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## MODELLING AND EVALUATION OF DETERIORATION PROCESS WITH MAINTENANCE ACTIVITIES

### MODELOWANIE I ANALIZA PROCESU STARZENIA MASZYN I URZĄDZEŃ PODDANYCH OKRESOWYM REMONTOM\*

*In this paper, we present an approach which allows evaluation of various possible maintenance scenarios with respect to both reliability and economic criteria. The method is based on the concept of a life curve and discounted cost used to study the effect of equipment aging under different maintenance strategies. The deterioration process is first described by a Markov model and then its various characteristics are used to develop the equipment life curve and to quantify other reliability parameters. Based on these data, effects of various “what-if” maintenance scenarios can be examined and their efficiency compared. Simple life curves are combined to model equipment deterioration undergoing diverse maintenance actions, while computing other parameters of the model allows evaluation of additional critical factors, such as the probability of equipment failure. Additionally, the paper deals with the problem of the model adjustment so that the computed repair frequencies are close to the historical values, which is very important in practical applications of the method. Moreover, we discuss the problems which may arise if automatic adjustment is used in cases when the hypothetical maintenance policies go beyond the conditions upon which the original model was built.*

**Keywords:** Deterioration modelling, probabilistic methods, maintenance policy, risk assessment.

*Przedmiotem artykułu jest modelowanie różnych możliwych scenariuszy eksploatacyjnych maszyn i urządzeń, które uwzględnia kryteria zarówno niezawodnościowe, jak i ekonomiczne. Metoda opiera się na zastosowaniu krzywych życia (ang. life curves) oraz kosztów zdyskontowanych (ang. discounted costs) do analizy wpływu, jaki różne strategie eksploatacyjne wywierają na starzenie się sprzętu. Punktem wyjścia jest opisanie procesu starzenia przez model Markowa, którego charakterystyki umożliwiają następnie wyznaczenie kształtu krzywej życia oraz obliczenie innych parametrów niezawodnościowych badanego sprzętu. W oparciu o uzyskane dane możliwa jest ocena różnych hipotetycznych scenariuszy eksploatacyjnych oraz porównanie ich efektywności. Proste krzywe życia mogą być łączone ze sobą w celu wizualizacji starzenia sprzętu poddawanego różnorodnym możliwym czynnościom naprawczym, natomiast obliczenie innych charakterystyk modelu pozwala wyznaczyć dodatkowe ważne parametry, takie jak prawdopodobieństwo uszkodzenia. Dodatkowo artykuł opisuje zagadnienie korygowania parametrów modelu, tak aby obliczane w nim częstości napraw sprzętu były bliskie wartościom znanym z jego historii eksploatacji, co jest bardzo ważne w praktycznych zastosowaniach metody. Omawiamy także problemy mogące pojawić się, gdy algorytm automatycznego korygowania modelu jest stosowany w analizach hipotetycznych strategii eksploatacyjnych wykraczających poza warunki, dla których model oryginalny został opracowany.*

**Słowa kluczowe:** modelowanie procesu starzenia, metoda probabilistyczna, polityka remontowa, ocena ryzyka.

#### 1. Introduction

Selection of an efficient maintenance strategy plays a very important role in the management of today's complex systems. When searching for an optimal strategy, numerous issues must be taken into account and, among them, reliability and economic factors are often equally important. On the one hand, for obvious reasons, in successful system operation failures should be avoided and this opts for extensive and frequent maintenance activities. On the other, superfluous maintenance may result in large and unnecessary costs. Finding a reasonable balance between these two factors is the key point in efficient maintenance management and to facilitate finding such a balance some measures should be available that allow for quantitative evaluation of the deterioration process of a system which is subjected to various maintenance actions (inspections, repairs, replacements, etc.).

The purpose of the development described in this work is to provide a computer tool for evaluating both the risks and the costs associated with the selection of various possible maintenance strategies. Rather than searching for a solution to a problem: “what maintenance

strategy would lead to the best reliability and dependability parameters of the system operation”, in this approach different maintenance scenarios can be examined in the “what-if” type of studies and then, using the tool, their reliability and economic effects can be automatically estimated so that the persons managing the maintenance is assisted in making informed decisions ([13, 34]). The mathematical approach that form the basis of this tool uses semi-Markov model first introduced in 1990 [4] and then improved and extended in [1 – 3, 5 – 6, 10 – 18, 22 – 32].

The method of maintenance evaluation which is the subject of this work has been presented initially in [8] and its specific extensions were further described in [26 – 28]. In this paper, after summarizing the current state of the development in Sections 2 and 3, we discuss one particular problem of automatic adjustment of the model which is required for representation of the deterioration process with modified repair frequencies (Section 4) and, finally, we include an original study of practical application that illustrates potential of this method in real-world situations (Section 5).

