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Theoretical and experimental research of supersonic missile ballistics

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Abstract. A mathematical-physical model of a supersonic missile was built, taking into account the mass and inertia properties. The model was implemented in the MathCAD14 simulation program. A numerical analysis of the missile ballistics was conducted and basic parameters were determined: range, altitude, velocity and acceleration, which enabled a test range program for manufactured missile models to be drawn up. Initial flight tests of missiles were carried out at the test range. Satisfactory accordance of experimental and theoretical dynamic parameters of the missile under study was obtained.

Key words: missile exterior ballistics, mathematical model, computer simulation, test range experiment.

Nomenclature

a, (g)	– Acceleration, (of gravity),
m	– Missile mass,
V	– Missile velocity,
ρ	– Air density,
F	– Thrust of a rocket motor,
c_x	– Aerodynamic coefficient,
α	– Angle of attack in elevation,
β	– Azimuth plane angle,
γ, κ	– Angle of inclination and deflection,
μ_a	– Roll angle,
p, q, r	– Velocity along *xyz* axis,
I_x, I_y, I_z	– Moment of inertia with respect to *xyz* axis,
L, M, N	– Roll, pitch and yaw torque of a missile,
Ψ, Φ, Θ	– Roll, pitch and yaw angle,
P_x, P_y, P_z	– Drag force, side force and aerodynamic lift.

1. Introduction

During the design process of supersonic missiles, computer programs are employed to determine initial ballistic parameters (velocity, acceleration, range) and to optimize the missile design. One of the more popular versatile programs for predicting the ballistic properties of fast objects is PRODAS [1], a licensed product of Arrow Tech Inc., USA. Although many commercial programs for the flight simulation of supersonic objects already exist, new programs are developed which take into account the dynamic aspects of various geometrical shapes of missiles and deal with the problem of aerodynamic stability [2, 3].

Drawing on the long-term experience of the Military University of Technology in designing ammunition and researching the external ballistics of missiles [4–8], the authors have developed their own original computer programme for the evaluation of supersonic missile ballistic parameters. To enable ballistic property analysis, a mathematical-physical model of a supersonic missile was built taking into account the mass and inertia properties. It was subsequently implemented in the MathCAD14 simulation program [9]. A numerical analysis of the missile ballistics was obtained, which enabled a test range program for the produced missile models to be drawn up.

The paper also presents the results of initial tests of the missile dynamics in test range conditions. The experimental flight parameters obtained at the test range served to verify the correctness of the computer simulation results, which is to be regarded as the verification of the constructed mathematical-physical model of the missile.

2. Physical model of the missile

The development process of the physical model of the missile took into account the flying object features, which significantly affect the analysed phenomenon. For the development of the physical model of the missile, it was first of all necessary to:

- assume the coordinate systems necessary to describe the missile movement and the forces affecting it;
- define the missile structure, that is, its
  - geometrical characteristics,
  - mass-inertia characteristics.

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- determine the components of external forces and the moments of those forces acting on the missile along with the functions which characterize them, that is,
  - gravity force and its moment,
  - aerodynamic force and its moment,
  - Coriolis force and its moment.
- determine the characteristics of the object's centre of motion, i.e. density, viscosity, temperature and pressure depending on the flight altitude.

A general view of the missile under study is presented in Fig. 1. Table 1 shows the basic geometrical and mass-inertia characteristics of the missile.

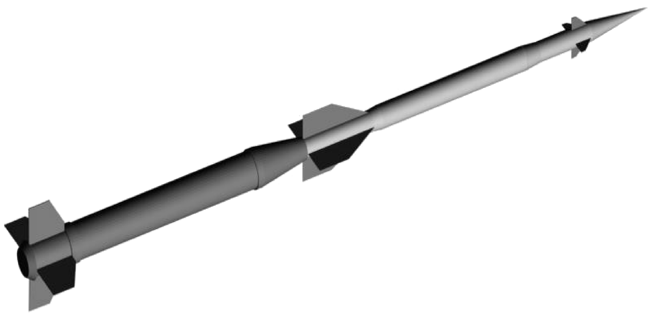

Fig. 1. General view of the missile under study

Table 1
Chosen characteristics of the missile

Missile dimensions and mass	Value
Initial mass M_0 [kg]	69.0
Total length L_p [m]	3.40
Diameter of first stage d_I [m]	0.170
Diameter of second stage d_{II} [m]	0.091
Area of max. cross-section S_{II} [m^2]	0.0023
Span of fins L_{st} [m]	0.40

