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The theoretical model of the transient behaviour for the micro combined heat and power organic Rankine cycle

This article presents results of calculations of the organic Rankine cycle (ORC) in working conditions. The calculations make it possible to define the inertia of the evaporator and condenser while the heat loads change occurs. Calculations allow to get the inertia of the system, and the parameters of temperature at the inlet to the turbine and condenser for the R123 refrigerant. Very similar calculations results of the parameters were obtained from own model for ORC cycle, and experiment results performed independently gave basis to conclude that the model well reflects the real systems. It should be emphasized that the dynamics calculations for the cycle prepared by this model was conducted for different working fluids (R123, HFE7100, propane). Good quality of the results from these calculations, allows to expand the scope of the model applicability to other working fluids. Our own results of calculations were compared with other authors experimental studies of the ORC cycle obtaining a good complaiance.

Nomenclature

A – surface area, m^2
c_p – specific heat at constant pressure, J/(kgK)
D – derivative constant of the PID controller, s
g – standard acceleration of gravity, $m/(s^2)$
H – pump head, m
h – enthalpy, J/kg
I – integral constant of the PID controller, 1/s
i_n – signal from control panel
i_p – feedback signal

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k – heat transfer coefficient, W/(m^{2}K)
L – length of exchanger, m
T – temperature, °C
M – mass, kg
$\dot{m}$ – mass flow, kg/s
P – proportional constant of the PID controller
P_u – net power of the pump, W
p – pressure, Pa
ρ – mass density, kg/m^3
λ – thermal conductivity, W/(mK)
Δz – pressure drop, expressed in meters of water column, m
$wall$ – wall,
l – liquid,
v – vapour,
hfe – hfe7100 working medium,
c – condensation,
b – boiling,
$insul$ – insulation
air – air
ol – thermal oil,
s – Laplace mathematical operator
t – time, s

1 Introduction

Thermal cycles are equipped with automatic control systems that enable stable performance at variable load and ensures the safety in case of failure condition. In literature are presented the practical issues of controlling under operating conditions and eventual breakdown in the thermal cycles. Theoretical studies associated with physical and mathematical modeling dynamic effects occurring in the basic components of thermal plants such as boilers, turbines, heat exchangers, etc. are also known [1–8].

Paper presents transient model of the organic Rankine cycle (ORC) made for the three subsystems of the ORC system (Fig. 1):

- multifuel boiler system,
- micro CHP system,
- heating system.

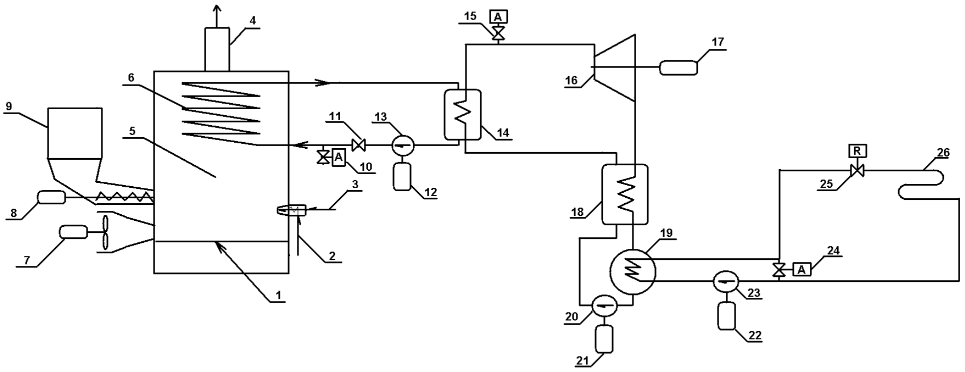

Figure 1. Circulation of micro combined heat and power (CHP): 1 – multifuel boiler grill, 2 – inlet gas fuel, 3 – inlet of liquid fuel, 4 – chimney, 5 – combustion chamber, 6 – multifuel boiler coil, 7 – air supply system, 8 – solid fuel screw drive, 9 – solid fuel store, 10 – pressure (relief) valve, 11 – valve, 12 – electric motor oil circulation pump, 13 – oil circulation pump, 14 – evaporator, 15 – pressure relief valve, 16 – microturbine, 17 – electric generator, 18 – regenerator, 19 – condenser, 20 – micro CHP circulation pump, 21 – micro CHP circulation pump engine, 22 – heating circuit pump engine, 23 – heating circuit pump, 24 – safety valve, 25 – thermostat, 26 – coil of heating system.

