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Modelling of Textile Composite Adapted for Roof Structure

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Abstract

3D structural components made of fibrous composite materials subjected to service loading are analysed. The laminate consists of a matrix reinforced with a ply of long and unidirectional fibres. Participation of the fibres and matrix can vary in each layer. The main goal is to create a composite roof plate transferring variable loads of wet snow or fir and to investigate its strength properties in respect of (i) the type of fibres and matrix, and (ii) the volume fraction of the matrix and fibres within each layer. Modelling of shape and mechanical properties is reduced to the final phase of structural design, omitting time-consuming, expensive experimental tests. The numerical procedure is solved using the ADINA environment and finite element method code.

Key words: *fibre reinforced composite, textile building materials, modelling, finite element method.*

Introduction

Modelling and structural analysis are integral components of architectural and engineering design. The selection of optimal load and appropriate material allows to analyse the internal forces and material stresses. The basic factors influencing the complex structure and building stability determine their different technical, economic and aesthetic aspects [1-3].

Composite materials are usually applied because of the mechanical, thermal and chemical properties e.g. high stiffness-to-weight and strength-to-weight ratios, corrosive resistance, low thermal expansion, and vibration damping. Composite characteristics (fibre orientation, material thickness, laminate configuration, type and volume fraction of the matrix etc.) influence properties like strength, thermal conductivity, electrical conductivity and the thermal expansion coeffcient [4, 5]. Laminates are applied to create the structural elements [6].

Analysis of composite behaviour caused by different loads is a part of the complex analysis of the material applied. The combination of fibre and matrix characteristics influences the global parameters of the laminate. The behaviour of composite structures composed of a matrix reinforced by different fibrous material has been discussed by many Authors, cf. [2, 7-10].

The distinctive characteristic of fibrous composite is sensitivity to the degradation process caused by long-lasting mechanical, thermal and chemical loads. It can cause changes in different properties i.e. internal friction in the material, the average value of material strength, the value of the elasticity coefficient, and a dispersion increase in its value. The various factors influence the degradation simultaneously, for example stress corrosion connected with thermal and mechanical fatigue [2, 6].

Theoretically speaking, the structural modelling and design of fibrous composites is unlimited and can be analysed using different methods and programs. Structural analysis and optimisation methods based on the evolutionary algorithm [11-13] or genetic algorithms [14, 15] can be applied, both supplemented by finite element code. Different optimisation techniques can also be introduced to design textile structures, for example the gradient-oriented group of deterministic methods [16, 17].

The main goal of the paper is to create a model of a composite roof plate which transfers variable loads of wet snow or fir and to investigate its strength properties in respect of (i) the type of fibres and matrix, and (ii) the volume fraction of the matrix and fibres in the structure. The real roof plate is a part of an exhibition pavilion which can be located in the environment of the characteristics prescribed. The plate is reinforced by long and unidirectional fibres which are usually applied to reinforce a flat surface of considerable strength (cf. sails, roof plates, paragliders etc.). A mathematical model is introduced according to Halpin and Tsai, supplemented by Adams and Doner [18-20]. A numerical solution is determined by means of the ADINA environment and finite element code.

Novelty elements are as follows: (i) the choice of a real object of the parameters prescribed i.e. the roof plate is a part of an exhibition pavilion located in the environment of the characteristics prescribed; (ii) the modelling introduces existing loads i.e. snowfall; and (iii) the introduction of long and unidirectional fibres in the matrix as the most resistant reinforcement of the composite.

Object of analysis

The object is a single-storey building without a basement and foundation, air-conditioned inside. East and west fronts have double entrance doors made of glass, which are the main and evacuation entries. The south facade is a brick wall made of clinker brick, whereas the north facade is a glass wall. The composite plate is located on the upper part of the multilayer roof and protects the structure from the adverse effects of climate controls. The external dimensions of the building are the following: height 4.5 m. length 15 m. width 5 m. and distance between trusses 1.44 m. Each entrance is secured by a little roof of length 1.44 m. According to design assumptions, the building is located in a flat park area covered by deciduous and coniferous trees as well as large and small shrubs. The pavilion

Figure 1. *Visualisation of the exhibition pavilion.*

is intended for exhibition purposes, cf. ***Figures 1-3***.

The roof plate is a composite structure made of a matrix reinforced with a ply of long and unidirectional fibres. The matrix material is homogeneous, isotropic and linear elastic, whereas the fibrous reinforcement is homogeneous, orthotropic and linear elastic. Fibres of global density ρ_f are regularly aligned and ideally connected with the matrix.

