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SIMULATION OF MULTICOMPONENT POLLUTION FLUID FILTERING PROCESS USING N-LAYER FILTERS

Andrij Safonyk, Oleksandr Naumchuk
National University of Water Management and Nature Resources Use, Department of Automation and Computer-Integrated Technologies, Rivne, Ukraine

Abstract. *In this article we considered and resolved the issue of incorporation of feedback of the process (the concentration of contamination of fluid and sediment) on the medium characteristics (porosity coefficient, filtration, diffusion, mass transfer, etc.) It was made during the simulation of cleaning fluid from multi impurities in n-ply sorption filter. We had retrieved algorithm numerically-asymptotic approximation solution of the corresponding model problem which is described by a system of nonlinear singularly perturbed differential equations "convection-diffusion-mass transfer." On this basis, we made a corresponding computer experiment.*

Keywords: filtration, reverse influence, sorption treatment, asymptotic upshots, nonlinear tasks

MODELOWANIE PROCESU FILTRACJI FILTREM N-WARSTWOWYM PŁYNU Z ZANIECZYSZCZENIAMI WIELOSKŁADNIKOWYMI

Streszczenie: *Rozpatrywane i rozwiązywany jest zagadnienie odwrotnego wpływu charakterystyk procesu (koncentracji zanieczyszczeń płynu i osadów) na charakterystyki ośrodka (współczynnik porowatości, filtracji, dyfuzji, przenoszenia masy, itp.) podczas modelowania procesu oczyszczania płynów z wieloskładnikowymi zanieczyszczeniami w N-warstwowym filtrze sorpcyjnym. Otrzymany algorytm asymptotycznie przybliża rozwiązanie odpowiedniego zadania modelowania, jest opisany układem równań nieliniowych „konwekcja-dyfuzja-transfer masy". Na jego podstawie przeprowadzono testy obliczeniowe.*

Słowa kluczowe: filtracja, wpływ odwrotny, oczyszczanie sorpcyjne, asymptotyczne rozwiązywanie zadań nieliniowych

Introduction

Due to the imperfection of existing mathematical models of filtration processes (forecasting, management and operational control methods), many of the relevant characteristic parameters are ignored or set arbitrarily. In particular, in many cases, neglect diffusion coefficient (which is not always practical), and its "traditional" accounting often leads to significant and unnecessary computational costs. Also, to date, is not enough developed, haphazard or, generally missing nonlinear model mechanisms that take into account the feedback effects of various characteristics of the process (of pollution concentration of liquid and sludge) on environmental characteristics (porosity coefficient, filtering of diffusion mass transfer etc). Almost missing is the work aimed at the development of software for automated control system of the filtration processes. Important is also constructing new models of filtering processes, by perturbations of existing models describing processes, but do not take into account a number of important characteristics of the environment. Many of the filtration processes in general are described only on the basis of experimental data and are not based on mathematical apparatus. No less urgent is the problem of mathematical description analysis of experimental data and justification of adequacy of the constructed models.

These questions, in spite of large volumes of liquids, filters used in this filter materials, their relatively high cost, the size of material losses due to the insufficient treatment of process liquids in various industries and especially in the energy sector, the expansion of existing and potential environmental problems are urgent and important (as from a theoretical point of view, and for water management and other industries).

1. Statement of the problem and its relationship to important scientific and practical tasks

1.1. Analysis of recent research and publications, which discuss current issues

Analysis of researched results [1-11, 13] indicates that the complex structure of interdependencies of different factors that determine the processes of filtration and filtration through a porous medium, which are not considered in traditional (classical, phenomenological) models of such systems. Consideration of different offs and additional factors were included in the basic model for a deeper study of the process and leads to the necessity of building bulky and inefficient (for numerical implementation and practical use) mathematical models. However, in many cases of practical importance, in the study of such processes can be applied modeling of various disturbances known as (idealized, averaged, baseline) backgrounds. At the same time filtering helps to reduce the equivalent of diameter of the granules downloaded - one of the universally accepted methods of improving the efficiency of filters [1]. In complex technological conditions change, optimal grain size load should depend on time. However, due to the complexity of implementation and operation in practice, filtering is not widely known even filters out "continuously" uneven loading. For these same reasons, actually limited to various approximations of optimal grain size load equivalent, grain diameter is "continuously" reduces in the direction of filtering by a specific law for the use-layer filters. The precision of approximation results, obviously, are the highest and the greatest number of filter layers. According to complexity of operating-layers filters, in particular, due to complications regeneration boot with growth rising. Because of the uncertainty of maximum economic benefits that can be gained in the operation of filters with optimal granulometric composition is currently the contradiction between its approximation accuracy and complexity filter operation which is decided in favor of reducing the latter. In other words, the practice of filtering the most common two-and n-layer filters.

