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Multiple Tuned Tunable Translational-rotational Vibration Absorbers in Beam

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Abstract

The paper deals with vibration of the beam with a system of the translational-rotational dynamic vibration absorbers attached. The beam is subjected to the distributed and concentrated harmonic excitation forces. Assuming small and linear vibration, an analytical Euler-Bernoulli model is applied and the solution to the problem is found with the use of Fourier method. Performing time-Laplace transformation the displacement amplitude of arbitrary point of the beam may be written in the frequency domain. The aim of the paper is to investigate the improvement of the efficiency of the translational-rotational absorbers compared with the translational ones in global vibration control problems. As an example reduction of the kinetic energy of the host structure is considered. Numerical simulations shows a considerable improvement of vibration reduction when the translational-rotational absorbers are utilized.

Keywords: dynamic vibration absorber, beam vibration, vibration reduction

1. Introduction

The primary task of dynamic vibration absorbers [DVA] – mainly the most common passive tuned mass dampers [TMD] – attached to the vibrating structure subjected to harmonic loading is to cease the steady-state oscillations at the point of attachment [1–4]. They are used both for damping of longitudinal and torsional vibration. Many theoretical studies have been devoted to methods of optimal choice of tuned mass dampers parameters for both linear and nonlinear problems [5–17].

Due to the number of possible applications in a wide variety of structures a lot of attention has been directed to the proper selection of TMD parameters in beam structures [20–26]. For continuous systems such as beams, usually the best location of a mass damper is a point of application of the load, but it might be technically impossible. In such situation and in the case of distributed loading, improperly chosen localization may increase the amplitude of vibration in certain areas of the system.

Depending on whether there is considered a local optimization problem – for example, minimization of the amplitude of the structure at a fixed point, or a global optimization problem – for example, minimization of the kinetic energy of the vibrating system, there may be obtained different optimal parameters of the damper and a key issue in global optimization problems is the right location of the damper [24,27].

To improve the efficiency of damping there are used systems of tuned mass dampers, tuned in the most general case for a single or several resonant frequencies for broadband excitation [17–19, 28] [20,22].

In this article a model based on Euler-Bernoulli theory is built for a beam subjected to the distributed and concentrated harmonic excitation forces, equipped with a system of the translational-rotational dynamic vibration absorbers. It is shown that the rotational vibration absorbers used together with the translational ones can significantly improve the effectiveness of vibration isolation.

2. Theoretical model

Figure 1 presents a system considered in the paper – a beam subjected to the distributed loading and p concentrated forces, with r translational-rotational vibration absorbers. The beam is of: length L, mass density ρ, cross-section area A, geometrical moment of inertia I, Young's modulus E.

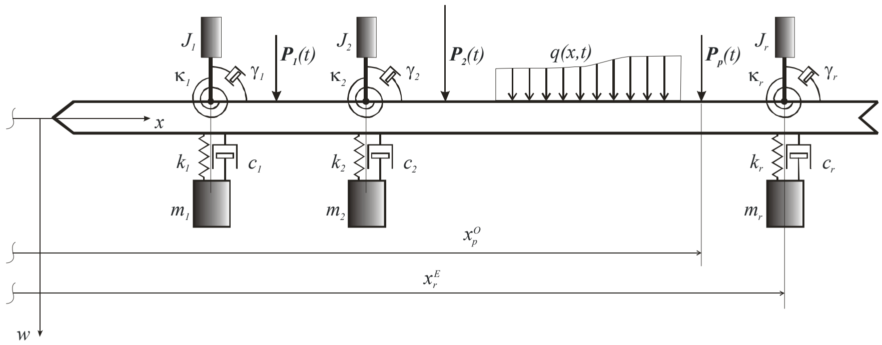

Figure 1. Beam with a system of translational-rotational vibration absorbers

Assuming small, linear vibrations of the Euler-Bernoulli beam with internal damping described by parameter α (Voigt-Kelvin model) the equation of motion takes the form [27]:

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI\alpha \frac{\partial^5 w}{\partial x^4 \partial t} + EI \frac{\partial^4 w}{\partial x^4} = q(x,t) + \sum_{j=1}^{p} P_j(t)\delta(x - x_j^O) + \sum_{j=1}^{r} F_j(t)\delta(x - x_j^E) + \sum_{j=1}^{r} \frac{\partial M_j(t)\delta(x - x_j^E)}{\partial x} \tag{1}$$

where:

$q(x,t)$ –distributed loading;

$P_j(t)$ – j-th concentrated force applied at the point of coordinate x_j^O;

$F_j(t)$ – j-th concentrated force applied from the translational vibration absorber at the location of coordinate x_j^E;

$M_j(t)$ – j-th concentrated torque applied from the rotational vibration absorber at the location of coordinate x_j^E;

m_j, c_j, k_j – mass, damping and stiffness coefficients of the j-th translational vibration absorber;

J_j, γ_j, κ_j – moment of inertia, damping and stiffness coefficients of the j-th rotational vibration absorber;

p – number of concentrated forces; r – number of translational-rotational vibration absorbers.