3. Mathematical model of the missile

The equations which make up the mathematical model of the missile have been derived from the impulse-momentum change theorems. The scalar equations describing the total spatial motion of the missile are provided below.

- scalar dynamic equations of a missile linear motion:

$$m \cdot \frac{dV}{dt} = F \cdot \cos\alpha \cdot \cos\beta - P_x - m \cdot g \cdot \sin\gamma, \quad (1)$$

$$\begin{gathered} m \cdot V \cdot \cos\gamma \cdot \frac{d\kappa}{dt} \\ = F \cdot (\sin\alpha \cdot \sin\mu_a + \cos\alpha \cdot \sin\beta \cdot \cos\mu_a) \\ + P_y \cdot \cos\mu_a + P_z \cdot \sin\mu_a, \end{gathered} \quad (2)$$

$$\begin{gathered} -m \cdot V \cdot \frac{d\gamma}{dt} \\ = -F \cdot (\cos\alpha \cdot \sin\beta \cdot \sin\mu_a + \sin\alpha \cdot \cos\mu_a) \\ + P_y \cdot \sin\mu_a - P_z \cdot \cos\mu_a + m \cdot g \cdot \cos\gamma, \end{gathered} \quad (3)$$

- scalar kinematic equations of a missile linear motion:

$$\frac{dx_g}{dt} = V \cdot \cos\gamma \cdot \cos\kappa, \quad (4)$$

$$\frac{dy_g}{dt} = V \cdot \cos\gamma \cdot \sin\kappa, \quad (5)$$

$$\frac{dz_g}{dt} = -V \cdot \sin\gamma, \quad (6)$$

- scalar dynamic equations of a missile rotational motion:

$$I_x \cdot \frac{dp}{dt} - (I_y - I_z) \cdot q \cdot r = L, \quad (7)$$

$$I_y \cdot \frac{dq}{dt} - (I_z - I_x) \cdot p \cdot r = M, \quad (8)$$

$$I_z \cdot \frac{dr}{dt} - (I_x - I_y) \cdot p \cdot q = N, \quad (9)$$

- scalar kinematic equations of a missile rotational motion:

$$\frac{d\Psi}{dt} = (q \cdot \sin\Phi + r \cdot \cos\Phi) \cdot \sec\Theta, \quad (10)$$

$$\frac{d\Phi}{dt} = p + \tan\Theta \cdot (q \cdot \sin\Phi + r \cdot \cos\Phi), \quad (11)$$

$$\frac{d\Theta}{dt} = q \cdot \cos\Phi - r \cdot \sin\Phi. \quad (12)$$

The system of differential equations should be supplemented with geometrical relations between the employed coordinate systems, the additional equations which describe the aerodynamic coefficients of aerodynamic forces and momentums, and the functions of the rocket motor mass and thrust changes in time.

Aerodynamic characteristics of anti-aircraft missile consist of aerodynamic forces and momentum coefficients, which influence on a missile moving in the air.

Described in relations (1)–(3) and (7)–(9) components of aerodynamic forces and aerodynamic forces momentum can be expressed by following formulas:

$$P_x = C_x \frac{\rho V^2}{2} S \quad \text{– drag force}, \quad (13)$$

$$P_y = C_y \frac{\rho V^2}{2} S \quad \text{– side force}, \quad (14)$$

$$P_z = C_z \frac{\rho V^2}{2} S \quad \text{– normal force}, \quad (15)$$

$$L^A = C_l^A \frac{\rho V^2}{2} S l_x \quad \text{– roll momentum}, \quad (16)$$

$$M^A = C_m^A \frac{\rho V^2}{2} S l_y \quad \text{– pitch momentum}, \quad (17)$$

$$N^A = C_n^A \frac{\rho V^2}{2} S l_z \quad \text{– yaw momentum}, \quad (18)$$

$l_{x,y,z}$ – characteristic parameter for building of roll, pitch and yaw momentum, correspondingly.