2 Subcycle models of micro combined heat and power

Assumptions and simplifications adopted in the model describing the failure working conditions [3,7]:

- lumped-parameter model,
- model describes the changes in time of thermodynamic parameters (transient model),
- lumped parameter model describing the heat exchangers, in which the phase change is realized, includes the heat losses to the environment,
- combustion model occupies an area associated with the convective heat transfer (the process of radiative heat transfer is skipped),
- ORC system controllers work in a negative feedback loop,
- pump control cycle is done by a proportional-integral-deriivative (PID) controller with delay,

- state variable, which is subject to regulation under failure conditions, is the fluid mass flow,
- system failure begins after reaching steady state (during operation) and it is modeled by the Heaviside step function
- model does not include the disappearance of boiling process in the cooled system.

Below are presented the equations describing the processes occurring in the heat exchangers realizing the phase change, formed by the basis of the taking assumptions:

- condensation process:

$$\frac{dT_c}{dt} = \frac{1}{\left(\frac{c_{p,v}+c_{p,l}}{2}\right) M_c}\left[A_{wall}k_{hfe-wall}\left(T_{wall}-T_c\right)+\dot{m}_{hfe}\left(h_v-h_l\right)\right] , \tag{1}$$

- heat transfer through the wall of the condenser:

$$\begin{aligned}\frac{dT_{wall}}{dt} = \frac{1}{M_{wall}c_{p,wall}}&\left[A_{wall}k_{hfe-wall}(T_c-T_{wall})+\right.\\ &\left.+A_{wall}k_{wall-water}(T_{water}-T_{wall})\right] ,\end{aligned} \tag{2}$$

- water cooling:

$$\begin{aligned}\frac{dT_{water}}{dt} = \frac{1}{M_{water}c_{p,water}}&\left[c_{p,water}\dot{m}_{water}(T_{water,in}-T_{water})+\right.\\ &\left.+\,k_{water-insul}A_{insul}(T_{insul}-T_{water})+k_{wall-water}A_{wall}(T_{wall}-T_{water})\right] ,\end{aligned} \tag{3}$$

- heat transfer through the insulation:

$$\begin{aligned}\frac{dT_{insul}}{dt} = \frac{1}{M_{insul}c_{p,insul}}&\left[A_{insul}k_{water-insul}(T_{water}-T_{insul})+\right.\\ &\left. .+\,A_{insul}k_{insul-air}(T_{air}-T_{insul})\right] .\end{aligned} \tag{4}$$

Below the equations describing the processes occurring in the evaporator are presented:

- evaporation process:

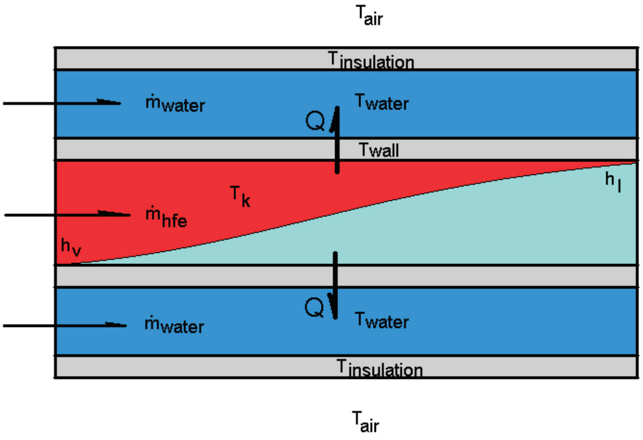

Figure 2. Graphical interpretation of condensation process in example heat exchanger.

$$\frac{dT_b}{dt} = \frac{1}{\left(\frac{c_{p,v}+c_{p,l}}{2}\right) M_b}\left[A_{wall}k_{hfe-wall}\left(T_{wall}-T_b\right) - \dot{m}_{hfe}\left(h_v-h_l\right)\right] \; , \tag{5}$$

