. If there is no singularity we assume zero velocity and acceleration at the end position of the effector:

$$\theta(0) = \theta_0,\ \dot{\theta}(0) = 0,\ \ddot{\theta}(0) = 0, \tag{2.1}$$

$$\theta(t_F) = \theta_F,\ \dot{\theta}(t_F) = 0,\ \ddot{\theta}(t_F) = 0. \tag{2.2}$$

From the equations, it is clear that neither linear nor quadratic interpolation holds between the initial and final positions because the slope vanishes at only one point. Therefore, higher order interpolation is required.

The trajectory is generated by an algorithm that, given general requirements, is able to optimize some parameters when moving a point from one position to another (Siciliano *et al.* [7], [8], Kuryło *et al.* [9], Kuric *et al.* [10]).

A third-order polynomial function can be used to generate the trajectory:

$$q(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0 \tag{2.3}$$

where the coefficients a_i are determined from the values of the initial and end position of the effector and the initial and final velocity, which is usually equal to zero. If we add the initial and end acceleration values to the conditions, the six boundary conditions must be satisfied. For the six boundary conditions, it is necessary to use a polynomial of at least fifth degree to plot the motion (Lenarčič *et al.* [11], Hroncová *et al.* [12], Frankovský *et al.* [13], Baressi *et al.* [14]). The time dependence of the motion for a general point is then given by the relation:

$$q(t) = a_1 t^5 + a_2 t^4 + a_3 t^3 + a_4 t^2 + a_5 t + a_6 \ , \tag{2.4}$$

whose coefficients a_i can be computed from the boundary conditions for the initial time $t = 0$ and the time at the final position $t = t_F$ for the variable $q(t)$ and for its first two derivatives. It is necessary to find a polynomial that will allow the joint variable θ_j to be represented over the whole range of motion:

$$\theta_j(t) = \theta_{j0} + \left(\theta_{jF} - \theta_{j0}\right) q(t) \tag{2.5}$$

where θ_{j0} and θ_{jF} are given the initial and final values of the *j*-th link variable. In vector form, Eq.(2.5) takes the form:

$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}_0 + \left(\boldsymbol{\theta}_F - \boldsymbol{\theta}_0\right) q(t) . \tag{2.6}$$

We plan the trajectory for the two-links planar manipulator according to Fig. 1. The manipulator's workspace is given in Cartesian coordinates in the plane, which are a function of time $x = x(t), y = y(t)$. The angular parameters of individual links that determine the position of the end-effector are θ_1 , θ_2 (Fig. 1) and are a function of time $\theta_1 = \theta_1(t)$, $\theta_2 = \theta_2(t)$.

We use the inverse kinematic transformation to determine the angular variables of individual links of the manipulator θ_1 , θ_2 . Coordinates of the end point are as follows:

$$x_L = x(t) = L_1 \cos\theta_1 + L_2 \cos\left(\theta_1 + \theta_2\right) , \tag{2.7}$$

$$y_L = y(t) = L_1 \sin\theta_1 + L_2 \sin\left(\theta_1 + \theta_2\right) . \tag{2.8}$$

From Fig.1 we can determine;

$$x_L^2 + y_L^2 = L_1^2 + L_2^2 + 2 L_1 L_2 \cos\theta_2 \tag{2.9}$$

and

$$\cos\theta_2 = \frac{x_L^2 + y_L^2 - L_1^2 - L_2^2}{2L_1L_2}. \tag{2.10}$$

The solution implies that $\cos\theta_2$ must be from the interval $\langle -1, 1\rangle$, otherwise the point in question would be outside the manipulator's workspace. Then

$$\sin\theta_2 = \pm\sqrt{1-\cos^2\theta_2} \tag{2.11}$$

where the positive sign represents the downward position of the arm and the negative sign represents the upward position of the arm. The angle θ_2 is calculated as:

$$\theta_2 = \mathrm{atan}2(\sin\theta_2, \cos\theta_2). \tag{2.12}$$

We then determine the angle θ_1 by inserting θ_2 into (2.7), (2.8) from which we get an algebraic system of two equations with two unknowns, whose solution is:

$$\sin\theta_1 = \frac{(L_1 + L_2\cos\theta_2)y_L - L_2\sin\theta_2 x_L}{x_L^2 + y_L^2}, \tag{2.13}$$

$$\cos\theta_1 = \frac{(L_1 + L_2\cos\theta_2)x_L - L_2\sin\theta_2 y_L}{x_L^2 + y_L^2}. \tag{2.14}$$

Then

$$\theta_1 = \mathrm{atan}2(\sin\theta_1, \cos\theta_1). \tag{2.15}$$

The angle of θ_1 depends on the angle of θ_2.