Existing in relationships (13)–(18) coefficients of aerodynamic forces and momentum C_x, C_y, C_z, C_l^A, C_m^A, C_n^A, are called aerodynamic characteristics of a missile. Formulas determining above mentioned coefficients are presented in relationships (19)–(24).

$$C_x = C_{x0}(Ma, Re) + C_{x\alpha^2}(Ma) \cdot \alpha^2 + C_{x\beta^2}(Ma) \cdot \beta^2, \tag{19}$$

$$C_y = C_{y0}(Ma) + C_{y\beta}(Ma) \cdot \beta, \tag{20}$$

$$C_z = C_{z0}(Ma) + C_{z\alpha}(Ma) \cdot \alpha, \tag{21}$$

$$C_l^A = C_{l0}^A(Ma) + C_{lp}^A(Ma) \cdot \overline{p}, \tag{22}$$

$$C_m^A = C_{m0}^A(Ma) + C_{m\alpha}^A(Ma) \cdot \alpha + C_{mq}^A(Ma) \cdot \overline{q}, \tag{23}$$

$$C_n^A = C_{n0}^A(Ma) + C_{n\beta}^A(Ma) \cdot \beta + C_{nr}^A(Ma) \cdot \overline{r}, \tag{24}$$

where $C_{x0}(Ma)$ – drag force coefficient for the angle $\alpha = \beta = 0$, $C_{x\alpha^2}(Ma)$ – induced drag coefficient for the angle $\alpha \neq 0$, $C_{x\beta^2}(Ma)$ – induced drag coefficient for the angle $\beta \neq 0$, $C_{z\alpha}(Ma)$ – normal force coefficient, $C_{y\beta}(Ma)$ – side force coefficient, $C_{l0}^A(Ma)$ – stabilizing roll moment coefficient, $C_{lp}^A(Ma)$ – dumping roll moment coefficient, $C_{m\alpha}^A(Ma)$ – stabilizing pitch moment coefficient, $C_{mq}^A(Ma)$ – dumping pitch moment coefficient, $C_{n\beta}^A(Ma)$ – stabilizing yaw moment coefficient, $C_{nr}^A(Ma)$ – dumping yaw moment coefficient.

Aerodynamic coefficients can be determined by both theoretical or experimental method. In the paper those characteristics of the missile under study were determined theoretically.

4. Numerical analysis of basic dynamic characteristics of missile

The constructed mathematical-physical model of the missile was implemented in the MathCAD14 calculation package. By applying the developed algorithm and the simulation software, the selected kinematic characteristics of the missile were determined. Figures 2, 3, and 4 feature diagrams of the basic kinematic characteristics (velocity, range, altitude) of the missile fired from the launcher at various angles.