The heterogeneous composite is next homogenised to create a homogeneous, orthotropic and linear elastic material. Due to the shape and material repeatability, the space problem can be further simplified to an optional plane cross-section, cf. ***Figure 4***.

Engineering constants of the composite can be determined vs. mechanical properties of the fibres and matrix as well as the fibre density. Substitute engineering constants of the composite are calculated according to the Halpin and Tsai approach [18], supplemented by Adams and Doner [19, 20]. The elastic constants of the composite materials calculated are the input data for numerical simulation of

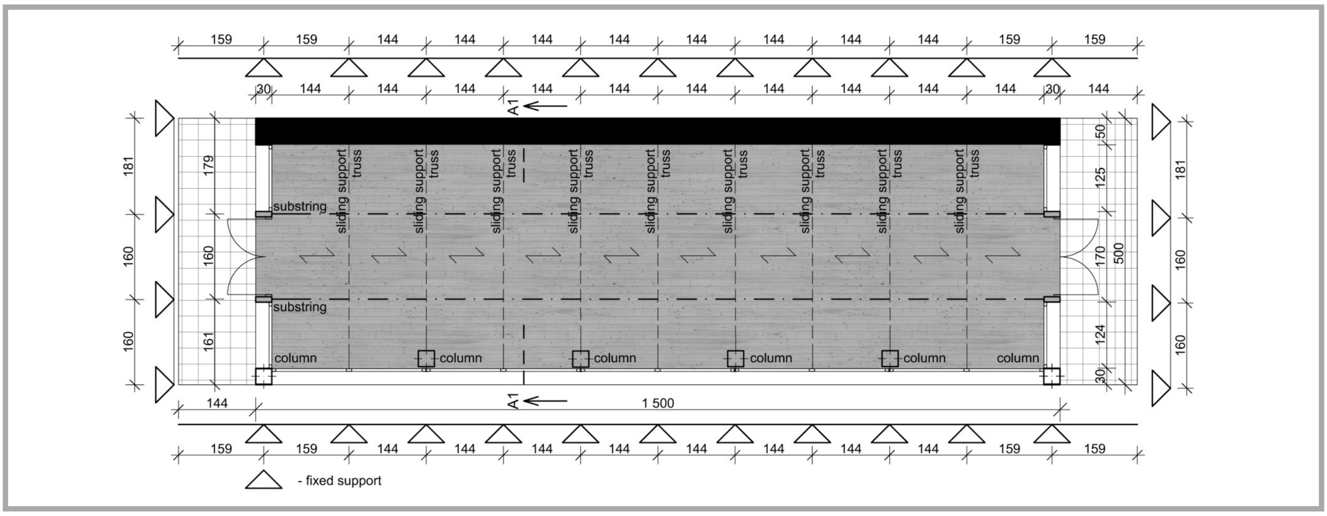


Figure 2. *Projection of the exhibition pavilion, scale 1:100 (dimensions in cm).*