3. Rendering of the manipulator trajectory

A model of a two-link manipulator coupled by a rotary kinematic pair is shown in Fig.1. The coordinate systems are stored in rotary kinematic pairs. The rotation angle of individual links is denoted by the angles θ_1, θ_2. The manipulator has *2* degrees of freedom of motion. Let us consider the arms of the manipulator as perfectly rigid bodies. For the lengths of the links $L_1 = 0.55\ m$ and $L_2 = 0.45\ m$ the angles at the initial position of the end effector *L* have been calculated.
At the initial position at time $t = 0$, the coordinates are $x_{L0} = 0.5\ m$, $y_{L0} = -0.86\ m$. At the final position at time $t_F = 2\ s$, the end-effector *L* has the coordinates $x_{LF} = -0.9\ m$, $y_{LF} = 0\ m$.

The angles were calculated using relations (2.7), (2.8), and (2.12), (2.15) for the initial and final effector positions, respectively. By calculating the angles for two possible positions and movements, we gets:

$$\theta_{1\,0} = [-54.5340°; -65.1190°],\ \theta_{2\,0} = [-11.7656°; 11.7656],$$

$$\theta_{1\,F} = [-156.8082°; 156.8082°],\ \theta_{2\,F} = [-51.9636°; 51.9636°].$$

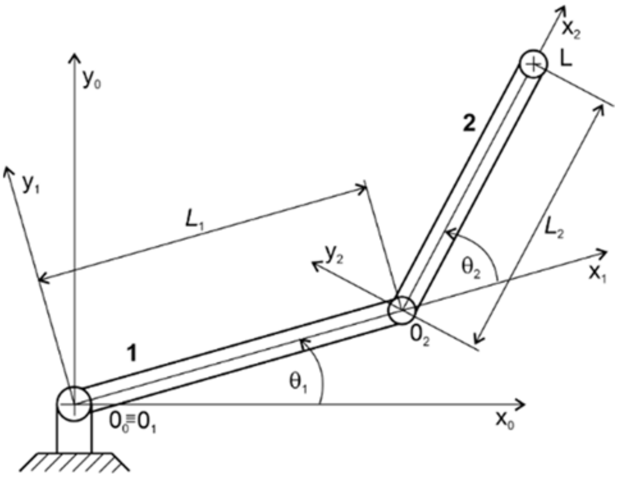


Fig.1. Two-link manipulator.

Figure 2 shows the positions of the arms at the start and end positions of the effector L for the calculated angles. The values of $\theta_{1\,0} = -65.1190°$, $\theta_{2\,0} = 11.7656$, $\theta_{1\,F} = 156.8082°$, $\theta_{2\,F} = 51.9636°$ were used to plot the endpoint trajectory, Fig.2.