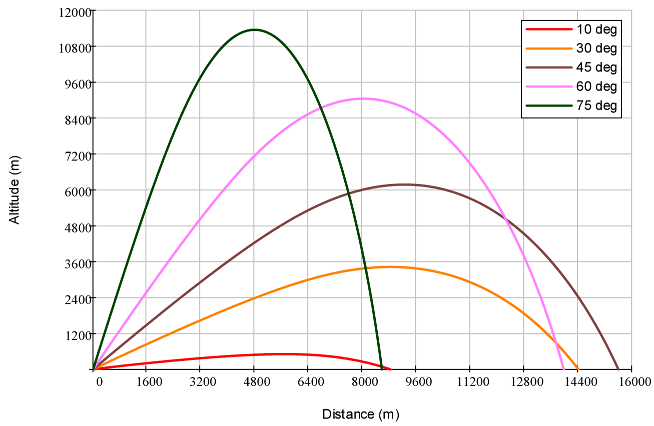

Fig. 2. Trajectory in elevation plane at varied launch angles

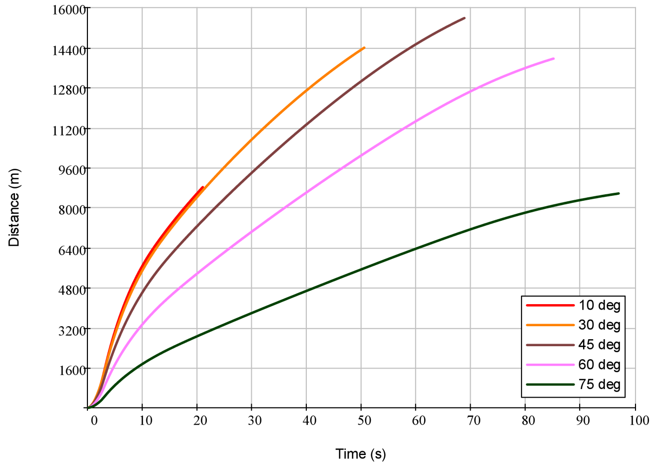

Fig. 3. Trajectory in azimuth plane at varied launch angles

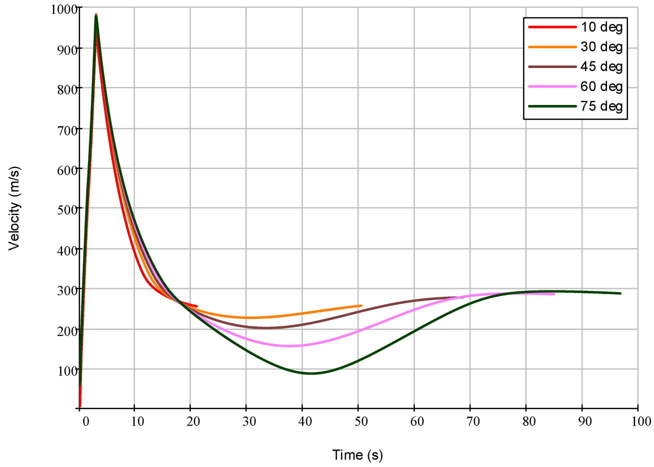

Fig. 4. Changes in missile velocity with time for varied launch angles

Numerical simulations of such dynamic characteristics of the missile flight as the drag coefficient (c_x) and the drag force (P_x) versus flight time and missile velocity expressed in Mach units (V_M) were also carried out. Those relations are presented in Sec. 5, where they are compared with the results obtained in experimental tests given a missile launch angle equal to 10°.

5. Experimental part

The dynamic tests were run at the naval test range. The missile was fired from a tube launcher set on a platform, enabling adjustment of the launcher elevation angle. Figure 5 shows the missile launching moment from the launcher inclined by 10°. A Weibel Doppler Radar with the SL 30031 antenna installed in the axis of the expected flight trajectory was used to measure the missile velocity. Based on data registered by the Doppler radar, the dependence of the missile velocity on time, $V = f(t)$, was determined (Fig. 6). The plot of velocity versus time, obtained from the computer simulation, was drawn in the same diagram. At a launching angle of 10°, the maximum flight altitude was 600 m, which, given a range of about 9 km, can be considered to be an approximately straight flight path.

The quantitative and qualitative coincidence of both plots in Fig. 6 is fully satisfactory.