- heat transfer through the wall of the evaporator:

$$\frac{dT_{wall}}{dt} = \frac{1}{M_{wall}c_{p,wall}}\left[A_{wall}k_{hfe-wall}\left(T_b-T_{wall}\right) + A_{wall}k_{wall-water}\left(T_{ol}-T_{wall}\right)\right] \; , \tag{6}$$

- heating oil:

$$\begin{aligned} \frac{dT_{ol}}{dt} = \frac{1}{M_{ol}c_{p,ol}}\big[&c_{p,ol}\dot{m}_{ol}(T_{ol,in}-T_{ol}) + \\ &+ kA_{air}(T_{insul}-T_{ol}) + kA_{wall-ol}(T_{wall}-T_{ol})\big] \; , \end{aligned} \tag{7}$$

- heat transfer through the insulation:

$$\begin{aligned} \frac{dT_{insul}}{dt} = \frac{1}{M_{insul}c_{p,insul}}\big[&A_{insul}k_{ol-insul}(T_{ol}-T_{insul}) + \\ &+ A_{insul}k_{insul-air}(T_{air}-T_{insul})\big] \; . \end{aligned} \tag{8}$$

A dynamic model describing the operation of boiler cycle (indirect heating system evaporator) is proposed

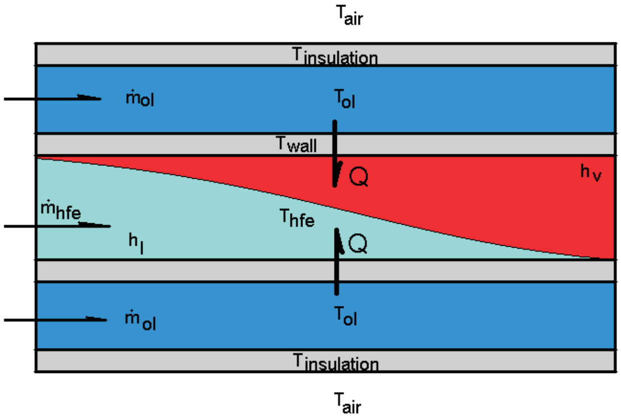

Figure 3. Graphical interpretation of the boiling process in example heat exchanger.

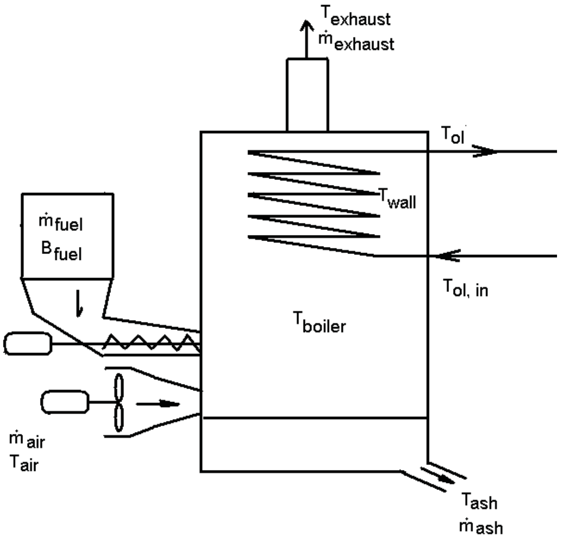

Figure 4. Diagram of the modelled multifuel boiler cycle.

- heat collection by thermal oil:

$$\frac{dT_{ol}}{dt} =$$

$$= \frac{1}{M_{ol} c_{p,ol}} \left[c_{p,ol} \dot{m}_{ol} \left(T_{ol,in} - T_{ol} \right) - k A_{wall-ol} \frac{\left(T_{boiler} - T_{ol,in} \right) - \left(T_{boiler} - T_{ol} \right)}{\ln \frac{\left(T_{boiler} - T_{ol,in} \right)}{\left(T_{boiler} - T_{ol} \right)}} \right] , \qquad (9)$$