1.2. Highlight of the unsolved aspects of the problem

According to the above studies, the work shall be considered and resolved the issue of incorporation of feedback of the process (the concentration of contamination of fluid and sediment) on the medium characteristics (porosity coefficient, filtration, diffusion, mass transfer, etc.) during the simulation of cleaning fluid from multi impurities in n-layer sorption filter.

1.3. Formulation of the problem

In this article we considered and resolved the issue of incorporation of feedback of the process (the concentration of contamination of fluid and sediment) on the medium characteristics (porosity coefficient, filtration, diffusion, mass transfer, etc.).

2. Statement of main research data with full justification of scientific results

We considered the one-dimensional spatial process of cleaning fluid filtration in n-ply filter layer thickness (Fig. 1), which is identified with a segment of the axis. We assumed [1] that the particle pollution can go from one state to another (processes of capture, separation, adsorption, desorption), while the concentration of pollution affects the considered layer.

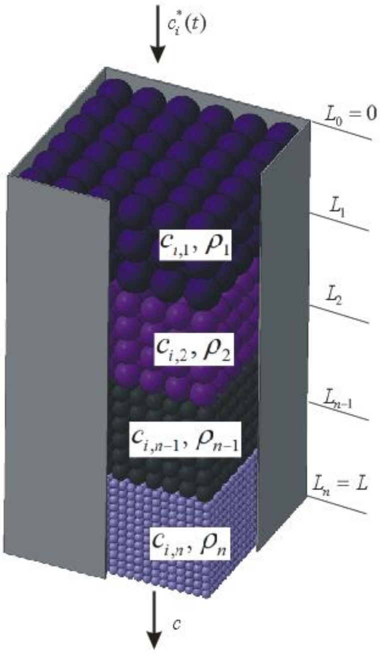

Fig. 1. Schematic representation of the n-layer filter

The concentration of pollution is a multicomponent, $c=c(x,t)=(c_1,...,c_m)=\left(c_1(x,t),...,c_m(x,t)\right)$, where are $c_i(x,t)$ – the component of the impurity concentration ($i=\overline{1,m}$) in liquid filtering medium. The corresponding process of filtering with inverse of the process (the concentration of fluid contamination and sediment) on the medium characteristics (porosity coefficient, filtration, diffusion, mass transfer, and so on. [1, 2]) can be expressed as the following model problem:

$$\begin{cases} \dfrac{\partial(\sigma(\rho)c_i)}{\partial t}+\dfrac{\partial\rho}{\partial t}+\dfrac{\partial(vc_i)}{\partial x}=D_i\dfrac{\partial^2 c_i}{\partial x^2}, \\ i=\overline{1,m},\ (x,t)\in G_k=\{x: L_{k-1}<x<L_k,\ 0<t<\infty\},\ k=\overline{1,l-1}, \\ \dfrac{\partial\rho}{\partial t}=\beta(\rho)\left(\sum_{i=1}^{m}q_ic_i\right)-\alpha(\rho)\rho+D_*\dfrac{\partial^2\rho}{\partial x^2}, \end{cases} \quad (1)$$

$$c_i\big|_{x=0}=c_i^*(t),\ \rho\big|_{x=0}=\rho^*(t),\ \left.\frac{\partial c_i}{\partial x}\right|_{x=L}=0,\ \left.\frac{\partial\rho}{\partial x}\right|_{x=L}=0,$$

$$c_i\big|_{t=0}=c_{*i}^*(x),\ \rho\big|_{t=0}=\rho_*^*(x), \quad (2)$$

$$[c_i]_{x=L_k}=0,\ [\rho]_{x=L_k}=0,\ \left[D_{i,k}\frac{\partial c_i}{dx}+vc_i\right]_{x=L_k}=0,$$