To solve the equation of motion (1) the method of separation of variables is utilized:

$$w(x,t) = \sum_{i=1}^{\infty} q_i(t)\varphi_i(x) \tag{2}$$

In the above expression $\varphi_i(x)$ are the eigenfunctions of the beam without absorbers attached, which depend on the boundary conditions. The functions of time $q_i(t)$ need to be determined. It is assumed the following form of the distributed loading: $q(x,t) = h(t)g(x)$.

After substitution of the series (2) into equation (1) the time Laplace transformation is performed (with initial conditions equal to zero) and it is obtained:

$$\begin{aligned}\sum_{i=1}^{\infty}[\rho A s^2 Q_i(s) + EI\alpha\beta_i^4 s Q_i(s) + EI\beta_i^4 Q_i(s) - a_i H(s) - \sum_{j=1}^{p} d_{ji} P_j(s) - \\ \sum_{j=1}^{r} b_{ji} F_j(s) - \sum_{j=1}^{r} e_{ji} M_j(s)]\varphi_i(x) = 0\end{aligned} \tag{3}$$

Assuming that the eigenfunctions $\varphi_i(x)$ are orthogonal with the weight function $\eta(x)$, the numerical values of the coefficients in equation (3) can be expressed as:

$$a_i = \frac{\int_0^l g(x)\varphi_i(x)dx}{K_i^2}, \quad d_{ji} = \frac{\varphi_i(x_j^O)}{K_i^2}, \quad b_{ji} = \frac{\varphi_i(x_j^E)}{K_i^2}, \quad e_{ji} = \frac{-\varphi_i'(x_j^E)}{K_i^2} \tag{4}$$

where: $K_i^2 = \int_0^L \eta(x)\varphi_i^2(x)dx$, and additionally $\beta_i^4 = \frac{\rho A}{EI}\omega_i^2$; ω_i is the i-th resonance frequency of the beam without vibration absorbers attached and with the internal damping neglected ($\alpha = 0$). In equation (3) the symbols: $Q_i(s)$, $H(s)$, $P_j(s)$, $F_j(s)$, $M_j(s)$ denote the Laplace transforms of the: $q_i(t)$, $h(t)$, $P_j(t)$, $F_j(t)$, $M_j(t)$ respectively.

Taking into account the linear independence of the eigenfunctions $\varphi_i(x)$ it can be obtained from equation (3) an expression for the Laplace transform $W(x,s)$ of the beam deflection $w(x,t)$:

$$W(x,s)=\sum_{i=1}^{\infty}\frac{a_iH(s)+\sum_{j=1}^{p}d_{ji}P_j(s)+\sum_{j=1}^{r}b_{ji}F_j(s)+\sum_{j=1}^{r}e_{ji}M_j(s)}{\rho As^2+EI(1+\alpha s)\beta_i^4}\varphi_i(x) \tag{5}$$

and the Laplace transform of the beam slope

$$\frac{\partial W(x,s)}{\partial x}=\sum_{i=1}^{\infty}\frac{a_iH(s)+\sum_{j=1}^{p}d_{ji}P_j(s)+\sum_{j=1}^{r}b_{ji}F_j(s)+\sum_{j=1}^{r}e_{ji}M_j(s)}{\rho As^2+EI(1+\alpha s)\beta_i^4}\varphi_i'(x) \tag{6}$$

Transforms of the force $F_j(s)$ and torque $M_j(s)$, transmitted on the beam from the j-th translational-rotational vibration absorber, mounted at the point of coordinate x_j^E, are given by the expressions [27]:

$$F_j(s)=-W(x_j^E,s)\frac{(c_js+k_j)m_js^2}{m_js^2+c_js+k_j} \tag{7}$$

$$M_j(s)=-\Theta(x_j^E,s)\frac{(\gamma_js+\kappa_j)J_js^2}{J_js^2+\gamma_js+\kappa_j} \tag{8}$$

where it is introduced the symbol: $\Theta(x,s)=-\dfrac{\partial W(x,s)}{\partial x}$.