- the process of combustion in the boiler chamber:

$$\begin{aligned}
&\tfrac{dT_{boiler}}{dt} = \tfrac{1}{M_k c_{p,k}}\Big[c_{p,fuel}\dot{m}_{fuel}\left(\tfrac{B_{fuel}}{c_{p,fuel}} - T_{boiler}\right)\\
&+c_{p,air}\dot{m}_{air}\left(T_{air} - T_{boiler}\right)\\
&-kA_{wall-ol}\left(T_{boiler} - T_{ol}\right) - kA_{wall-iz}\left(T_{boiler} - T_{ot}\right)\\
&-c_{p,exhaust}\dot{m}_{exhaust}\left(T_{boiler} - T_{exhaust}\right) - c_{p,ash}\dot{m}_{ash}\left(T_{boiler} - T_{ash}\right)\Big]
\end{aligned} \tag{10}$$

- the dynamics of heat transfer for the return of cold liquid from the heating system is presented by the equation:

$$M_5 c_p \frac{dT_5}{dt} = \dot{m}_5 cp_5\left(T_4 - T_5\right) - kA\frac{(T_4 - T_3) - (T_5 - T_3)}{\ln\frac{(T_4 - T_3)}{(T_5 - T_3)}}\ . \tag{11}$$

The dynamics of heat transfer from the heated room with the environment describes the relationship

$$M_3 c_p \frac{dT_3}{dt} = kA\frac{(T_4 - T_3) - (T_5 - T_3)}{\ln\frac{(T_4 - T_3)}{(T_5 - T_3)}} - k_{ot}A_{ot}\left(T_3 - T_o\right)\ . \tag{12}$$

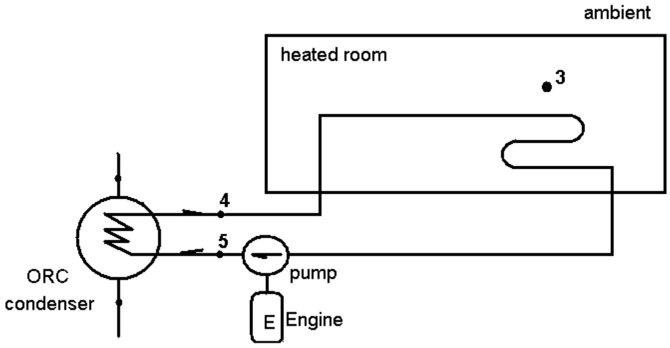

Figure 5. Diagram of the modelled heating system.

Additional equations describe the system pumps, and microturbines:

- net power for the pump:

$$P_u = \dot{Q}H\rho g\ , \tag{13}$$

- pump heat:

$$H = \frac{p_t - p_s}{\rho g} + \Delta z \ , \tag{14}$$

- temperature at the outlet of the microturbine:

$$T_k = \left[(T_b + 273,15) \left(\frac{P_k}{P_b} \right)^{\frac{\frac{c_{p,v}(P_k)}{c_{v,v}(P_k)} - 1}{\frac{c_{p,v}(P_k)}{c_{v,v}(P_k)}}} \right] - 273.15 \ . \tag{15}$$

Model of the dynamics of micro CHP in the closed control system uses a classical continuous controller based on the rules of PID. The relationship describing the regulator is expressed in the form

$$P_u\left(s\right) = 0 \leq \left| \left(P + I\frac{1}{s} + Ds \right) (i_n - i_p) \right| \leq \max \ . \tag{16}$$

For this control system was introduced stepwise (function Heaviside'a) changes related to simulating a power failure of the circulation pump.

3 The results of calculations

In order to verify complaisance of our own model with experimental studies performed by Pei G. *et al.* [9] the results of own calculations for time of reaching the set conditions were compared. Figures 6 and 7 present a comparison of experimental results by Pei G. *et al.* [9] with own numerical calculations for vapour temperature at the inlet to the turbine and to the condenser for the R123 refrigerant in the ORC cycle. Experimental studies were made for 16 kW heat flux in the condenser [9]. For such conditions model of ORC system presented in [7] was modified for R123 refrigerant properties, and heat load in condenser was determined at 16 kW level. Very similar results (Figs. 6 and 7) of the calculations for parameters obtained from our own model for ORC cycle with the experiment performed independently gave basis to conclude that the model well reflects the real systems.