$$\left[D_{*k}\frac{\partial\rho}{dx}\right]_{x=L_k}=0, \quad (3)$$

$$v=\kappa(\rho)\cdot grad\ P, \quad (4)$$

where $\rho(x,t)$ – concentration of impurities trapped filter filling; $\beta(\rho)$ – coefficient is characterizing the volume of sediment contaminants per unit time, $\left(\beta(\rho)=\beta_0-\varepsilon\beta_*\rho(x,t)\right)$; $\alpha(\rho)$ – coefficient is characterizing the volume of particles separated by the same time, from grain filling, $\left(\alpha(\rho)=\alpha_0+\varepsilon\alpha_*\rho(x,t)\right)$; $c_i^*(t)$ – impurity concentration at the inlet filter; $\sigma(\rho)$ – porosity filtering attachments, $\sigma(\rho)=\sigma_0-\varepsilon\sigma_*\rho(x,t)$, where σ_0 – initial porosity attachments, $\kappa(\rho)$ – filtration coefficient, $\kappa(\rho)=\kappa_0-\varepsilon\gamma\rho(x,t)$ ($x\in[L_{k-1},L_k]$); $D_i=\begin{bmatrix} D_{i,1}=b_1\varepsilon, \\ ... \\ D_{i,l}=b_l\varepsilon, \end{bmatrix}$, $D_*=\begin{bmatrix} D_{*1}=b_{*1}\varepsilon, \\ ... \\ D_{*l}=b_{*l}\varepsilon, \end{bmatrix}$, $\beta_0,\beta_*,\alpha_0,\alpha_*,\sigma_*,b_k,b_{*k},q_i,\kappa_0,\varepsilon$ – solid parameters characterizing the corresponding coefficients; $\beta(\rho),\alpha(\rho),\sigma(\rho),\kappa(\rho)$ – soft parameters found experimental method; ε – a small parameter; v – speed filtering, $[L_{k-1},L_k]$ – k-th layer filter ($k=1,2,...,l$); P – pressure in the equations (3) [ ] – growth of the corresponding function at that point. The pressure $P=P(x,t)$ is determined by solving the equation $\dfrac{\partial}{\partial x}\left(\kappa(\rho)\dfrac{\partial P}{\partial x}\right)=\dfrac{\partial\sigma(\rho)P}{\partial t}$, is obtained on the basis of the law of motion and the equation of state $div\,v=\dfrac{\partial\sigma(\rho)P}{\partial t}$, at the boundary, $P(0,t)=P_*(t)$, $P(L,t)=P^*(t)$ $0<t<\infty$ and primary $P(x,0)=P_*^*(x)$, $0<x<L$, conditions, where $P_*(t)$, $P^*(t)$, $P_*^*(x)$ – gives sufficiently smooth and coordinated in the corner points of the field G_k functions.

To simplify the presentation, we considered importance of the practice of case the velocity field $v=v(x,t)$ is given.

Asymptotic approximation of solution of a model problem

$$c_i(x,t)=\begin{cases} c_{i,1}(x,t),\ L_0=0\le x<L_1, \\ c_{i,2}(x,t),\ L_1\le x<L_2, \\ ... \\ c_{i,l}(x,t),\ L_{l-1}\le x<L_l=L, \end{cases},$$

$$\rho(x,t)=\begin{cases} \rho_1(x,t),\ L_0=L\le x<L_1, \\ \rho_2(x,t),\ L_1\le x<L_2, \\ ... \\ \rho_l(x,t),\ L_{l-1}\le x<L_l=L, \end{cases}$$ found in the form of asymptotic series [1, 2]:

$$c_{i,k}(x,t)=c_{i,k,0}(x,t)+\sum_{j=1}^{n}\varepsilon^jc_{i,k,j}(x,t)+\sum_{j=0}^{n+1}\varepsilon^j\underset{\sim}{M}_{i,k,j}\left(\underset{\%}{\xi},t\right)+$$
$$+\sum_{j=0}^{n+1}\varepsilon^j\tilde{M}_{i,k,j}(\tilde{\xi},t)+\sum_{j=0}^{m+1}\varepsilon^jA_{i,l,j}(\xi,t)+R_{ci,k}(x,t,\varepsilon),$$
$$\rho_k(x,t)=\rho_{k,0}(x,t)+\sum_{j=1}^{n}\varepsilon^j\rho_{k,j}(x,t)+\sum_{j=0}^{2n+1}\varepsilon^{j/2}\underset{\sim}{P}_{k,j}\left(\underset{\sim}{\mu},t\right)+ \quad (5)$$
$$+\sum_{j=0}^{2n+1}\varepsilon^{j/2}\tilde{P}_{k,j}(\tilde{\mu},t)+\sum_{j=0}^{m+1}\varepsilon^{j/2}B_{l,j}(\mu,t)+R_{\rho,k}(x,t,\varepsilon),$$