The transforms given by formulas (7) and (8) should be substituted into expressions (5) and (6). The resulting transforms of the line deflection and slope of the beam should be satisfied at the points where the translational-rotational absorbers are attached to the beam. These conditions furnish with the system of linear equations to determine $W(x_k^E,s)$ and $\Theta(x_k^E,s)$ (k = 1,2,..r).

In order to simplify the notation, the following symbols are introduced:

$$W(x_j^E,s)=W_j,\ \Theta(x_j^E,s)=\Theta_j,\ \varphi_i(x_j^E)=\varphi_{ij},\ \varphi_i'(x_j^E)=\varepsilon_{ij},\ a_iH(s)+\sum_{j=1}^{p}d_{ji}P_j(s)=A_i$$

$$\rho As^2+EI(1+\alpha s)\beta_i^4=B_i,\quad \frac{(c_js+k_j)m_js^2}{m_js^2+c_js+k_j}b_{ji}=D_{ji},\quad \frac{(\gamma_js+\kappa_j)J_js^2}{J_js^2+\gamma_js+\kappa_j}e_{ji}=E_{ji} \tag{9}$$

System of $2r$ linear equations for the unknown W_k, Θ_k (k = 1,2,..r) takes the form:

$$\begin{aligned}
&W_k\left[1+\sum_{i=1}^{\infty}D_{ki}\frac{\varphi_{ik}}{B_i}\right]+\sum_{j=1,j\neq k}^{r}\sum_{i=1}^{\infty}W_jD_{ji}\frac{\varphi_{ik}}{B_i}+\sum_{j=1}^{r}\sum_{i=1}^{\infty}\Theta_jE_{ji}\frac{\varphi_{ik}}{B_i}=\sum_{i=1}^{\infty}A_i\frac{\varphi_{ik}}{B_i}\\
&\Theta_k\left[\sum_{i=1}^{\infty}D_{ki}\frac{\varepsilon_{ik}}{B_i}-1\right]+\sum_{j=1,j\neq k}^{r}\sum_{i=1}^{\infty}\Theta_jE_{ji}\frac{\varepsilon_{ik}}{B_i}+\sum_{j=1}^{r}\sum_{i=1}^{\infty}W_jD_{ji}\frac{\varepsilon_{ik}}{B_i}=\sum_{i=1}^{\infty}A_i\frac{\varepsilon_{ik}}{B_i}
\end{aligned} \tag{10}$$

Having solved the system (10) the transforms of the forces $F_j(s)$ and torques $M_j(s)$ may be obtained from expressions (7–8) and utilized then to calculate from formulas (5) and (6) the transforms of the deflection and slope of the beam. Assuming steady state vibration, after substituting $s = j\omega$ ($j = \sqrt{-1}$), it may be obtained the expressions for the deflection and slope of the beam in the frequency domain.

3. Numerical results: tunable translational-rotational vibration absorber – global control problem

It has been built a numerical algorithm which determines in s-domain the transforms of the deflection and slope for arbitrary boundary conditions at the ends of the beam. When harmonic excitation is considered the algorithm allows to obtain the amplitude-frequency characteristics at any cross-section of the beam for the deflection and slope respectively, for the bending moment and transverse force, the time-averaged kinetic energy of the system or its part can also be determined.

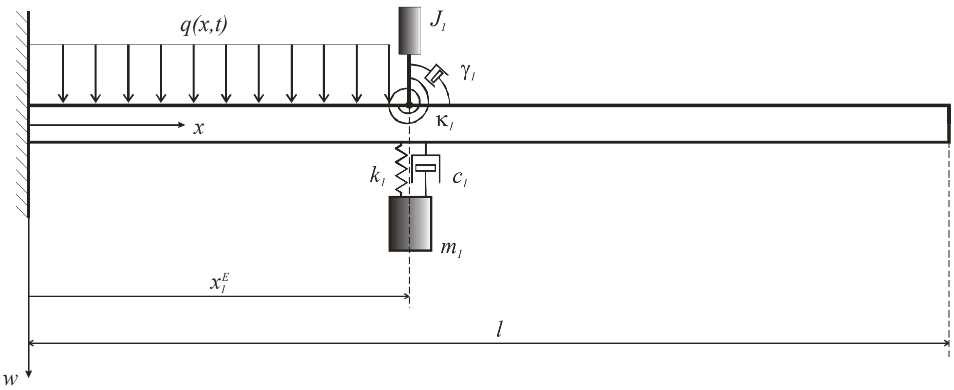

Figure 2. Cantilever beam of length l excited by a uniform distributed harmonic loading with a tunable translational-rotational absorber attached

