Steady-state and transient heat transfer through fins of complex geometry

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Abstract   Various methods for steady-state and transient analysis of temperature distribution and efficiency of continuous-plate fins are presented. For a constant heat transfer coefficient over the fin surface, the plate fin can be divided into imaginary rectangular or hexangular fins. At first approximate methods for determining the steady-state fin efficiency like the method of equivalent circular fin and the sector method are discussed. When the fin geometry is complex, thus transient temperature distribution and fin efficiency can be determined using numerical methods. A numerical method for transient analysis of fins with complex geometry is developed. Transient temperature distributions in continuous fins attached to oval tubes is computed using the finite volume – finite element methods. The developed method can be used in the transient analysis of compact heat exchangers to calculate correctly the heat flow rate transferred from the finned tubes to the fluid.

Keywords: Fin of complex geometry; Fin efficiency; Finite volume method – finite element method

Nomenclature

A –   surface area, m^2
c –   specific heat capacity, J/(kg K)

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h	–	heat transfer coefficient, $\mathrm{W/m^2 K}$)
k	–	thermal conductivity, $\mathrm{W/(m\ K)}$
m	–	fin parameter, $\mathrm{1/m}$
$\mathbf{n}$	–	outward unit normal vector
P	–	pitch of tube arrangement, m
$\dot{\mathbf{q}}$	–	heat flux vector
$\dot{Q}$	–	heat flow rate, W
r	–	radius, m
s	–	fin pitch, m
t	–	time, s
T	–	temperature, $\mathrm{^oC}$ or K
$\bar{T}$	–	mean temperature, , $\mathrm{^oC}$ or K
V	–	volume, $\mathrm{m^3}$
x, y, z	–	Cartesian coordinates

Greek symbols

δ_f	–	fin thickness, m
ρ	–	density, $\mathrm{kg/m^3}$
η	–	fin efficiency

Subscripts

b	–	base
e	–	equivalent
f	–	fluid
i	–	number of finite element or fin section
in	–	inner
l	–	lateral
max	–	maximum value
o	–	outer

1   Introduction

Plate fin and tube heat exchangers are used as economizers in steam power boilers, air-conditioning coils, convectors for home heating, waste-heat recovery systems for gas turbines, cooling towers, air-fin coolers, car radiators and heater cores, which are used to heat the air within the car passenger compartment. Compact heat exchangers with plate fins can be manufactured by electrical or laser welding of the fins to the tube or can also be extruded directly from the tube wall metal. Continuous-plate fins are used extensively in plate fin and tube heat exchangers.

A recurring component of extended surface is that of a single sheet of metal pierced by round or oval tubes in either an in-line or a staggered arrangement. In this type of heat exchangers each fin extends from tube to tube. Marking the symmetry planes between the tubes gives a rectangle or

a hexagon around the tube. It is not possible to find an exact solution for this type of fins. Numerical methods such as finite element method or finite volume methods can be used to determine the temperature distribution or the fin efficiency.

At first the approximate methods, like the method of the equivalent radius or the sector method for determination of the fin efficiency will be discussed. When the fin geometry is complex, the temperature distribution and fin efficiency can be determined using finite element method or finite control volume method. Commercial computational fluid dynamics CFD software can be used for this purpose. To demonstrate the effectiveness of this approach the efficiency of the rectangular fin attached to the circular tube will be computed using the ANSYS Fluent software [26].

A numerical method for the transient analysis of fins with complex geometry has been developed. Transient temperature distributions in continuous fins attached to oval tubes will be computed using the finite volume method – finite element method (FVM-FEM). The temperature distribution for a step temperature rise in the fin base temperature will be calculated. The fin model will be divided into triangular elements and then finite volumes will be formed around the nodes by connecting triangle gravity centers with the side centers of triangles. A system of differential equations of the first order for transient temperature at the nodes will be solved using the Euler explicit method. The efficiency of the fin can be calculated for steady-state and transient conditions. The developed method can be used in the transient analysis of compact heat exchangers to calculate correctly the heat flow rate transferred from the finned tubes to the fluid. The agreement of the results obtained by the developed method and commercial software are good despite the coarse finite volume mesh used in the FVM-FEM.