where $R_{cj,k}, R_{\rho,k}$ – remaining members; $c_{i,k,j}(x,t)$, $\rho_{k,j}(x,t)$, ($i=\overline{1,m}$; $j=\overline{0,n}$; $k=\overline{0,l}$) - members of the regular parts asymptotics; $\underset{\sim}{M}_{i,k,j}\left(\underset{\sim}{\xi},t\right)$, $\tilde{M}_{i,k,j}(\tilde{\xi},t)$, ($i=\overline{1,m}$, $j=\overline{0,n+1}$), $\underset{\sim}{P}_{k,j}\left(\underset{\sim}{\mu},t\right)$, $\tilde{P}_{k,j}(\tilde{\mu},t)$, ($j=\overline{0,2n+1}$, $k=\overline{0,l-1}$) – functions such as boundary layer in the neighborhood of $x=L_k$ (adjustment for the transition flow filtration with one of the k-th layer filter in the next), $A_{i,l,j}(\xi,t)$, $B_{l,j}(\mu,t)$ ($j=\overline{0,m+1}$) – functions such as boundary layer in the neighborhood of $x=L$ (Amendment output flow filtration), $\tilde{\xi}=x\cdot\varepsilon^{-1}$, $\tilde{\mu}=x\cdot\varepsilon^{-1/2}$,

$\tilde{\xi}=(L-x)\cdot\varepsilon^{-1}$, $\tilde{\mu}=(L-x)\cdot\varepsilon^{-1/2}$, $\xi=(L-x)\cdot\varepsilon^{-1}$, $\mu=(L-x)\cdot\varepsilon^{-1/2}$ – appropriate regularization transformations.. After substitution (5) (1) we used the standard procedures to determine the function equating $c_{i,k,j}$, $\rho_{k,j}$, $j=\overline{0,n}$ [13], get

$$\begin{cases}\sigma_0\dfrac{\partial c_{i,k,0}}{\partial t}+v\dfrac{\partial c_{i,k,0}}{\partial x}+q_i c_{i,k,0}=0,\\ \dfrac{\partial\rho_0}{\partial t}=\beta_0\left(\sum_{i=1}^{m}q_i c_{i,k,0}\right)-\alpha_0\rho_{k,0},\\ c_{i,k,0}\big|_{x=0}=\bar{c}_{i,k}(t),\ \rho_{k,0}\big|_{x=0}=\bar{\rho}_k(t),\\ c_{i,k,0}\big|_{t=0}=\bar{c}_{i,k}(x),\ \rho_{k,0}\big|_{t=0}=\bar{\rho}_k(x),\end{cases}$$

where $\bar{c}_{i,k}(t)=c_i^*(t),\ \bar{\rho}_k(t)=\rho^*(t),$ while $k=0$, $\bar{c}_{i,k}(t)=c_{i,k-1,0}(L_{k-1},t),\bar{\rho}_k(t)=\rho_{k-1,0}(L_{k-1},t),$ while $k=\overline{1,l}$;

$$\begin{cases}-\sigma_*\rho_{k,j-1}\dfrac{\partial c_{i,k,j}}{\partial t}+v\dfrac{\partial c_{i,k,j}}{\partial x}-q_i\sigma_*\dfrac{\partial\rho_{k,j-1}}{\partial t}c_{i,j}=g_{i,k,j},\\ \dfrac{\partial\rho_{k,j}}{\partial t}=-\beta_*\rho_{k,j-1}\left(\sum_{i=1}^{m}q_i c_{i,k,j}\right)-\alpha_*\rho_{k,j-1}\rho_{k,j},\\ c_{1,k,j}\big|_{x=0}=0,\ c_{2,k,j}\big|_{x=0}=0,\ \rho_{k,j}\big|_{x=0}=0,\ c_{1,k,j}\big|_{t=0}=0,\\ c_{2,k,j}\big|_{t=0}=0,\ \rho_{k,j}\big|_{t=0}=0,\ i=\overline{1,m},\ j=\overline{1,n},\ k=\overline{1,l-1};\end{cases}$$

$$\begin{cases}b_{i,k-1}\underset{\sim}{M}''_{i,k,0\underset{\sim}{\xi}\underset{\sim}{\xi}}(\underset{\sim}{\xi},t)+\underset{\sim}{M}'_{i,k,0\underset{\sim}{\xi}}(\underset{\sim}{\xi},t)=0,\ \underset{\sim}{M}_{i,k,0}(\underset{\sim}{\xi},t)\underset{\underset{\sim}{\xi}\to\infty}{\to}0,\\ b_{i,k}\tilde{M}''_{i,k,0\tilde{\xi}\tilde{\xi}}(\tilde{\xi},t)-\tilde{M}'_{i,k,0\tilde{\xi}}(\tilde{\xi},t)=0,\ \tilde{M}_{i,k,0}(\tilde{\xi},t)\underset{\tilde{\xi}\to-\infty}{\to}0,\\ c_{i,k-1,j}(L_{k-},t)+\underset{\sim}{M}_{i,k,0}(0_-,t)=c_{i,k,j}(L_{k+},t)+\tilde{M}_{i,k,0}(0_+,t),\\ \left(c'_{i,k-1,0x}(L_{k-},t)+\underset{\sim}{M}'_{i,k,0\underset{\sim}{\xi}}(0_-,t)\right)=\\ =\dfrac{b_{i,k}}{b_{i,k-1}}\left(c'_{i,k,0x}(L_{k-},t)+\tilde{M}'_{i,k,0\tilde{\xi}}(0_+,t)\right);\end{cases}$$