(\*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie [www.ein.org.pl](http://www.ein.org.pl)

## 2. Modelling deterioration and maintenance

Probabilistic maintenance models [1 – 8, 11 – 12, 14, 21 – 25, 31] are the preferred tools for quantifying the effect of inspection and maintenance on reliability and costs. Their important advantage, apart from relative simplicity even when applied for complex technical systems or elements, is the ability to incorporate uncertainties associated with the deterioration of equipment and the outcomes of inspection and maintenance. The impact which maintenance makes on performance of the system – on both its reliability parameters and operational effectiveness – can be analysed with various performance measures, including: cost of performing inspection, maintenance and repair [8, 11 – 12, 15, 18, 22 – 23, 25, 31], unavailability (or availability) [2, 11, 25, 31], frequency of failure [15], first passage time (FPT) [18, 21], cost of interruption or cost of lost revenue [15]. As investigated in [32], additional care must be paid when analysing maintenance with non-periodic inspections because classic models may be unable to provide accurate results in such situations. To handle them appropriately new probabilistic models have been proposed in [1, 3].

In the typical approach, maximizing the performance measures becomes the objective of maintenance optimization like, for example in [12, 23], when single objective optimization is aimed at minimization of the operation and maintenance cost. In the more comprehensive solutions, the optimization has two objectives, e.g., to maximize the availability and to minimize the cost [11, 31] or to maximize the FPT while minimizing life cycle cost and unavailability [25]. In the latter solution, the objective function is formulated by assigning different weight factors for FPT, unavailability and life cycle cost.

In general, tuning parameters of the maintenance policy in the search of the optimal configuration can be realized using sensitivity analyses or optimization techniques. For example, modification of the inspection rates is used in [2, 18, 22] where sensitivity analyses were applied to investigate the behaviour of reliability and cost measures. In [11, 25, 31] the task is solved using optimization methods based on simulated annealing algorithm and Markov decision process.

At the heart of the methodology proposed in this paper is the probabilistic model that assumes that the equipment will deteriorate in time and, if not maintained, will eventually fail. If the deterioration process is discovered, preventive maintenance is performed which can restore the condition of the equipment. Such a maintenance activity will return the system to a specific state of deterioration, whereas repair after failure will restore to “as new” condition [5, 17]. The maintenance policy components that must be recognized are: monitoring or inspection (how the equipment state is determined), the decision process (which determines the outcome of the decision), and finally, the maintenance actions (or possible decision outcomes).

### 2.1. Construction of the model

All the necessary assumptions about the aging process and maintenance activities can be incorporated in an appropriate state-space (Markov) model [11, 14, 16, 19, 24, 33]. It consists of the states the equipment can assume in the process, and the possible transitions between them. In a Markov model, the rates associated with the transitions are assumed to be constant in time.

The method described in this work uses a model of the Asset Maintenance Planner (AMP) [6–7]. The AMP model is designed for equipment exposed to deterioration but undergoing maintenance at prescribed times. It computes the probabilities, frequencies and mean durations of the states of such equipment. The basic ideas in the AMP model are the probabilistic representation of the deterioration process through discrete stages, and the provision of a link between deterioration and maintenance. For a structure of a typical AMP model see Fig. 1.

In the model, the deterioration progress is represented by a chain of deterioration states  $D1 \dots DK$  which leads to the state  $F$  symbolizing

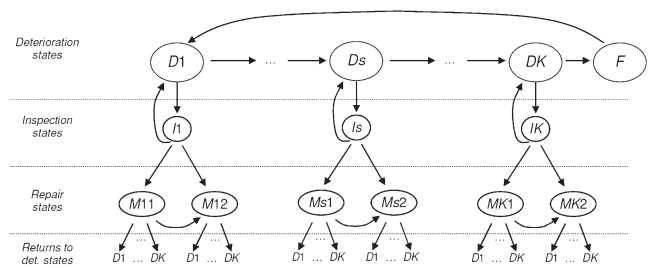


Fig. 1. Structure of the state-transition model which represents the deterioration process together with the inspection and repair events (an example with two types of repairs is shown).

occurrence of a failure. In most situations, it is sufficient to represent deterioration by three stages: an initial ( $D1$ ), a minor ( $D2$ ), and a major ( $D3$ ) stage ( $K = 3$ ). This last is followed, in due time, by equipment failure ( $F$ ) which requires extensive repair or replacement.

In order to slow deterioration and thereby extend equipment lifetime, the operator will carry out maintenance according to some predefined policy. In the model of Fig. 1, regular inspections ( $I_s$ ) are performed which result in decisions to continue with minor ( $Ms1$ ) or major ( $Ms2$ ) maintenance or do nothing (more than two types of repairs can also be included). The expected result of all maintenance activities is a single-step improvement in the deterioration chain; however, allowances are made for cases where no improvement is achieved or even where some damage is done through human error in carrying out the maintenance, which results in returning to the stage of more advanced deterioration.

The choice probabilities (at transitions from inspection) and the probabilities associated with the various possible outcomes are based on user input and can be estimated, e.g., from historical records or operator expertise.