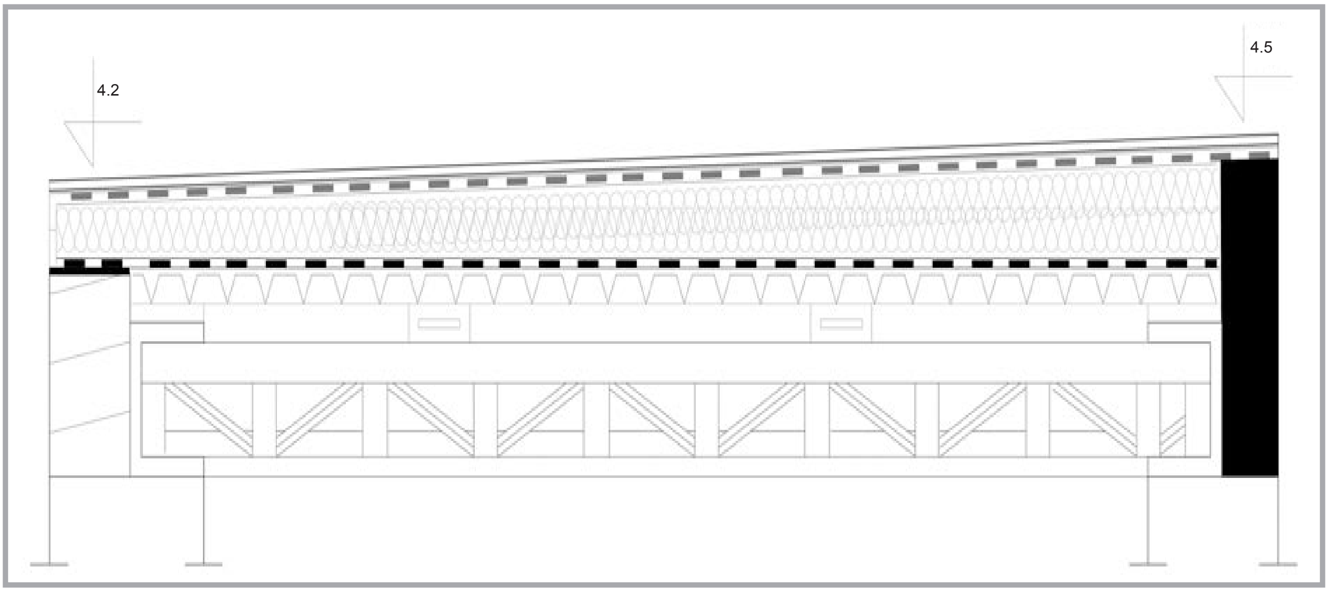


Figure 3. *Cross-section of the A1-A1 pavilion, scale 1:20 (dimensions in m).*

the composite behaviour. The reinforcement factor ξ characterises the shape and arrangement method of fibres [21] and has a different form in respect of the cross-section of fibres. Adams and Doner determined the circular shape by a square array, and Foye analysed the square and rectangular cross-sections. Material constants of the lamina model according to Halpin and Tsai are the ***Equation (1)*** [18-20].

We can introduce the following notations.

E_1, E_{1m}, E_{1f} – Young's moduli in the direction of the fibre for the composite material, matrix and fibres in GPa;

E_2, E_{2m}, E_{2f} – Young's moduli in the direction perpendicular to fibres for the composite material, matrix and fibres in GPa;

G_{12}, G_{12m}, G_{12f} – shear moduli for the composite material, matrix and fibres in GPa;

G_{2m}, G_{23} – shear moduli for the matrix, shear moduli for the composite material in GPa;

v_{12}, v_{12m}, v_{12f} – major Poisson's ratios for the composite material, matrix and fibres in –;

v_{21} – minor Poisson's ratios of the composite material in –;

ρ_m, ρ_f – density saturation of the composite material with matrix and fibres in –;

ξ – reinforcement factor of the composite in dependance on the cross-section of fibers and packing geometry in –.

The modelling process of a multilayer composite made of the equivalent homogeneous material is composed of two steps: (i) substitute engineering constants of the lamina are described within a particular single layer; (ii) the constants are determined for a stack of layers i.e. for a complex structure. The typical rule of mixture is applied to determine the substitute material parameters, which is the most efficient method in simple cases of homogenisation. The layer can be homogenized using different methods, for example image analysis, and the image system of a point can be simplied to the volume fraction of each component.

$$E_1 = E_{1f}\rho_f + E_{1m}\rho_m; \qquad E_2 = E_3 = E_{2m}\frac{1+\xi\chi\rho_f}{1-\xi\chi\rho_f}$$

$$\chi = \frac{E_{2f} - E_{2m}}{E_{2f} + \xi E_{2m}}; \qquad G_{12} = G_{13} = E_{12m}\frac{1+\xi\chi\rho_f}{1-\xi\chi\rho_f}$$

$$G_{20} = \frac{E_{2m}}{2(1+\nu_{12m})}; \qquad \chi = \frac{G_{12f} - G_{12m}}{G_{12f} + \xi G_{12m}};$$

$$G_{23} = \frac{E_f}{2\left[(1+\nu_f)\rho_f + \frac{E_f}{E_m}(1-\nu_m)(1-\rho_f)\right]}$$

$$\nu_{21} = \left(\frac{E_2}{E_1}\right)\nu_{12}; \qquad \nu_{12} = \nu_{13} = \nu_{12f}\rho_f + \nu_{12m}\rho_m;$$

$$\xi_{for\ G_{12}} = \begin{cases} 1 \\ (1+40\rho_w^{10})\ for\ \rho_f > 0.5 \end{cases}; \qquad \xi_{for\ E_2} = 2;$$