$$\begin{cases}b_{*k-1}\underset{\sim}{P}''_{k,0\underset{\sim}{\mu}\underset{\sim}{\mu}}(\underset{\sim}{\mu},t)+\underset{\sim}{P}'_{k,0\underset{\sim}{\mu}}(\underset{\sim}{\mu},t)=0,\ \underset{\sim}{P}_{k,0}(\underset{\sim}{\mu},t)\underset{\underset{\sim}{\mu}\to\infty}{\to}0,\\ b_{*k}\tilde{P}''_{k,0\tilde{\mu}\tilde{\mu}}(\tilde{\mu},t)-\tilde{P}'_{k,0\tilde{\mu}}(\tilde{\mu},t)=0,\ \tilde{P}_{k,0}(\tilde{\mu},t)\underset{\tilde{\mu}\to-\infty}{\to}0,\\ \rho_{k-1,j}(L_{k-},t)+\underset{\sim}{P}_{k,0}(0_-,t)=\rho_{k,j}(L_{k+},t)+\tilde{P}_{k,0}(0_+,t),\\ \left(\rho'_{k-1,0x}(L_{k-},t)+\underset{\sim}{P}'_{k,0\underset{\sim}{\mu}}(0_-,t)\right)=\dfrac{b_{*k}}{b_{*k-1}}\left(\rho'_{k,0x}(L_{k-},t)+\tilde{P}'_{k,0\tilde{\mu}}(0_+,t)\right);\end{cases}$$

$$\begin{cases}b_{i,k-1}\underset{\sim}{M}''_{i,k,0\underset{\sim}{\xi}\underset{\sim}{\xi}}(\underset{\sim}{\xi},t)+\underset{\sim}{M}'_{i,k,0\underset{\sim}{\xi}}(\underset{\sim}{\xi},t)=\underset{\sim}{m}_{i,k}(\underset{\sim}{\xi},t),\ \underset{\sim}{M}_{k,0}(\underset{\sim}{\xi},t)\underset{\underset{\sim}{\xi}\to\infty}{\to}0,\\ b_{i,k}\tilde{M}''_{i,k,0\tilde{\xi}\tilde{\xi}}(\tilde{\xi},t)-\tilde{M}'_{i,k,0\tilde{\xi}}(\tilde{\xi},t)=\tilde{m}_{i,k}(\tilde{\xi},t),\ \tilde{M}_{i,k,0}(\tilde{\xi},t)\underset{\tilde{\xi}\to-\infty}{\to}0,\\ c_{i,k-1,j}(L_{k-},t)+\underset{\sim}{M}_{i,k,0}(0_-,t)=c_{i,k,j}(L_{k+},t)+\tilde{M}_{i,k,0}(0_+,t),\\ \left(c'_{i,k-1,0x}(L_{k-},t)+\underset{\sim}{M}'_{i,k,0\underset{\sim}{\xi}}(0_-,t)\right)=\\ =\dfrac{b_{i,k}}{b_{i,k-1}}\left(c'_{i,k,0x}(L_{k-},t)+\tilde{M}'_{i,k,0\tilde{\xi}}(0_+,t)\right),\ i=\overline{1,n};\end{cases}$$

$$\begin{cases}b_{*k-1}\underset{\sim}{P}''_{k,0\underset{\sim}{\mu}\underset{\sim}{\mu}}(\underset{\sim}{\mu},t)+\underset{\sim}{P}'_{k,0\underset{\sim}{\mu}}(\underset{\sim}{\mu},t)=\underset{\sim}{p}_k(\underset{\sim}{\mu},t),\ \underset{\sim}{P}_{k,0}(\underset{\sim}{\mu},t)\underset{\underset{\sim}{\mu}\to\infty}{\to}0,\\ b_{*k}\tilde{P}''_{k,0\tilde{\mu}\tilde{\mu}}(\tilde{\mu},t)-\tilde{P}'_{k,0\tilde{\mu}}(\tilde{\mu},t)=\tilde{p}_k(\tilde{\mu},t),\ \tilde{P}_{k,0}(\tilde{\mu},t)\underset{\tilde{\mu}\to-\infty}{\to}0,\\ \rho_{k-1,j}(L_{k-},t)+\underset{\sim}{P}_{k,0}(0_-,t)=\rho_{k,j}(L_{k+},t)+\tilde{P}_{k,0}(0_+,t),\\ \left(\rho'_{k-1,0x}(L_{k-},t)+\underset{\sim}{P}'_{k,0\underset{\sim}{\mu}}(0_-,t)\right)=\\ =\dfrac{b_{*k}}{b_{*k-1}}\left(\rho'_{k,0x}(L_{k-},t)+\tilde{P}'_{k,0\tilde{\mu}}(0_+,t)\right),\ i=\overline{1,n},\end{cases}$$