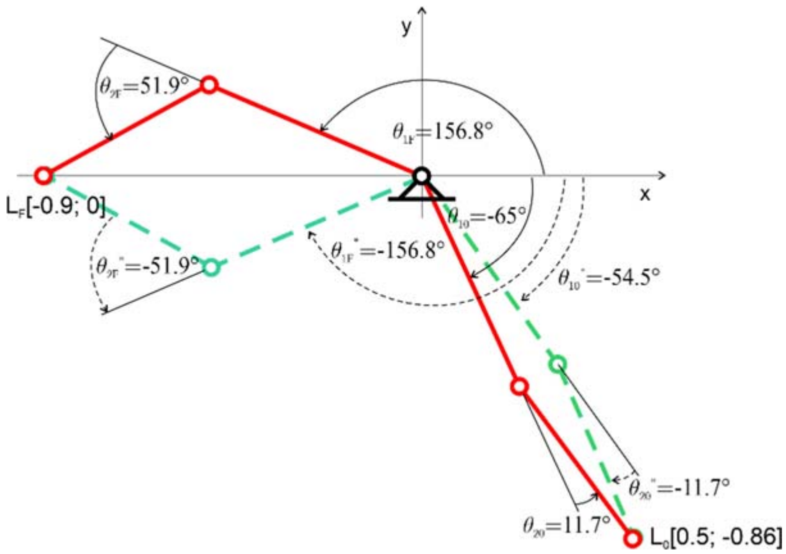


Fig.2. Manipulator with given start and end position of point *L*.

The time history of the angular rotations θ_1, θ_2 of individual arms is found in the form of a polynomial of degree five by using relation (2.4):

$$\theta_1(t) = a_1 t^5 + a_2 t^4 + a_3 t^3 + a_4 t^2 + a_5 t + a_6 \ , \tag{3.1}$$

$$\theta_2(t) = b_1 t^5 + b_2 t^4 + b_3 t^3 + b_4 t^2 + b_5 t + b_6 \tag{3.2}$$

where the coefficients a_6 and b_6 correspond to the magnitudes of the angles at the initial position of the arms at times $t = 0$, $a_6 = \theta_{10}$ and $b_6 = \theta_{20}$. Expressed in radians, they are:

$$a_6 = \theta_{10} = \theta_1(t=0) = -65.1190\left(\frac{\pi}{180}\right) = -1.1365\,[\text{rad}]\,; \tag{3.3}$$

$$b_6 = \theta_{20} = \theta_2(t=0) = 11.7656\left(\frac{\pi}{180}\right) = 0.2053\,[\text{rad}]\,. \tag{3.4}$$

To ensure the desired movement of the individual terms, we express the other coefficients in polynomials, which can be written in vector form:

$$\boldsymbol{a} = [\,a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6\,]^{\mathrm{T}}\,; \boldsymbol{b} = [\,b_1 \quad b_2 \quad b_3 \quad b_4 \quad b_5 \quad b_6\,]^{\mathrm{T}}\,. \tag{3.5}$$

These coefficients are used to ensure the desired endpoint motion and are determined from the initial and endpoint conditions of the monitored point. The degree of the polynomial is chosen according to the number of prescribed conditions to allow their calculation.

At the desired start and end positions of the arms and the end point, respectively, the velocity and acceleration will be zero. Given this, the angular velocity and angular acceleration will also be zero at these positions, which serves as a condition for calculating the coefficients of the polynomial.

The angular velocity of the first term in time t is the derivative of the angle $\theta_1(t)$ with respect to time:

$$\frac{d}{dt}\left(\theta_1(t)\right) = \dot{\theta}_1(t) = 5a_1t^4 + 4a_2t^3 + 3a_3t^2 + 2a_4t + a_5\;. \tag{3.6}$$

Due to the zero angular velocity of $\dot{\theta}_1(t) = 0$ at the time $t = 0$ the parameters will be $a_5 = 0$. The second derivative of the rotation angle $\theta_1(t)$ with respect to time is the angular acceleration:

$$\frac{d^2}{dt^2}\left(\theta_1(t)\right) = \ddot{\theta}_1(t) = 20a_1t^3 + 12a_2t^2 + 6a_3t + 2a_4\;. \tag{3.7}$$

Given zero angular acceleration $\ddot{\theta}_1(t) = 0$ at time $t = 0$ the parameters will be $a_4 = 0$. Similarly, for the angle $\theta_2(t)$, given zero angular velocity and acceleration at the initial and final positions, the parameters will be $b_5 = 0$, $b_4 = 0$.