Mathematically, the model in Fig. 1 can be represented by a semi-Markov process, and solved by the well-known procedures. The solution will yield all the state probabilities, frequencies and mean durations. Moreover, the model can be further analysed using Monte Carlo methods: starting from any given state  $Dk$ , transitions of the system are simulated until the failure state  $F$  is reached and the corresponding time moments are recorded as the values of the first passage times (FPT) to failure. These times are subsequently taken as the estimates of the mean remaining lifetimes in each deterioration state. Specific issues arising in numerical implementation of this idea are discussed in [20].

### 2.2. Using the model to estimate the life curve and the probability of failure

A convenient way to represent the deterioration process is by the *life curve* of the equipment [5]. Such a curve shows the relationship between asset condition, expressed in either engineering or financial terms, and time. For examples please refer to Fig. 3 in Section 5 where life curves will be used in a case study presenting various types of analysis carried out for evaluation of the maintenance scenarios.

As pointed out above, computing the average first passage time (FPT) from the first deterioration state ( $D1$ ) to the failure state ( $F$ ) yields an average lifetime of the equipment, i.e., the length of its life curve. On the other hand, solving the model for the state probabilities makes possible computing the expected state durations, which are used to determine the shape of the curve (some additional decisions are required as to how the deterioration states are mapped to the ranges of the asset condition values, which is discussed in [8] and [29]). Simple life curves obtained for different maintenance policies can be later combined if constructing composite life curves which de-

scribe various maintenance scenarios are required (as an example see Fig. 4 in Section 5).

Having the model and the life curve, one can compute the probability of failure (*PoF*) within given time period *T* for the equipment which is in some specific asset condition. The procedure is as follows:

- 1° For the current asset condition (an input parameter), find from the life curve the corresponding deterioration state *D<sub>s</sub>* and then compute a state progress *SP* (%), i.e., estimate how long the equipment has already been in the *D<sub>s</sub>* state (this is calculated with the assumption that the value of asset condition decreases with constant rate when the system remains in *D<sub>s</sub>*, hence simple proportionality rule can be used).
- 2° Running FPT analysis on the model, find the distributions *D<sub>s</sub>(t)* and *D<sub>s+1</sub>(t)* of the first passage time from the current state *D<sub>s</sub>* and the subsequent deterioration state *D(s+1)*, to the failure state *F*.
- 3° Interpreting the state progress as a weight which balances the current equipment condition between *D<sub>s</sub>* and *D(s+1)*, estimate the final value of the probability as:

$$PoF = D_s(T) \cdot (1 - SP) + D_{s+1}(T) \cdot SP \quad (1)$$

### 3. Automatic adjustment of the model

Preparing the Markov model for some specific equipment is not an easy task and requires expert intervention. The goal is to create the model representing closely the real-life deterioration process known from the records that usually describe equipment operation under a regular maintenance policy with some specific frequencies of inspections and repairs. The model itself permits calculation of the repair frequencies and compliance of the computed and recorded frequencies is a very desirable feature that verifies the trustworthiness of the model.

At this point, we will describe briefly a method of model adjustment proposed in [26] and [29] that aims at reaching such a compliance. It can be used also for a different task: fully automatic generation of a model for a new maintenance policy with modified frequencies of repairs which is very often required during the evaluation of various hypothetical maintenance options.

#### 3.1. The method

Let *K* represents the number of deterioration states and *R* the number of repairs in the model under consideration. Also, let *P<sup>sr</sup>* = probability of selecting maintenance *r* in state *s* (assigned to decision after state *I<sub>s</sub>*) and *P<sup>s0</sup>* = probability of returning to state *D<sub>s</sub>* from inspection *I<sub>s</sub>* (situation when no maintenance is scheduled as a result of the inspection). Then, for all states *s* = 1 ... *K*:

$$P^{s0} + \sum_r P^{sr} = 1 \quad (2)$$

Let *F<sup>r</sup>* represents the frequency of repair *r* acquired through solving the model. The problem of model tuning can be formulated as follows:

Given an initial Markov model *M<sub>0</sub>*, constructed as above and producing the frequencies of repairs  $\mathbf{F}_0 = [F_0^1, F_0^2, \dots, F_0^R]$ , adjust the probabilities *P<sup>sr</sup>* so that some goal frequencies *F<sub>G</sub>* are achieved.

Since the model presented in Fig. 1 has many parameters, one could devise different approaches manipulating their values to achieve the desired effect. We have selected to vary the probability values

since these are usually guessed by an expert whereas the repair rates and their durations are largely based on historical records.

The vector *F<sub>G</sub>* usually corresponds to the observed historical values of the frequencies of various repairs but can also represent new hypothetical repair frequencies of some possible maintenance policy. In the proposed solution, a sequence of tuned models *M<sub>0</sub>, M<sub>1</sub>, M<sub>2</sub>, ... M<sub>N</sub>* is evaluated with each consecutive model approximating desired goal with a better accuracy. Starting with *i* = 0 the procedure consists of the following steps:

- 1° For model *M<sub>i</sub>* compute the vector of repair frequencies *F<sub>i</sub>*.
- 2° Evaluate an error of *M<sub>i</sub>* as a distance between vectors *F<sub>G</sub>* and *F<sub>i</sub>*.
- 3° If the error is within the user-defined limit, consider *M<sub>i</sub>* as the final model and stop the procedure (*N* = *i*); otherwise proceed to the next step.
- 4° Create a new model *M<sub>i+1</sub>* through tuning values of *P<sub>i</sub><sup>sr</sup>*, then correct *P<sub>i</sub><sup>s0</sup>* according to (2).
- 5° Proceed to step 1° with the next iteration.