$$\begin{cases}b_{i,l}A''_{i,l,j,\xi\xi}+vA'_{i,l,j,\xi}=I(j)B'_{i,l,j-1,t}+\\ +I(j)\varepsilon^{\frac{j}{2}}B'_{i,l,j,t}+I(j+1)B'_{i,l,j,t}+\sigma_0A'_{i,l,j-1,t},\\ A_{i,l,j}\underset{\xi\to\infty}{\to}0,\ A'_{i,l,j,\xi}(L_j,t)=K_j(t);\end{cases}$$

$$b_{*l}(x)B''_{l,j,\mu\mu}-\alpha_j(x)B_{l,j}-B'_{l,j,t}=0,$$

$$B_{l,j}\underset{\mu\to\infty}{\to}0,\ B'_{l,j,\mu}(L_i,t)=H_j(t);$$

$$I(a)=\begin{cases}0,\ if\ \ a-is\ an\ even\ number,\\ 1,\ if\ \ a-is\ an\ odd\ number,\end{cases}$$

$$K_j(t)=\begin{cases}0,\ j=m+1,\\ -c'_{l,l,j,x}(L_j,t),\ j=0,...,m,\end{cases}$$

$$H_j(t)=\begin{cases}0,\ j=m+1,\\ -\rho'_{l,j,x}(L_j,t),\ j=0,...,m.\end{cases}$$

As a result, solving problems (5), (6) we found:

$$c_{i,k,0}(x,t)=\begin{cases}\bar{c}^*_{i,k}\left(t-\dfrac{\sigma_0 x}{v}\right)\cdot e^{\frac{q_i x}{v}},\ t\ge\dfrac{\sigma_0 x}{v},\\ \bar{c}^*_{i,k}\left(x-\dfrac{vt}{\sigma_0}\right)\cdot e^{q_i t},\ t<\dfrac{\sigma_0 x}{v},\end{cases}$$

$$\rho_{k,0}(x,t)=\beta_0 e^{-\alpha_0 t}\int_0^t\left(\sum_{i=1}^{m}q_i c_{i,k,0}(x,\tilde{t})\right)e^{\alpha_0\tilde{t}}d\tilde{t}+\bar{\rho}^*_{k*}(x),$$

$$c_{i,k,j}(x,t)=\begin{cases}-\dfrac{\sigma_* e^{-\int_0^x\lambda_{k,j}(\tilde{\tilde{x}},f(\tilde{\tilde{x}})+t-f(x))d\tilde{\tilde{x}}}}{v}\times\\ \times\displaystyle\int_0^x\frac{g_{i,k,j}(\tilde{x},f(\tilde{x})+t-f(x))e^{\int_0^{\tilde{x}}\lambda_{k,j}(\tilde{\tilde{x}},f(\tilde{\tilde{x}})+t-f(x))d\tilde{\tilde{x}}}}{\rho_{k,j-1}(\tilde{x},f(\tilde{x})+t-f(x))}d\tilde{x},\\ t\ge f(x),\\ e^{-\int_0^t\lambda_{k,j}(f^{-1}(\tilde{t}+f(x)-t),\tilde{t})d\tilde{t}}\times\\ \times\displaystyle\int_0^t e^{-\int_0^{\tilde{t}}\lambda_{k,j}(f^{-1}(\tilde{\tilde{t}}+f(x)-t),\tilde{\tilde{t}})d\tilde{\tilde{t}}}g_{i,k,j}(f^{-1}(\tilde{t}+f(x)-t),\tilde{t})d\tilde{t},\\ t<f(x),\end{cases}$$

$$\rho_{k,j}(x,t)=-\beta_* e^{-\alpha_*\int_0^t\rho_{k,j-1}(x,\tilde{t})d\tilde{t}}\times\int_0^t\rho_{k,j-1}(x,\tilde{\tilde{t}})\left(\sum_{i=1}^{2}c_{i,k,j}(x,\tilde{\tilde{t}})\right)e^{\alpha_*\int_0^{\tilde{\tilde{t}}}\rho_{k,j-1}(x,\tilde{t})d\tilde{t}}d\tilde{\tilde{t}},$$