By adjusting the polynomials, we obtain a system of three equations of three unknowns at the initial and final times. Angle $\theta_1(t=0)$, $\theta_1(t=t_F)$;

$$\theta_1(t=t_F) - \theta_1(t=0) = a_1t_F^5 + a_2t_F^4 + a_3t_F^3\;, \tag{3.8}$$

$$0 = 5a_1t_F^4 + 4a_2t_F^3 + 3a_3t_F^2\;, \tag{3.9}$$

$$0 = 20a_1t_F^3 + 12a_2t_F^2 + 6a_3t_F\;. \tag{3.10}$$

Angle $\theta_2(t=0)$, $\theta_2(t=t_F)$;

$$\theta_2(t=t_F) - \theta_2(t=0) = b_1t_F^5 + b_2t_F^4 + b_3t_F^3\;, \tag{3.11}$$

$$0 = 5b_1 t_F^4 + 4b_2 t_F^3 + 3b_3 t_F^2 \ , \tag{3.12}$$

$$0 = 20b_1 t_F^3 + 12b_2 t_F^2 + 6b_3 t_F \ . \tag{3.13}$$

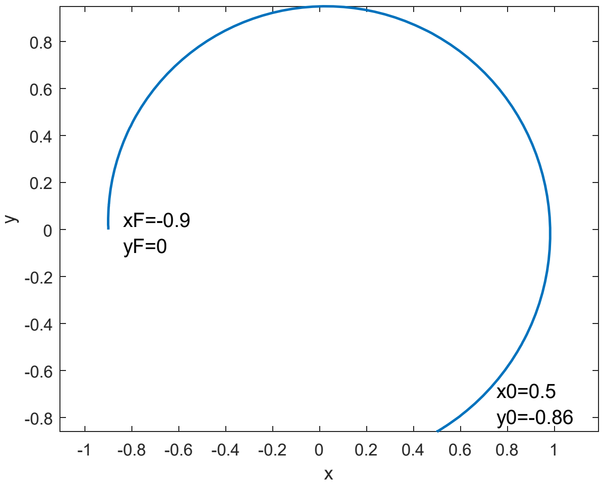


Fig.3. Trajectory of the endpoint from the selected start position to the end position.

To simplify the solution, these equations can be written in matrix form

$$\begin{bmatrix} t_F^5 & t_F^4 & t_F^3 \\ 5t_F^4 & 4t_F^3 & 3t_F^2 \\ 20t_F^3 & 12t_F^2 & 6t_F \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \theta_1(t = t_F) - \theta_1(t = 0) \\ 0 \\ 0 \end{bmatrix}, \tag{3.14}$$

$$\begin{bmatrix} t_F^5 & t_F^4 & t_F^3 \\ 5t_F^4 & 4t_F^3 & 3t_F^2 \\ 20t_F^3 & 12t_F^2 & 6t_F \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \theta_2(t = t_f) - \theta_2(t = 0) \\ 0 \\ 0 \end{bmatrix}. \tag{3.15}$$

The solution gives the coefficients for the angle function in matrix form:

$$\boldsymbol{a} = \begin{bmatrix} 0.7263 & -3.6313 & 4.8417 & 0 & 0 & -1.1365 \end{bmatrix}^{\mathrm{T}}, \tag{3.16}$$

$$\boldsymbol{b} = \begin{bmatrix} 0.1315 & -0.6577 & 0.8770 & 0 & 0 & 0.2053 \end{bmatrix}^{\mathrm{T}}. \tag{3.17}$$

Substituting the only sensitive coefficients (3.16), (3.17) into relations (3.1), (3.2), we obtain the time-dependent angular momentum of the arms, on the basis of which the trajectory of the end-effector *L* can be plotted. The trajectory of the effector *L* is plotted in Fig.3. In Fig.4, the time-dependent trajectories of the rotation angles, angular velocities, and angular accelerations of the manipulator links can be seen.