$$\upsilon_{23} = \frac{\left[\nu_f\rho_f + \frac{E_f}{E_m}\nu_m(1-\rho_f)\right]\left[1+\left(\frac{E_f}{E_m}-1\right)\rho_f\right] + \left(\frac{E_f}{E_m}\nu_m - \nu_f\right)^2\rho_f(1-\rho_f)}{\left[\rho_f + \frac{E_f}{E_m}(1-\rho_f)\right]\left[1+\left(\frac{E_f}{E_m}-1\right)\rho_f\right] - \left(\frac{E_f}{E_m}\nu_m - \nu_f\right)^2\rho_f(1-\rho_f)}$$

Equation 1.

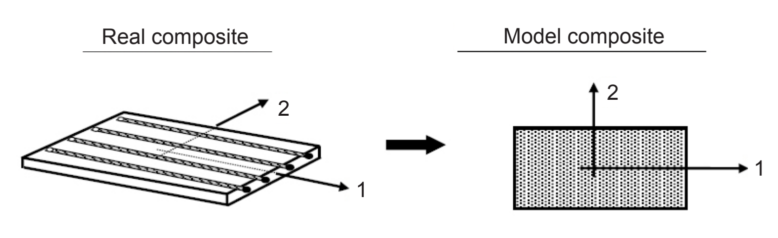


Figure 4. *Model of the composite layer reinforced with fibres.*

Table 1. *Material characteristics of composite components [21].*

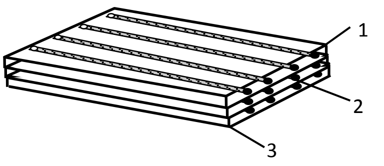
Layer number		Composite material	Young's module *E*, GPa	Poisson's ratio, ν
	1	ceramic fibres epoxy matrix	120.00 3.43	0.35 0.35
	2	glass E fibres polyester matrix	75.00 3.20	0.25 0.41
	3	glass E fibres epoxy matrix	75.00 3.43	0.25 0.35

Table 2. *Substitute elasticity constants for the assumed participation of components.*

Substitute engineering constants of the composite		Percentage participation of components (*f/m*)	
		30/70	**70/30**
Ceramics fibres (f), epoxy matrix (m)	longitudinal Young's modulus, GPa	38.40	85.03
	transverse Young's modulus, GPa	7.35	21.98
	major Poisson's ratio	0.35	0.35
	shear modulus, GPa	2.27	8.37
Glass E fibres (f), polyester matrix (m)	longitudinal Young's modulus, GPa	24.74	53.46
	transverse Young's modulus, GPa	6.65	18.69
	major Poisson's ratio	0.36	0.30
	shear modulus, GPa	2.01	7.01
Glass E fibres (f), epoxy matrix (m)	longitudinal Young's modulus, GPa	24.90	53.53
	transverse Young's modulus, GPa	7.09	19.66
	major Poisson's ratio	0.32	0.28
	shear modulus, GPa	2.24	7.62

Numerical calculation of snow load on a roof of uniform inclination

The roof plate of variable participation of matrix and reinforcement presented is subjected to different loading. The com-

Table 3. *Density and calculation of snow load [22, 26].*

Type of snow	Snow density, kN/m^3	Calculation load of snow cover zone I (Lodz, Poland), kN/m^2
Fresh snow	0.5 – 0.7	0.16 – 0.23
Damp fresh snow	1.0 – 2.0	0.32 – 0.65
Stabilized snow, deep permafrost	2.0 – 3.0	0.65 – 0.97
Snow thickened by the action of wind	3.5 – 4.0	1.14 – 1.30
Firn	4.0 – 8.5	1.30 – 2.76

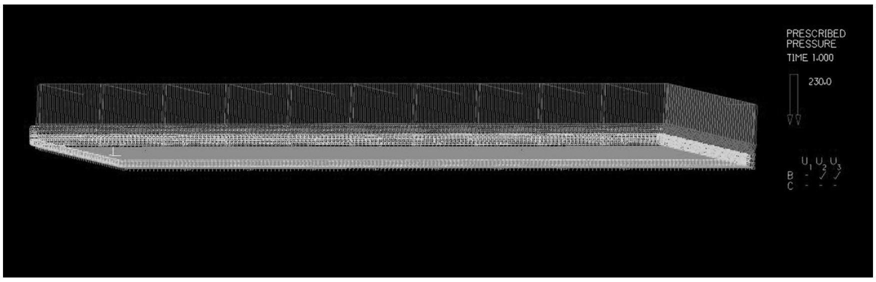


Figure 5. *Composite structural element after discretisation using the ADINA system.*