#### 3.2. Approximating model probabilities

Of all the steps outlined in the previous section, it is clear that tuning the probabilities *P<sub>i</sub><sup>sr</sup>* in step 4° is the heart of the whole procedure.

In general, the probabilities represent *K*·*R* free parameters and their uncontrolled modification could lead to a serious deformation of the model. To avoid this, a restrictive assumption is made: if the probability of some particular maintenance must be modified, it is modified proportionally in all deterioration states, so that at all times

$$P_0^{1r} : P_0^{2r} : \dots : P_0^{Kr} \sim P_i^{1r} : P_i^{2r} : \dots : P_i^{Kr} \quad (3)$$

for all repairs (*r* = 1 ... *R*).

This assumption also significantly reduces dimensionality of the problem, as now only *R* scaling factors  $\mathbf{X}_{i+1} = [X_{i+1}^1, X_{i+1}^2, \dots, X_{i+1}^R]$  must be found to get all new probabilities for the model *M<sub>i+1</sub>*:

$$P_{i+1}^{sr} = X_{i+1}^r \cdot P_0^{sr}, \quad r = 1 \dots R, \quad s = 1 \dots K \quad (4)$$

Moreover, although the frequency of a repair *r* depends on the probabilities of all repairs (modifying probability of one repair changes, among others, state durations in the whole model; thus, it changes the frequency of all states) it can be assumed that, in a case of a single-step small adjustment, its dependence on repairs other than *r* can be considered negligible and

$$F_i^r = F_i^r (X_i^1, X_i^2, \dots, X_i^R) \approx F_i^r (X_i^r) \quad (5)$$

With these assumptions, generation of a new model is reduced to the problem of solving *R* non-linear equations in the form of

$F_i^r (X_i^r) = F_G^r$ . This can be accomplished with one of the standard root-finding algorithms.

One point of the procedure requires additional attention, though: applying equation (4) with *X<sub>i+1</sub><sup>r</sup>* > 1 may violate the condition

$$\sum_{r=1}^R P_{i+1}^{sr} \leq 1 \quad (6)$$



in some deterioration state  $s$ . This situation needs special tests that would detect such illegal probability values and reduce them proportionally so that their sum does not exceed 1: a so called *scale-down transformation* needs to be applied. As practical studies show, such conditions do occur during model tuning towards repair frequencies that are remarkably higher than  $F_0^r$  from the initial model  $M_0$ . In its simplest form, the scale-down operation consists in dividing each probability  $P^{sr}$  in the offending state  $s$  by the sum of all repair probabilities in this state:

$$P^{sr} = P^{sr} / S_{Ds}, \quad S_{Ds} = \sum_{r=1}^R P^{sr} \quad (7)$$

This will also imply that  $P^{s0} = 0$  which means that every inspection ends with some repair and there are no direct returns from  $I_s$  state to  $D_s$ . Moreover, this obligatory correction mechanism can result in a violation of the proportionality rule (3), as an inevitable side effect.

### 3.3. Numerical implementation

The following three approximation algorithms were implemented in the task of solving equation (5): Newton method working on a linear approximation of  $F^r$  functions (the NOLA method), the secant method and the false position (*falsi*) method. For their detailed presentation please refer to [26] and [29].

Generally, if the scale-down operation (7) does not disturb the iteration flow, any of the approximation algorithms can arrive at the requested goal frequencies with just a few steps, even if imposed precision margins are very narrow. The iterative scheme is very efficient with regard to this aspect. Moreover, practical tests have shown that although simplifications of the NOLA solution may seem critical, it is reasonably efficient and stable in the real-world cases because it has one advantage over its more sophisticated rivals: since it does not depend on previous approximations, selection of the starting point is not so important and the accuracy during the first iterations is often better than in the secant or *falsi* methods. Superiority of the latter methods, especially of the *falsi* algorithm, manifests itself in the later stages of the approximation when the potential problems with initial selection of the starting point have been diminished.

## 4. Correction of the adjustment procedure for saturated models

Adjusting the model to the repair frequencies that are substantially higher than the original ones may lead to the *model saturation* – a condition in which repair probabilities reach the limit (6) in every state  $D_s$  and there is no room for further increase if the adjustment procedure is limited only to the simple probability scaling as expressed in equation (4). In this situation, bringing together the two requirements: tuning the model towards high repair frequencies and, at the same time, keeping the modifications of the internal structure within a safe range that does not break proper relation with the original, is a challenge and is discussed in this section.