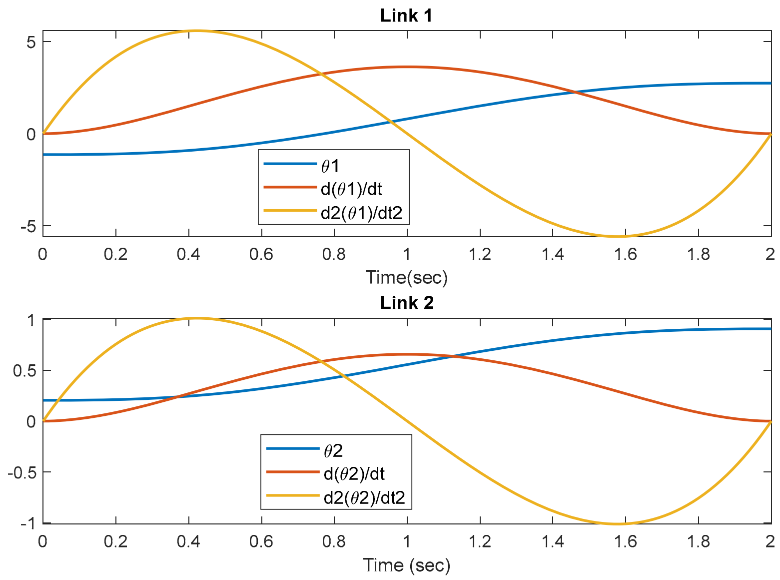


Fig.4. The course of angular quantities of individual manipulator links.

By determining the angular variables of the individual links of the manipulator, the working point of the manipulator can be determined. The workspace is a subspace of the space of all positions that the end-effector can occupy, taking into account the arm lengths. The solvability of the problem is judged in this workspace. The size and shape of the workspace is defined by the characteristics of the manipulator:

- geometric dimensions of the manipulator,
- by restricting the wedge variables.

The workspace of the manipulator arms can be plotted as workspace coordinates for various combinations of angles $\theta_1(t)$, $\theta_2(t)$. Figure 5 plots the manipulator workspace with the end effector trajectory plotted for a given start and end effector position.

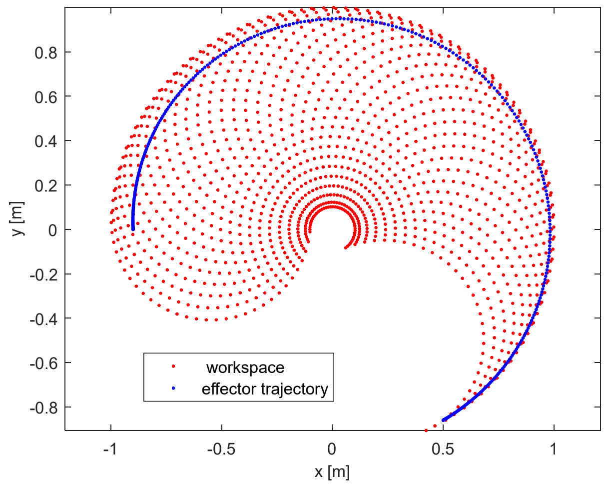
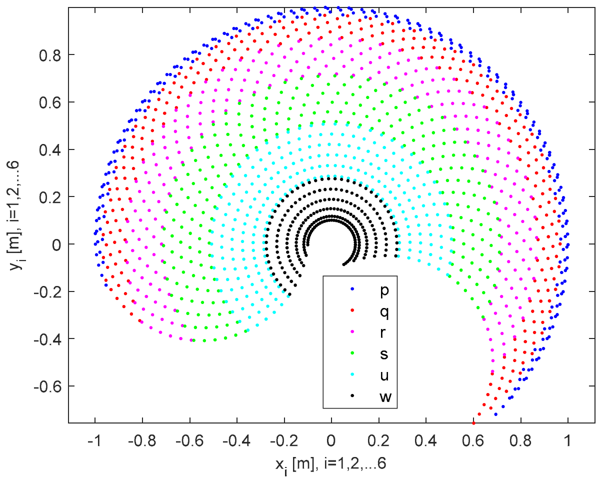


Fig.5. Manipulator workspace with marked effector trajectory.



Mark	Angle restriction θ_2
p	$0° \le \theta_2 \le 30°$
q	$30° \le \theta_2 \le 60°$
r	$60° \le \theta_2 \le 90°$
s	$90° \le \theta_2 \le 120°$
u	$120° \le \theta_2 \le 150°$
w	$150° \le \theta_2 \le 180°$

Fig.6. Manipulator workspace at angular constraints.