A cantilever steel beam is considered excited by a uniform distributed harmonic loading in the region: $0 < x < x_1^E$ (Figure 2), with parameters: $l = 1.0\,\mathrm{m}$, $x_1^E = 0.3l$, $E = 2.1 \cdot 10^{11}\ \mathrm{N/m^2}$, $\rho = 7800\,\mathrm{kg/m^3}$. The internal damping of the beam is neglected. The beam has a rectangular cross-section with a width of $b = 0.05\,\mathrm{m}$ and a height of $h = 0.005\,\mathrm{m}$. There is only one translational-rotational absorber of mass $m_1 = 0.1\,\mathrm{kg}$ and moment of inertia $J_1 = 0.0001\,\mathrm{kgm^2}$ attached at $x_1^E = 0.3l$ – the right border of the loading. The aim of the absorber is isolation of vibration transferred from the loaded to the unloaded area of the beam. As a global measure of vibration is used the time-averaged kinetic energy in the unloaded region: $x_1^E < x < l$.

Because a simple control algorithm can be used, from the practical point of view it is preferable to use the tunable dampers [24]. The first four natural frequencies of the presented beam are: f_1 = 4.191 Hz, f_2 = 26.264 Hz, f_3 = 73.541 Hz, f_4 = 144.110 Hz.

The calculated time-averaged kinetic energy of the unloaded region of the beam is shown in Figure 2, for comparison, for three cases:

- only the translational vibration absorber attached to the beam;
- the translational-rotational vibration absorber attached to the beam;
- the beam alone, without any vibration absorber attached.

It is assumed in further calculations that the absorbers attached are tuned so that they are resonant at each single frequency and do not have energy dissipating appliances ($c_1 = 0$, $\gamma_1 = 0$).

It can been seen from the graph in Figure 2 that there is a much improvement in the efficiency of the translational-rotational absorber compared to the translational one – a reduction of the kinetic energy in the range of frequency $\langle 1.0\,\text{Hz},\ 8.0\,\text{Hz}\rangle$ is almost of two orders of magnitude (around eighty times for the frequency equal 8.0 Hz).

The drawback of the tunable absorbers is that they cause an increase in global vibration at the new natural frequencies of the resulting structure which coincide with the excitation frequency. In this case the better performance may be obtained by detuning the absorber [24].

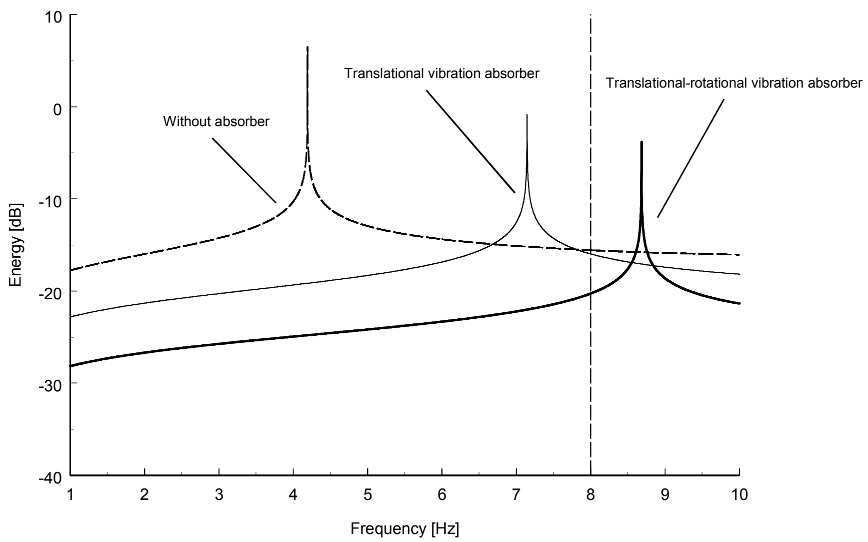

Figure 3. Kinetic energy of the unloaded region of the cantilever beam: without any absorber; with the translational absorber; with the translational-rotational absorber – the absorbers attached are tuned to be resonant at each frequency

4. Conclusions

The computational model presented can be used in local and global problems of optimal choice of position and parameters of the system of translational-rotational vibration absorbers in beams. Theoretical calculations are illustrated by an example of the use of tunable translational-rotational absorber in global control of vibration. The numerical results obtained demonstrate the possible significantly improved effectiveness of the translational-rotational absorber compared to the translational one, due it can absorb and isolate both the translational and rotational motion of the beam.

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