### 4.1. The problem

For practical illustration of the problem we will use two real-world Markov models, A and B that are especially prone to probability saturation. Both models have the same general structure with  $K = R = 3$ , i.e., they include three deterioration states ( $D1 \div D3$ ) and three repairs: minor (index = 1), medium (2) and major (3). The main difference between them lies in the distributions of the repair probabilities  $P^{sr}$  in the deterioration states (or, strictly speaking, in inspection states  $I1 \div I3$  associated with the deterioration states, as in Fig. 1).

The model A has been created with an assumption that although there are no repairs in the first state  $D1$ , when the equipment is in subsequent states  $D2$  and  $D3$  every inspection leads to some sort of repair and the totals  $S_{D2} = S_{D3} = 1$  ( $P^{20} = P^{30} = 0$ ). Actual probability distribution in each state is chosen so that in the medium deterioration state  $D2$  the minor repair is the most common ( $P^{21} = 0.80$ ) while in the major deterioration  $D3$  the distribution is more balanced with medium repair taking half of the chances ( $P^{32} = 0.50$ ).

The model B is a sibling of A with just one difference: repair probabilities in  $D2$  and  $D3$  are lowered by, respectively, 20% and 10%, which means that after inspections  $I2$  and  $I3$  it is possible to return to  $D_s$  without undertaking any repair ( $P^{20} = 0.2$  and  $P^{30} = 0.1$ ). From the point of view of the current discussion, model B, as opposite to model A, has more potential for the probability growth.

In the following analysis, a series of models for the goal frequencies will be generated in cases A and B

$$F_G = [\alpha \cdot F_0^1, F_0^2, F_0^3]$$

with factor  $\alpha$  increasing from 0.5 (frequency of the minor repair reduced by half) to 2.0 (minor repair performed twice as often) in steps of 0.1. Values of  $\alpha$  will be expressed as %. Frequency of the minor repair (no. 1) was selected as the varying parameter in  $F_G$  just as an example with frequencies of the other repairs remaining constant, but equivalent results could be demonstrated with changing the frequencies of medium or major repairs.

As it was discussed with greater detail in [30], both models can be successfully adjusted only up to the point of saturation which is reached for  $\alpha = 100\%$  for model A (i.e., the initial model is already saturated) and 130% for model B. As it turns out, in this particular case the values  $P^{20} = 0.2$  and  $P^{30} = 0.1$  in model B leave enough room for approximately 30% increase of  $F^1$ . In both saturation situations probabilities in the states  $D2$  and  $D3$  sum up to unity and cannot be further increased, while in  $D1$  the  $P^{11}$  is zero and applying the scaling factor as in equation (4) cannot produce any increase. On the other hand, the procedure has no problems with an adjustment towards frequencies lower than the saturated ones and, in such cases, the probabilities are scaled accordingly.

### 4.2. Modification of the adjustment procedure

The above examples of unsuccessful tuning can be used for illustration of the proposed extension to the algorithm: if the model gets saturated after some adjustment iteration but there is still a state with null repair probability, the process can be continued in the same iterative way after some non-zero probability is added in this state. Such modification, though, goes beyond the restrictive assumption expressed by equation (3) and, being a more serious invasion into the model structure, must be applied in a cautious and thoughtful manner.

In particular, the following two issues must be taken into account: (1) forcing non-zero probability in some state before it is not absolutely necessary, i.e., prior to the model saturation, instantly changes reaction to the adjustment iterations; hence, may change the final result of the tuning also in cases when the standard procedure would be able to produce the correct result; (2) replacing the null value of  $P^{sr}$ , even if delayed up to the moment of saturation, but with probability which is too high for the actual needs, also may affect the final result in a way that is against the general idea of conservative tuning which should try to preserve the structure of the original model with minimal possible modifications. Consequences of the improper modifications that violate the above rules were presented in [30].

After analyses of numerous case studies like the above two examples, the following modification of the adjustment procedure has been found to be the most flexible and efficient solution that gives optimal results in a broad range of practical cases. Its main idea is not only

to delay the increase of null probability until the moment of model saturation, but also to scale its value adequately.

The modification does not amend the general iterative scheme defined in point 3.1 in steps 1° ÷ 5°; the changes are limited only to the internal details of step 4°, which computes new probability values for the next model  $M_{i+1}$ . The modified implementation of this operation detects and deals differently with the following two cases:

- If the model is not saturated, i.e., there is a state with  $0 < S_{D_s} < 1$ , the standard approach is applied: in all states the values of  $P^{sr}$  are multiplied by the scaling factors  $X^r$  (equation (4)) and then, if required, they are scaled down as in equation (7).
- If the model is saturated but there is a state with  $P^{sr} = 0$  (a chance for probability increase), this particular null probability is replaced with a predicted average increase of  $P^{sr}$  in other states computed by the regular method as described above; after this the model is no longer saturated and the iterative scaling of this probability can be continued with the standard algorithm.

It should be noted that in case (b) the new value that replaces the null probability is computed as an average of the predicted *actual* increases of probabilities for a given repair in other states: these increases will be scaled down with equation (7) because these states, by virtue of the method, will be saturated. As a result, the applied value of the increase will be proportional to the needs of particular situation but, at the same time, it will be additionally constrained.