where $g_{i,k,j}(x,t)=b_{i,k}\dfrac{\partial^2 c_{i,k,j-1}}{\partial x^2}+q_i\rho_{k,j-1}$, $\lambda_{k,j}(x,t)=-q_i\sigma_*\dfrac{\partial\rho_{k,j-1}}{\partial t}$,

$$\underset{\sim}{m}_{i,k}(\underset{\sim}{\xi},t)=I(j)\underset{\sim}{P}'_{k,j-1t}+I(j)\varepsilon^{\frac{j}{2}}\underset{\sim}{P}'_{k,jt}+I(j+1)\underset{\sim}{P}'_{k,jt}+\sigma(x)\underset{\sim}{M}'_{i,k,j-1t},$$

$$\tilde{m}_{i,k}(\underset{\sim}{\xi},t)=I(j)\tilde{P}'_{k,j-1t}+I(j)\varepsilon^{\frac{j}{2}}\tilde{P}'_{k,jt}+I(j+1)\tilde{P}'_{k,jt}+\sigma_0\tilde{M}'_{i,k,j-1t},$$

$$\underset{\sim}{p}_k(\underset{\sim}{\mu},t)=-\alpha_0\underset{\sim}{P}_k,\ \tilde{p}_k(\underset{\sim}{\mu},t)=-\alpha_0\tilde{P}_k,$$

$$I(a)=\begin{cases}0,\ if\ \ a-is\ an\ even\ number,\\ 1,\ if\ \ a-is\ an\ odd\ number,\end{cases}$$

Approximate values of the functions $f_j(x)$ found by [1] interpolation array (x_i,t_i), $i=\overline{1,n}$, where $x_i=\Delta x\cdot i$, $t_{i+1}=t_i+\dfrac{\Delta x}{v}\sigma_*\rho_{j-1}(x_i,t_i)$. The score of remaining members are helding the same [9].

Functions $\underset{\sim}{M}_{i,k,j}(\underset{\sim}{\xi},t)$, $\tilde{M}_{i,k,j}(\tilde{\xi},t)$, $\underset{\sim}{P}_{k,j}(\underset{\sim}{\mu},t)$, $\tilde{P}_{k,j}(\tilde{\mu},t)$, $A_{i,l,j}(\xi,t)$, $B_{l,j}(\mu,t)$ are similar as in the [10].

The numerical calculations. For simplicity, we assumed that the concentration of pollution is a two-component, then the system (1) - (3) can be rewritten as:

$$\begin{cases} \dfrac{\partial\left(\sigma(x,t)c_1(x,t)\right)}{\partial t}+\dfrac{\partial\rho(x,t)}{\partial t}+\dfrac{\partial\left(v(x,t)c_1(x,t)\right)}{\partial x}=D_1\dfrac{\partial^2 c_1}{\partial x^2}, \\ \dfrac{\partial\left(\sigma(x,t)c_2(x,t)\right)}{\partial t}+\dfrac{\partial\rho(x,t)}{\partial t}+\dfrac{\partial\left(v(x,t)c_2(x,t)\right)}{\partial x}=D_2\dfrac{\partial^2 c_2}{\partial x^2}, \\ \dfrac{\partial\rho(x,t)}{\partial t}=\beta(\rho)\left(q_1c_1(x,t)+q_2c_2(x,t)\right)-\varepsilon\alpha(\rho)\rho(x,t)+D_*\dfrac{\partial^2\rho}{\partial x^2}, \end{cases} \quad (6)$$

$$c_1\big|_{x=0}=c_1^*(t),\ c_1\big|_{t=0}=0,\ c_2\big|_{x=0}=c_2^*(t),\ c_2\big|_{t=0}=0,$$
$$\left.\frac{\partial c_1}{\partial x}\right|_{x=L}=0,\ \left.\frac{\partial c_2}{\partial x}\right|_{x=L}=0,\ \rho\big|_{x=0}=0,\ \rho\big|_{t=0}=0,\ \left.\frac{\partial\rho}{\partial x}\right|_{x=L}=0, \quad (7)$$
$$\left[c_1\right]_{x=L_k}=0,\ \left[c_2\right]_{x=L_k}=0,\ \left[\rho\right]_{x=L_k}=0,$$

$$\left[D_{1,k}\frac{\partial c_1}{dx}+vc_1\right]_{x=L_k}=0,\ \left[D_{2,k}\frac{\partial c_2}{dx}+vc_2\right]_{x=L_k}=0\ \left[D_{*k}\frac{\partial\rho}{dx}\right]_{x=L_k}=0. \quad (8)$$

Solutions of differential equations (12) under conditions (13) - (14) looked similar to the overall problem in the form of asymptotic series (see [1, 2]).