2 Approximate analytical methods for determining fin efficiency of complex geometry

Plate fin and tube heat exchangers are used as economizers in steam power boilers, air-conditioning coils, convectors for home heating, waste-heat recovery Systems for gas turbines, cooling towers, air-fin coolers, car radiators and heater cores, which are used to heat the air within the car passenger compartment [1–9]. Compact heat exchangers with plate fins can be manufactured by electrical or laser welding of the fins to the tube or can also be extruded directly from the tube wall metal. Continuous-plate fins are

used extensively in plate fin and tube heat exchangers [1, 3–6]. A recurring component of extended surface is that of a single sheet of metal pierced by round or oval tubes in either an in-line (Fig. 1) or a staggered (Fig. 2) arrangement [4]. For the constant heat transfer coefficient over the fin surface, the plate can be divided into imaginary rectangular (Fig. 1b) or hexangular fins (Fig. 2b) [4,9,13,14,17,18]. Because of the symmetry, the outer circumference of the polygonal fin is thermally insulated.
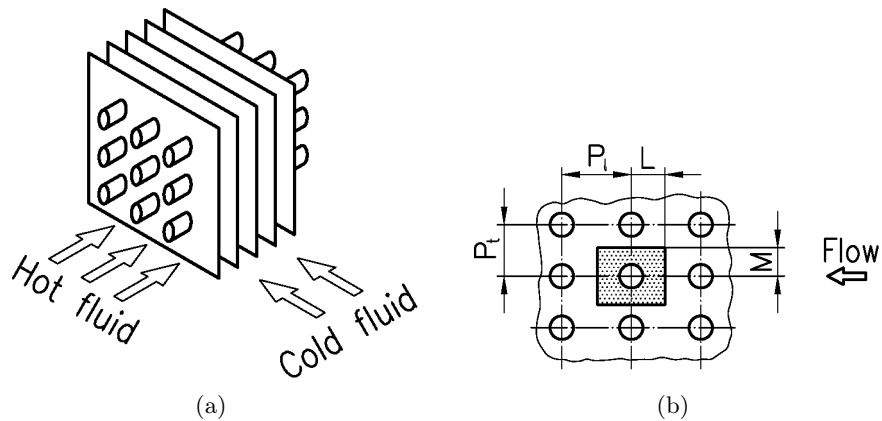
Figure 1: Plate fin and tube heat exchanger with an in-line tube arrangement: (a) cross flow heat exchanger, (b) an in-line tube arrangement.
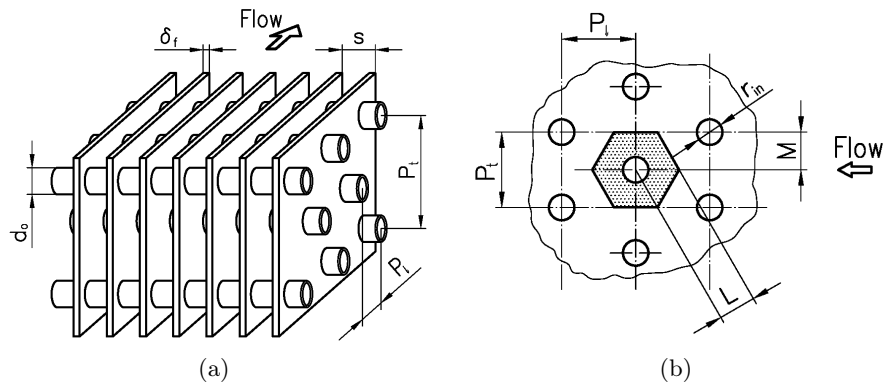
Figure 2: Plate fin and tube heat exchanger with an staggered tube arrangement: (a) cross flow heat exchanger, (b) an staggered tube arrangement.

Zabronsky has proposed that the efficiency of the square fin is equal approximately to the efficiency of the circular fin of equal surface area [13]. The accuracy of the method of equivalent circular fin is not always satisfactory, Especially for rectangular fins. Carrier and Anderson have shown that the accuracy of the sector method is better [14]. Rich prepared graphs to facilitate the application of the sector method [15]. Schmidt [12,16] has found mathematical correlations which are accurate and simple. The Schmidt method is based on determining a radius, $r_{o,e}$, of a circular fin that has the same fin efficiency as the rectangular or hexangular fin. The circular fin efficiency, η, is given by

$$\eta = \frac{\tanh(mr_{in}\phi)}{mr_{in}\phi} \ , \tag{1}$$

where the parameters m and ϕ are defined as

$$m = \left(\frac{2h}{k\delta_f}\right)^{\frac{1}{2}} \ , \quad \phi = \left(\frac{r_o}{r_{in}} - 1\right)\left[1 + 0.35\ln\left(\frac{r_o}{r_{in}}\right)\right] \ . \tag{2}$$