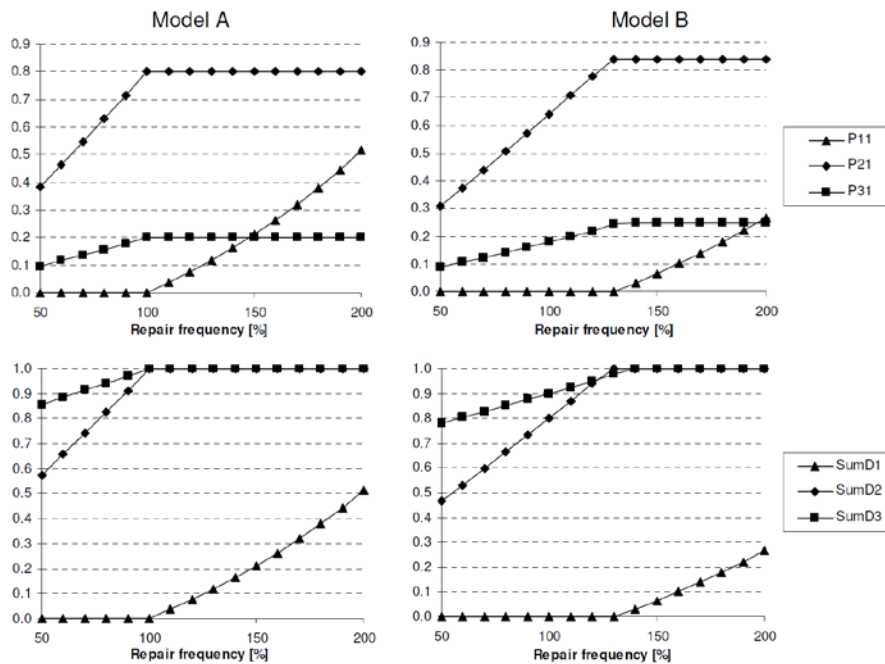


Fig. 2. Successful tuning of the models A (left) and B (right) beyond the point of model saturation by the proposed extension of the adjustment procedure.

Figure 2 presents the results obtained after application of this extended procedure to the models A and B: the upper graphs shows probabilities of the minor repair in all three deterioration states ( $P^{11}$ ) and the lower graphs – sum of all repair probabilities in every state ( $S_{D_s}$ ). For both models, the adjustment can be successfully completed beyond the point of saturation, i.e., up to the doubled frequency of the minor repair, while for goal frequencies without model saturation ( $\alpha < 100\%$  for model A and  $\alpha < 130\%$  for model B) the results are identical to the outcomes of the standard (unmodified) procedure.

Moreover, the graphs unveil the actual mechanism of model adjustment. Before saturation  $P^{11} = S_{D_1} = 0$  and scaling only  $P^{21}$  and  $P^{31}$  is enough for reaching the goal frequencies. At the point when this becomes insufficient ( $\alpha = 110\%$  and  $\alpha = 140\%$ ) the null values of  $P^{11}$

are increased and further growth is limited to the  $D_1$  state with the other two remaining saturated.

## 5. Evaluating reliability and cost for different maintenance strategies

The methodology presented in the two previous sections will be now illustrated by a practical example of maintenance evaluation. The example is based on a real-world piece of equipment with a model created and fine-tuned so that it represents the actual reliability and maintenance parameters found in the historical records. According to them, the average equipment life has been found to reach 18.7 years of operation before failure. The model includes three deterioration states and represents the default maintenance policy with three possible repair types corresponding to, respectively, minor, medium and major repairs.

### 5.1. Life curves

Fig. 3 presents life curves computed for this equipment with various repair policies. The rightmost one represents the standard (historical) policy with all three repairs implemented with their typical frequencies, while the leftmost one – corresponding to the average equipment life of approx. 10 years – has been created from the model with all repairs removed (so called “do nothing” policy). As it is shown, in this specific case, turning off all the maintenance actions results in shortening of the equipment life by 46% and this fact can be compared to expected economic savings. The other three curves represent the following mid-range scenarios which were selected in this work as typical examples of the solutions that may be considered in the real-world applications:

- turning off the major repair without changing the frequencies of the remaining two ones (minor and medium), which has been evaluated to reduce the average equipment life to 14.7 years (i.e. by 21%),
- keeping only the medium repair with minor and major ones removed (equipment life reduced by 28% to 13.4 years),
- reducing by half the frequencies of all three repairs (equipment life reduced by 40% to 11.3 years).

It should be stressed that in the three mid-range cases the curves have been computed using models that were tuned to required repair frequencies with the numerical procedure described earlier in this paper.

Having such models not only the shape and length of the curves can be evaluated, but also other significant reliability characteristics, with the probability of failure within the specific time horizon being one of the most important in further analysis.