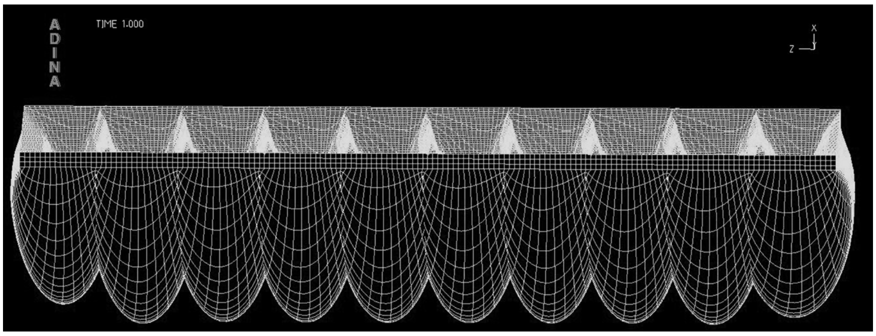


Figure 6. *Exaggerated deformation of composite structure at $\rho_f = 0.3$, snow load 230 N/m^2.*

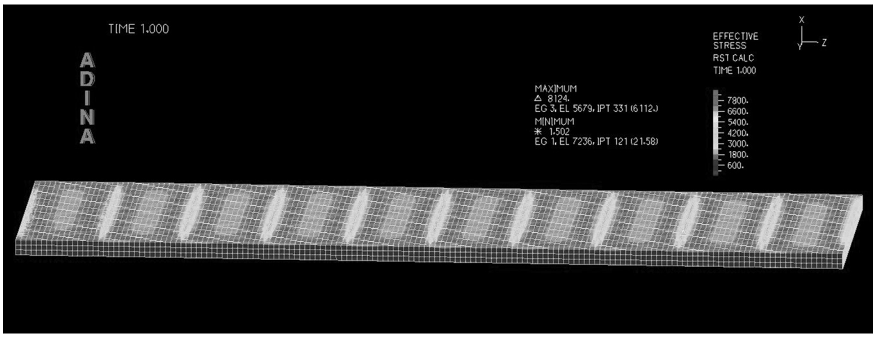


Figure 7. *Distribution of reduced stresses at $\rho_f = 0.3$, snow load 230 N/m^2.*

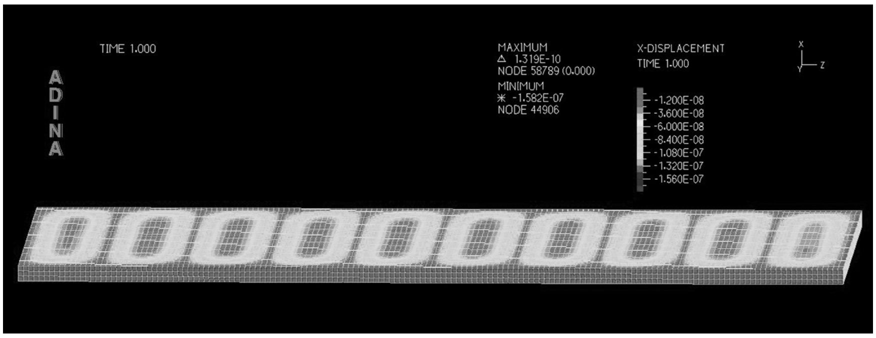


Figure 8. *Deformation in vertical direction at $\rho_f = 0.3$, snow load 230 N/m^2.*

posite is created by three layers of the material characteristics shown in ***Table 1***. The substitute elasticity constants are specified in ***Table 2***.

Snow is the main load of the roof, defined according to the standard [22]. This load is different and depends on the climatic zone and surrounding temperature. The load changes in respect of the roof area. The acting load should always be within the range of the permissible value which influences the structural durability. Of course, the snow load can be permanently monitored to secure an adequate safety level, and the snow should be removed from the roof if necessary [22-25]. To illustrate the problem, some numerical examples are presented and solved using ADINA software and finite element code.