Fig. 5. View of a missile launched at a 10° angle

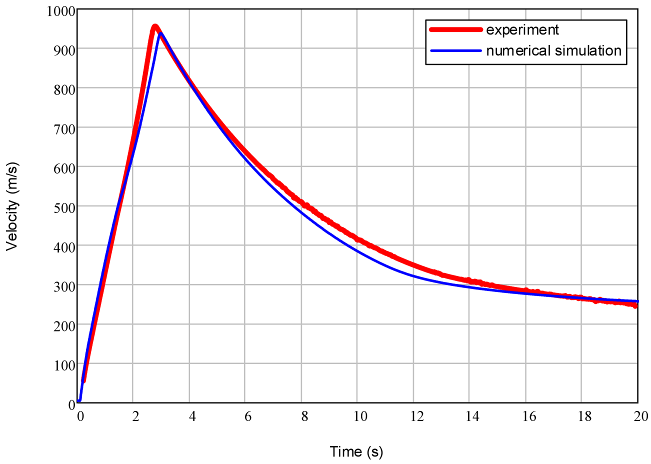

Fig. 6. Velocity vs. time for a missile launched at a 10° angle

The flight with the rocket engine running lasted about 2.7 seconds. After the thrust stopped, and the first stage separated, the missile can be regarded as a constant mass body (approx. 40 kg). Having differentiated the experimental dependence $V = f(t)$ (Fig. 6), we obtained the acceleration dependence versus time $a = f(t)$. Figure 7 shows the missile acceleration plots based on both experimental data and theoretical calculations. A good coincidence of the two plots was obtained. Having integrated the experimental dependence $V = f(t)$ (Fig. 6), we obtained the distance dependence versus time $D = f(t)$. The runs of both experimental and calculated functions are similar (Fig. 8).

A missile in flight is affected by aerodynamic drag (force) depending on velocity. The pattern of aerodynamic drag dependence on time $P_x = f(t)$ will be similar to dependence $V = f(t)$, as shown in Fig. 9. The $P_x = f(t)$ function can also be presented as the dependence of drag force on missile velocity $P = f(V)$. Figure 10 shows this dependence for both experimental data and the data calculated in the computer simulation. The plots are characteristic for this physical dependence and differ not more than 7%.