### 5.2. Maintenance scenarios

The models and the life curves for different repair policies can be used for evaluation of various maintenance scenarios. As examples, we will consider a situation when, with an initial equipment deterioration estimated as 80% of “as new” condition, some specific actions – a repair or just a change in maintenance policy – will take place after a 3 year delay while the effects will be evaluated for a 10 year time period. The actions in the scenarios will be as follows:

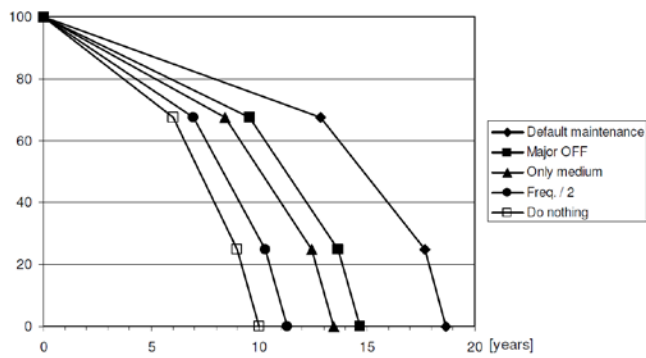


Fig. 3. Life curves for equipment with different repair policies

- adopting “do nothing” policy, which means just stopping all inspections and repairs; in case of failure the equipment will be refurbished and its condition restored to 85%,
- replacing the equipment with “as new” one and then switching to the “do nothing” policy,
- performing a major refurbishment of the equipment which restores its condition to 85% and then continuing with a medium repair only.

Fig. 4 shows the composite life curves created over a period of 10 years for the above scenarios and compares them to the “continue as before” policy. The composite curves were constructed with the appropriate segments of the basic curves from Fig. 3. Starting from the initial asset condition of 80% of the initial asset value, which corresponds to the equipment ca. 8 year old, the curves run down to 72% during the first three years and then split at the moment of the action. For the “do nothing” action deterioration rate speeds up, while for the two other actions the asset condition is first increased as a result of the replacement or refurbishment and, then, a new reduced repair policy is applied, which again causes a higher rate of deterioration. The shapes of the curves make possible a quantitative comparison of these processes and allow evaluation of their effects.

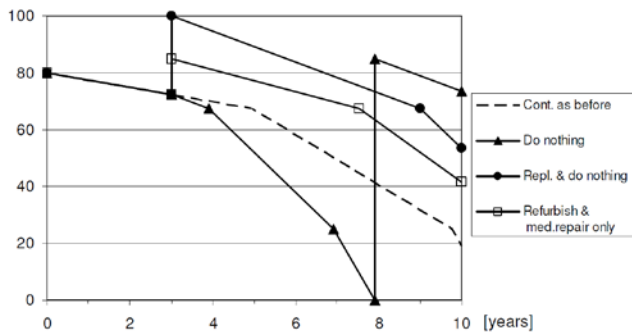


Fig. 4. Life curves for different maintenance scenarios over a time horizon of 10 years

It can be noted that, in the case of “do nothing” action, it is predicted that the equipment will fail within the time horizon under consideration. While in such a case, different actions (repairs or replacements) may take place, in this specific scenario it is assumed that the equipment will be repaired with its condition restored to 85%, but other courses of action can also be modelled.

### 5.3. Probability of failure

Probability of failure within the time horizon computed for the strategies under consideration is shown in Fig. 5. Values on the graphs are presented as functions of the action delay time (100% = 3 years)

and they are compared against the probability of failure for the unmodified standard maintenance (“continue as before”). The value of this probability has been computed to be 42%.

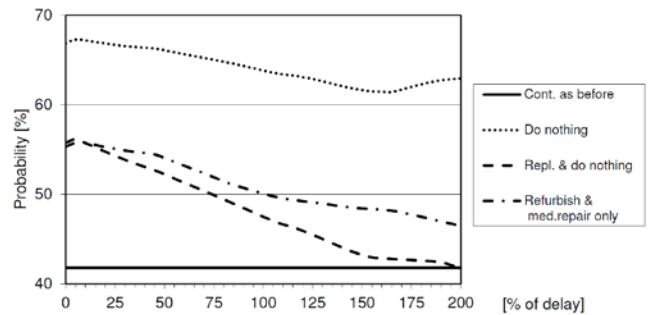


Fig. 5. Probability of equipment failure within a period of 10 years as a function of action delay

It can be seen in case of all three scenarios that, since the new maintenance policy after the action is more or less reduced, the more the action is delayed, the less probable equipment failure becomes. For evident reasons adopting “do nothing” policy leads to the highest values of the failure probability, while replacing the equipment and “doing nothing” afterwards turned out to be a less dangerous strategy (in terms of failure probability) than refurbishing and then keeping only the medium repair. Whether the differences in the economic expenses of these two possible strategies justify this discrepancy in the reliability parameter or not – remains an open question and generally depends on the costs associated with the equipment failures.