The virtual roof plate is made of three-layer composite material of the engineering constants listed in ***Table 2***. The total density of fibres is $\rho_f = 0.3$. Fresh snow implies a calculation load of 230 N/m^2. ***Figure 5*** shows the composite structural element discretised using the ADINA system and ***Figure 6*** – the structural deformation. Exaggerated deformation is used to show the nature of the phenomenon. ***Figure 7*** illustrates the distribution of reduced stresses and ***Figure 8*** – deformation of the composite in the vertical direction.

The next calculation sequence applies to the total density of fibres $\rho_f = 0.7$. The results of computer simulations: distribution of reduced stresses and the deformation along the vertical direction are presented in ***Figures 9*** and ***10***, respectively.

The results obtained show that the maximal values of stresses (equal to 8.124 kPa and 7.611 kPa, respectively) and displacements (equal to $1.319 \cdot 10^{-10}$ m and $6.628 \cdot 10^{-11}$ m) are under the level of permissible parameters required. The value of maximal stress calculated according to plane truss theory is equal to 0.188 MPa, and the displacements $3.140 \cdot 10^{-10}$ m, i.e. many times greater than the value according to Halpin and Tsai.

Concluding remarks

The model of a roof plate is characterised by a set of substitute material parameters which are determined in respect of the matrix reinforced with long and unidirectional fibres. The material characteristics describe the mechanical properties of the composite with sufficient accuracy. The state of stress, deformation and displacement of the structural element are closely related to both the structure of the composite and material characteristics.

Static properties of the composite plate were investigated in respect of the type of fibres and matrix as well as the volume fraction of matrix and fibres in the structure. The results obtained prove that the structure proposed is resistant enough to transmit snow loads without destruction. The reduced stress is always below the critical level and does not cause significant material effort.

The model introduces long and unidirectional fibres as the most efficient reinforcement of the composite. The structural shape of the composite can be further optimized, for example (i) the shape of reinforced fibres can be elliptical and characterised by the two semi-axes of the cross-section, and (ii) a particular layer can be defined by the different arrangement direction etc. The structure can also be optimised by various placements of layers within the package, which is often applied in clothing laminates/inlayers [26]. The results discussed can initiate the optimal design of a composite structure subjected to a tracking load. This problem will be discussed in the consecutive paper. Thus the results obtained constitute the starting point for further analysis of the behaviour of multi-layer composite subjected to complex cases of thermal and mechanical loads.

The different composite behaviour of material filled with fibres of various type and volume fraction of components in each layer, gives the opportunity to design this structure in respect of any desired mechanical response.

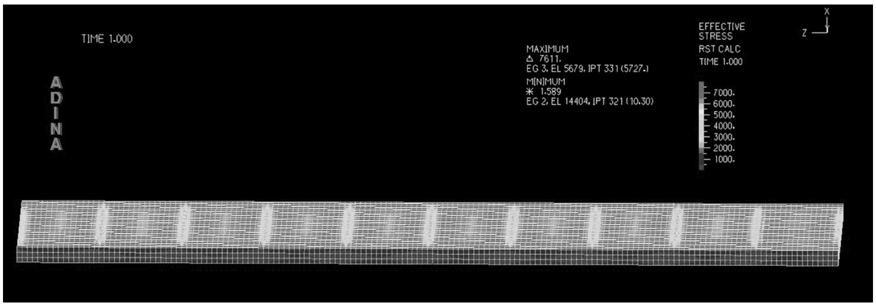


Figure 9. *Distribution of reduced stresses at $\rho_f = 0.7$, snow load 230 N/m^2.*

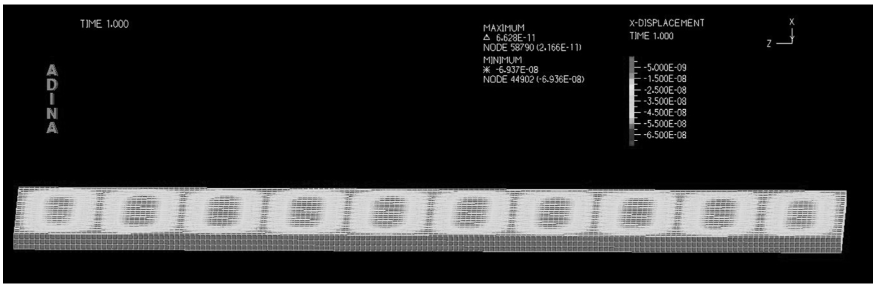


Figure 10. *Deformation in vertical direction at $\rho_f = 0.7$, snow load 230 N/m^2.*

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