4 Summary

This article presents comparison between own theoretical model with independent experimental results for R123 refrigerant medium applied in micro-ORC cycle [9]. Micro-ORC system were described by lumped-parameters model. The proposed

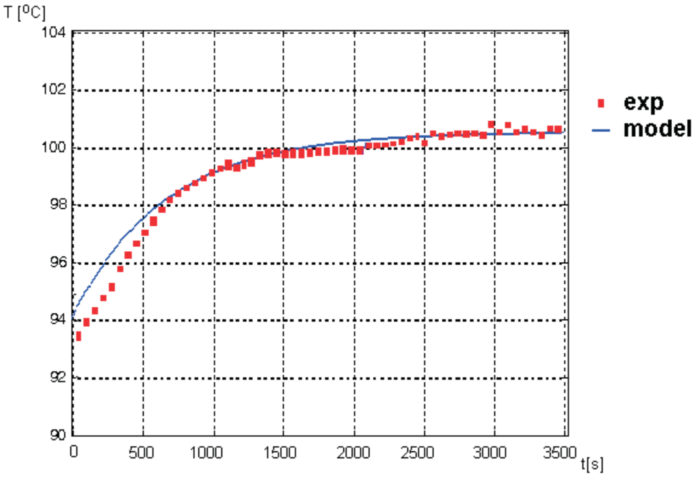

Figure 6. Comparison of the results of temperature on the inlet to the turbine (own model and experimental studies for R123 refrigerant according to [9]).

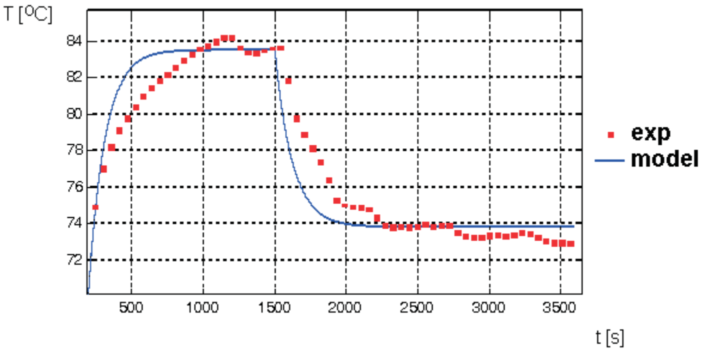

Figure 7. Comparison of the results of temperature at the inlet to the condenser (own model and experimental studies for R123 refrigerant according to [9]).

model of micro ORC systems allows to predict the behaviour of the system in case of disturbances, and changes in the expected operating conditions (adjustment on the controller). It also allows to determine of inertia of the system in case of failure. Similar results of the calculations for parameters obtained from our own model for ORC cycle with the experiment performed independently gave basis to conclude that the model well reflects the real systems. Own theoretical model reflects quite well the ORC systems operating on different working fluids.

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Teoretyczny model ciągły dla obiegu kogeneracyjnego ORC

Streszczenie

W pracy przedstawiono wyniki obliczeń obiegu ORC w warunkach eksploatacyjnych. Przeprowadzone obliczenia umożliwiają określenie bezwładności układu podczas zmiany obciążeń cieplnych parownika i skraplacza. Z obliczeń uzyskano czasy bezwładności oraz parametry temperatury na dolocie do turbiny i skraplacza dla czynnika R123. Bardzo zbliżone wyniki obliczeń parametrów uzyskanych z własnego modelu obiegu ORC do przeprowadzonego niezależnie eksperymentu dają podstawy do stwierdzenia, że model dobrze odzwierciedla układy rzeczywiste. Podkreślić należy fakt, że obliczenia dynamiki obiegu za pomocą tego modelu przeprowadzono dla różnych czynników roboczych (R123, HFE7100, propan) uzyskując dobre jakościowo wyniki, co pozwala poszerzyć zakres jego stosowalności również na inne czynniki robocze. Wyniki własnych obliczeń porównano z badaniami eksperymentalnymi obiegu ORC innych autorów uzyskując dobrą zgodność.