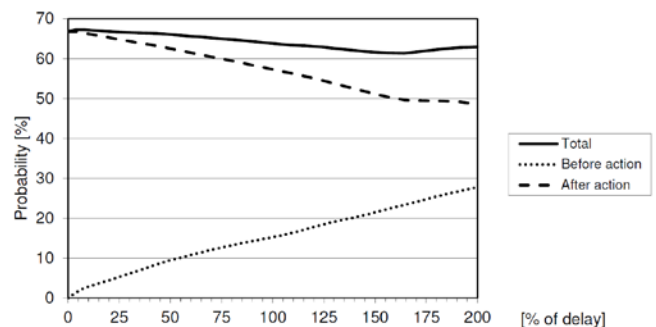


Fig. 6. Probability of equipment failure before and after the action for “do nothing” scenario

One interesting observation can be made about the curve for “do nothing” strategy: its decrease is not strictly monotonic and there is a local minimum at the level of 61% for the delay equal to 164% (4.9 years) after which the probability begins to rise slowly. To explain this rise, the two components: the probability of failure before and after the action should be investigated and they are shown in Fig. 6. In general, these two components behave as expected: the later the action takes place, the higher the probability of failure before and the lower probability of failure after the action but the rates of these two flows – increasing and decreasing – are not constant and do not sum up into a monotonic decrease. In this case, the probability of failure after the action falls down to some extent slower after the point of 164% and this causes the local minimum in the total probability of failure.

### 5.4. Cost analysis

In financial evaluations, the costs are expressed as the present value (PV) quantities and this approach should also be used in this kind of studies because maintenance decisions on aging equipment include

timing, and the time value of money is an important consideration in any decision analysis. The cost difference is often referred to as the Net Present Value (NPV). In the case of maintenance, the NPV can be obtained for several re-investment options which are compared with the “Continue as before” policy.

Cost evaluation for any maintenance scenario involves calculation of the following three fundamental classes of components:

1. cost of the maintenance activities,
2. cost of the selected action (i.e., refurbishment or replacement),
3. cost associated with failures (cost of repairs, system cost, penalties).

To compute the PV, inflation and discount rates are required for the specified time horizon. The cost of maintenance over the time horizon is the sum of the maintenance costs incurred by the original maintenance policy for the duration of the delay period (up to the action), and the costs incurred by the new policy for the remainder of the time horizon (after the action). The costs associated with the equipment failure over the time horizon can be computed similarly except that the failure costs before and after the action should be multiplied by the respective probabilities of failures, and the two products added.

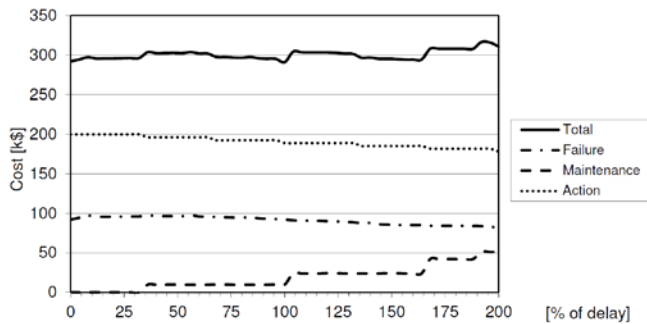


Fig. 7. Estimated cost of “replace & do nothing” scenario (the total value and the three components)

Fig. 7 presents the plots showing the cost analysis for the exemplary scenario “replace and then do nothing”. Again (as it was in the case of the probability of failure) the values are visualized as functions of the action delay varying in the range  $0 \div 200\%$  of the user-specified reference value. In this particular case, this value was 3 years and the

costs correspond to the estimated expenses over the period of 10 years (i.e., for 150% delay one can read the costs incurred over the period of 10 years evaluated for situation when the replacement was delayed for 4.5 years).

The “Maintenance” component that can be seen in this figure includes inspection and repair costs that were incorporated in the model and, since in this scenario there is no maintenance after the replacement, for 0% delay (the action done immediately) all the maintenance activities are suspended from the start of the time horizon and the value of this component falls to zero. Only after delaying the action by 35% the first repair is expected to be performed (incurring some non-zero cost) while the further increase of this delay causes more and more repairs to take place – hence several noticeable jumps appear in the flow of this curve. As for the cost of the replacement itself (“Action”), although it does not depend on the delay, is not constant due to the PV calculations. Also cost of the failure (loss of equipment, penalties, loss of revenue, repair cost, etc.) although assumed to be constant for each specific scenario, in this analysis fluctuates due to changes in probabilities of failure (estimated separately for the periods before and after the action) and, to a lesser extent, also due to the PV calculations.

## 6. Conclusions

The purpose of the method presented in this paper is to help the maintenance supervisor in choosing an effective yet cost-efficient maintenance policy. Based on the Markov models representing deterioration process, the equipment life curve and other reliability parameters can be evaluated. Once a database of equipment models is prepared, the end-user can perform various studies with different maintenance strategies and compare expected outcomes. As the results are visualized through the relatively simple concept of a life curve, no detailed expert knowledge about internal reliability parameters or configuration is required.

Additionally, we have presented a method for automatic adjustment of a given deterioration model to the requested new repair frequencies. Such a task arises often either in fine-tuning of the model to historical records of equipment operation or during analyses of the possible hypothetical maintenance options. The proposed adjustment method strives to be as conservative as possible with regard to the amount of alterations introduced to the existing model in order to avoid its deformation and, consequently, corruption of the produced results.

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