The workspaces of the manipulator expressed by the coordinates of the points under angular constraints, where the angle of θ_1 link 1 was varied in the interval $\langle -65°; 180° \rangle$ and the angle of θ_2 link 2 was varied under angular constraints in the interval $\langle 0°; 180° \rangle$, can be seen in the color resolution in Fig.6.

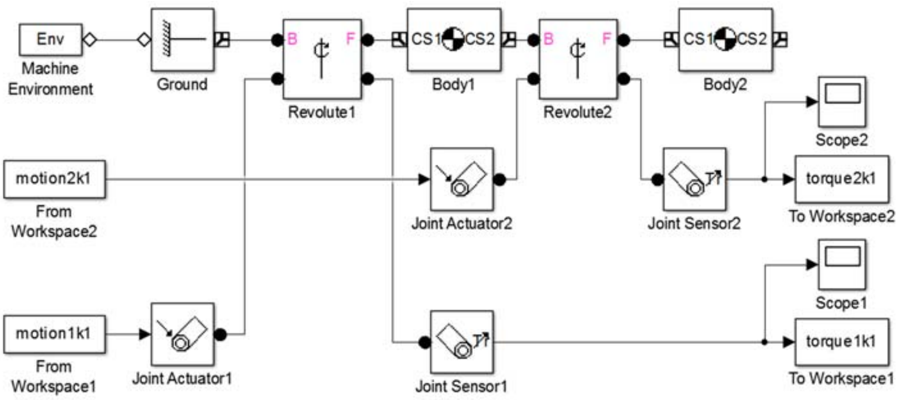


Fig.7. Scheme for solving the inverse problem of dynamics.

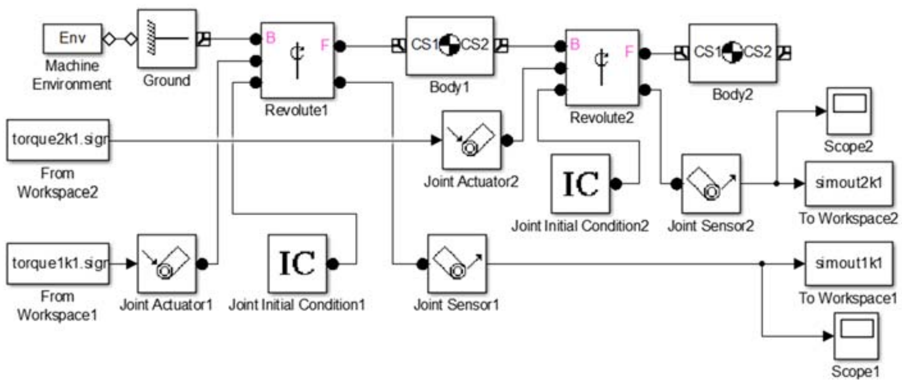


Fig.8. Scheme for solving the direct dynamics problem.

The inverse problem of dynamics can be used to obtain the torques of individual links of the manipulator during motion. In the SimMechanics program, based on the obtained workspace quantities, the torques of the two links in motion have been determined (Hroncová *et al.* [15], Vavro *et al.* [16], Grepl [17], [18], Frankovsky *et al.* [19]). Figure 7 shows the flowchart in SimMechanics for the inverse dynamics problem. The obtained results of the torques of the inverse problem were used in the direct dynamics problem (Fig.8). The results of solving the direct dynamics problem in SimMechanics are identical to the angular quantities in Fig.4.
Figure 9 plots the torque waveforms of individual manipulator links at different end link loads: $m_{21} = 0.45\ kg$, $m_{22} = 0.95\ kg$, $m_{23} = 1.45\ kg$, $m_{24} = 1.95\ kg$, $m_{25} = 2.45\ kg$, $m_{26} = 3.45\ kg$. The mass of the first manipulator link was considered $m_1 = 0.55\ kg$.