The results of calculations by formulas (4) at $c_1^*(t)=170$ mg/l, $c_2^*(t)=35$ mg/l, $L=0.8$ m; $v=1/360$ m/s; $\beta_0=0.3\ \text{s}^{-1}$; $q_1=q_2=1$; $\alpha_0=0.0056\ \text{s}^{-1}$; $\sigma_0=0.5$; $\alpha_*=1$; $\beta_*=1$; $\sigma_*=1$; $b=b_*=1$; $\varepsilon=0.001$.

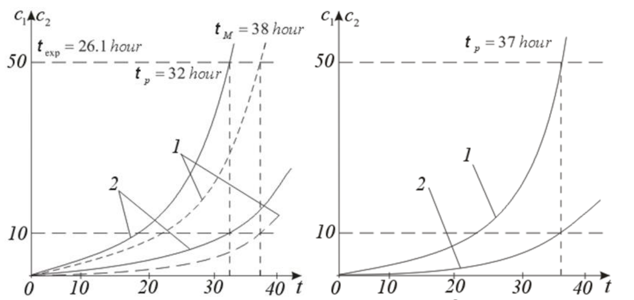

Fig. 2. Charts pollution concentration distribution at the output filter at time t: 1 - for Myntsom; 2 – of the formulas (5) when k = 1 – (a) and the formula (5) for k = 2 – (b)

This is the data which is obtained from the experiments (corresponding index "exp") in accordance with the classical model Mintsa (corresponding index "M") [12] and calculated by formulas (5) (corresponding to the index "p"). We see that the calculations by formulas (5) are more accurate compared with the classical model Mintsa. In Figure 3 we see that for k = 3 and k = 4, while the protective effect does not change, so 3 layers are sufficient to ensure the maximum effectiveness of cleaning on these criteria, which is used a 3-bed filters in practice. These results make it possible to calculate the dynamics of promoting pollution concentrations and deposition along the filter.

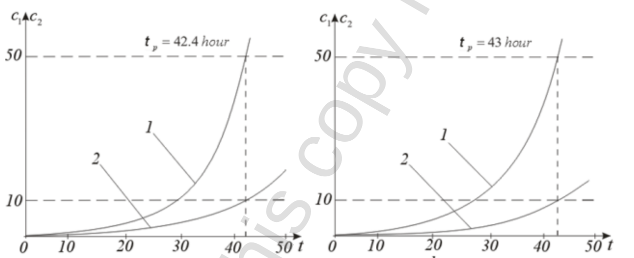

Fig. 3. Charts pollution concentration distribution at the output filter at time t by the formulas (5) when k = 3 – (a) and k = 4 – (b)

3. Conclusions

In the work we formed spatial mathematical model that takes into account the reverse effect determining factors (concentration of fluid and sediment contamination) on the medium characteristics (porosity coefficient, filtration, diffusion - diffusion, mass transfer, etc..) during the simulation of cleaning liquids from multicomponent pollution n sorption-layer filters.

There was offered the algorithm for solving the corresponding model problem. There were shown the results of calculations of the distribution of impurity concentration and mass volume house ¬ shock height porous filter loading for different points in time and for different amounts of layers. We made the comparative characterization data which is obtained experimentally by calculated based on the classic model Mintsa and gave us the result of calculations (in particular, according to data presented in Fig. 2) we see that the accuracy of the results of our calculations are significantly higher compared with the calculations obtained according to the classical model Mintsa, and to ensure maximum efficiency of the filter is enough 3 layers of backfill.

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Ph.D. Andrij P. Safonyk
e-mail: safonik@ukr.net

Associate professor of the Department of automation, electrical engineering and computer integrated technologies of the Institute of Automation, Cybernetics and Computer Engineering, National University of Water Management and Nature Resources Use, Rivne, Ukraine.
Engaged in scientific mathematical modeling of natural and technological processes, computer simulation of technological processes, computer techniques and computer technologies, programming.

Ph.D. Oleksandr M. Naumchuk
e-mail: alexnu@ukr.net

Associate professor of the Department of automation, electrical engineering and computer integrated technologies of the Institute of Automation, Cybernetics and Computer Engineering, National University of Water Management and Nature Resources Use, Rivne, Ukraine.
Engaged in scientific research of effects of electromagnetic radiation from the mobile communication equipment on the environment.

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