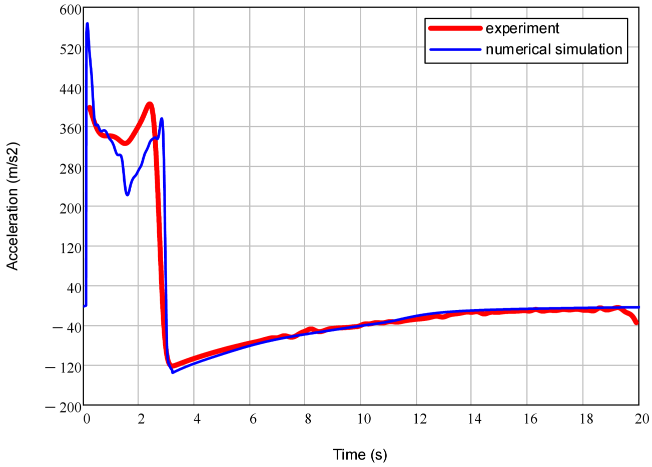

Fig. 7. Acceleration of a missile in time

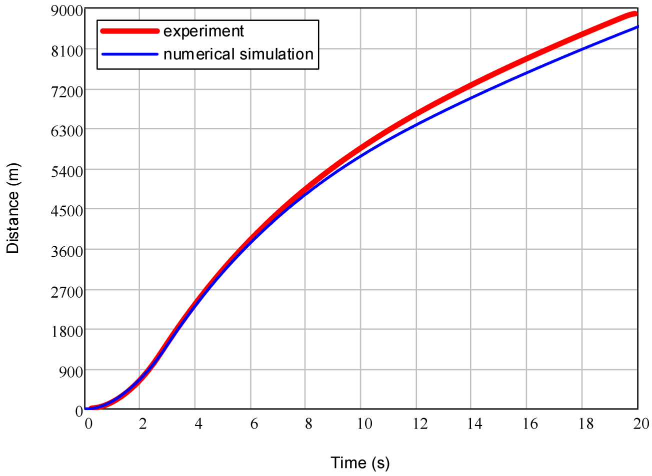

Fig. 8. Distance vs. time for a missile launched at a 10° angle

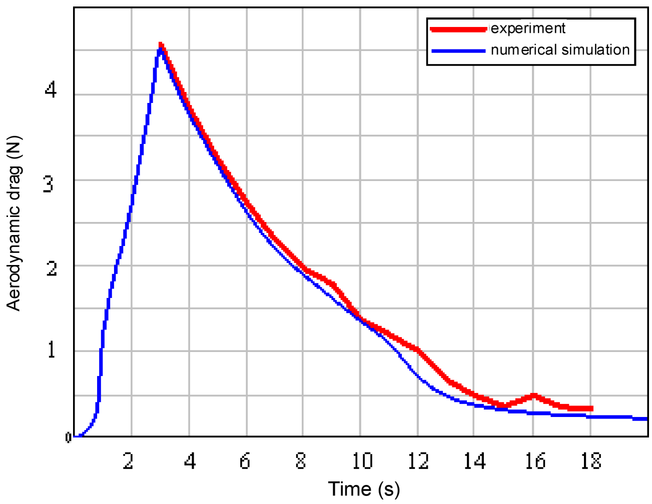

Fig. 9. Change in aerodynamic drag with time of flight

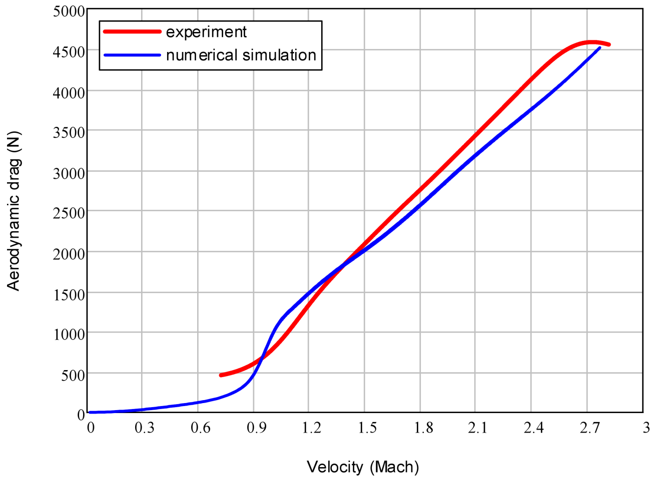

Fig. 10. Aerodynamic drag force as a function of missile velocity, expressed as the Mach number

By taking the forces into account and referring them to velocity, a diagram of the drag coefficient can be prepared and compared with the theoretical coefficient. Figure 11 shows the dependence of the drag coefficient on velocity, expressed as the Mach number.

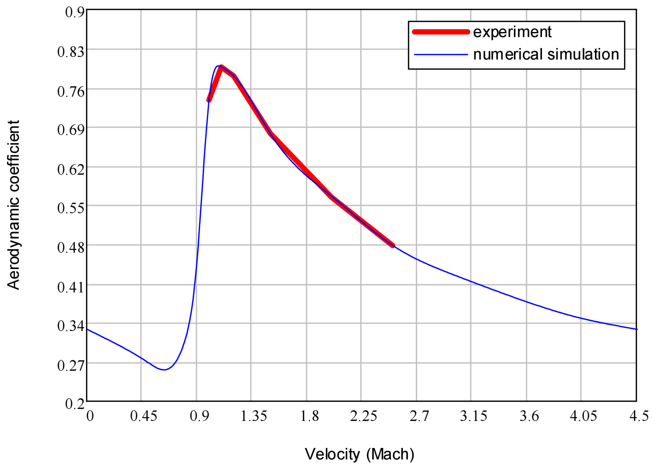

Fig. 11. Aerodynamic drag force coefficient as a function of missile velocity, expressed as the Mach number

6. Conclusions

A mathematical-physical model of the studied missile was developed and implemented in the MathCAD14 calculation package. The selected kinematic characteristics of the missile were determined using the algorithm developed and simulation software. The results of computer simulations were presented as diagrams of the basic kinematic characteristics (velocity, range, altitude) of the missile launched at various angles. Simulations of the dependence of such dynamic parameters as drag force and drag coefficient versus velocity of the missile were also carried out. The results of this effort showed great usability of the MathCAD14 calculation package in determining the dynamic characteristics of supersonic objects.

Based on the results of initial site tests of the studied missile, the experimental data were compared with computer simulation results. A high accordance of experimental and theoretical kinematic characteristics was obtained (velocity, range, altitude), as well as a satisfactory coincidence of dynamic results (drag force and drag coefficient).

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