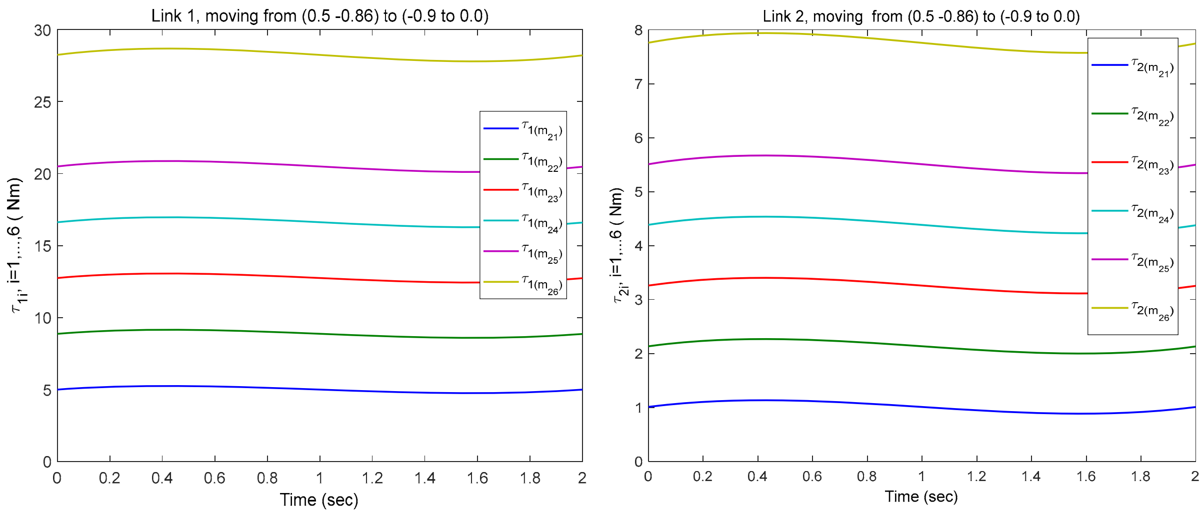


Fig.9. Torques of links for different effector loads.

The maximum values of the torques for the individual end point loads of link 2 are given in Tab.1.
Based on the obtained results of the torques, it is possible to oversize the drive size of individual links of the manipulator.

Table 1. Maximum torque values.

Maximum torque	Weight of link 2					
	m_{21}	m_{22}	m_{23}	m_{24}	m_{25}	m_{26}
τ_1 *Nm*	*5.2438*	*9.1493*	*13.0547*	*16.9602*	*20.8656*	*28.6765*
τ_2 *Nm*	*1.1356*	*2.2695*	*3.4034*	*4.5372*	*5.6711*	*7.9389*

4. Conclusion

The article dealt with the trajectory planning of the end-effector of a bipartite manipulator. The MATLAB program was used in the solution. The inverse kinematics was used to determine the rotation angles of the manipulator arms, based on which the program plotted the trajectory of the end effector given its start and end positions. At the same time, it was possible to determine the time dependence of the angular quantities of individual manipulator links. On the basis of the results, obtained it was possible to plot the workspace. The workspace was plotted for the angular constraints chosen by us. However, the angular constraints can be changed depending on the need when manipulating objects.

At the same time, on the basis of the catch quantities obtained, the moments of rotation of the manipulator arms can be determined by inverse dynamics. The courses of the moments of rotation were determined in the SimMechanics program. The results were verified according to the outputs of the direct dynamics problem from the SimMechanics program.

Based on the outputs, it is possible to design actuators according to the purpose of the manipulator, looking at the maximum load when handling objects of different weights. When handling different objects, it is necessary to oversize the actuator to meet all the requirements for its purpose.

It is also necessary to design the control for the manipulator in order for it to work properly. The control provides stability in the execution of individual movements.

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Nomenclature

a_i – coefficient functions of a polynomial

$\boldsymbol{p}$ – position vector

$q(t)$ – variable function

t – time [s]

τ_{ji} – torque of the j-th link [Nm]

θ_F – angle of rotation of the effector in the final position (at time) t_F

θ_{jF} – finite values of the angular variable of the j-th link

θ_{j0} – initial values of the angular variable of the j-th link

θ_0 – angle of rotation of the effector at the initial position (at time) $t = 0$

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