The symbols in Eqs. (1) and (2) denote: h – heat transfer coefficient; k – thermal conductivity of the fin material, r_{in}, r_o – inner and outer radius of the fin, δ_f – fin thickness. For the rectangular fin, Schmidt has developed the following correlation

$$\frac{r_{o,e}}{r_{in}} = 1.28\psi(\beta - 0.2)^{1/2} \ , \quad \psi = \frac{M}{r_{in}} \ , \quad \beta = \frac{L}{M} \ , \tag{3}$$

distances length L and M are as in Fig. 1b, where L is always selected to be greater than or equal to M. The parameter ϕ defined by Eq. (2) is evaluated using the equivalent outer radius $r_{o,e}$ instead of r_o. Similar correlation has been developed for the hexagonal fin

$$\frac{r_{o,e}}{r_{in}} = 1.27\psi(\beta - 0.3)^{1/2} \ , \tag{4}$$

where L and M are defined in Fig. 2b. The dimensions are selected in such a way that $L \leq M$. The sector method is more accurate, but very laborious [9,15,17,18]. It was developed in 1940s [4] and is used to date. Rich prepared graphs included in ASHRAE Handbook [15] which make it easier to determine the efficiency of complex shape fins by the sector method. Rectangular and hexagonal fins may be analyzed by the sector method. In this method, the smallest symmetrical section of the fin (Figs. 3a and 4a) is

divided into n sectors (Figs. 3b and 4b). The surface area of the ith sector is A_i $(i = 1, \ldots, n)$. Each sector is considered as a circular fin with the radius $r_{o,i}$. The outer radius $r_{o,i}$ of each sector is determined by equating the surface area of the sector with the area of the equivalent circular sector. The fin efficiency η_i of the ith sector is then calculated using the formula for the annular fin efficiency.
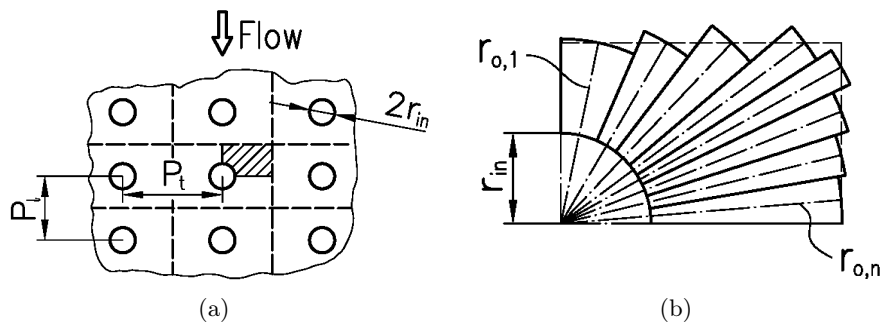
Figure 3: Illustration of the sector method for an in-line tube arrangement; (a) division of the plate fin into imaginary rectangular fins, (b) division of the symmetrical part of the fin into sectors.
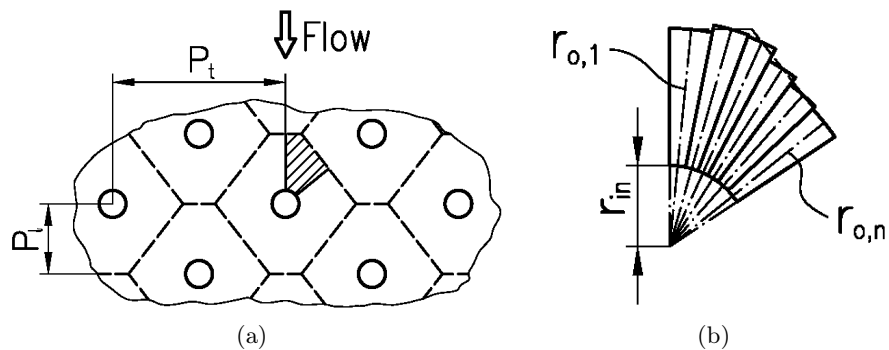
Figure 4: Illustration of the sector method for a staggered tube arrangement; (a) division of the plate fin into imaginary hexagonal fins, (b) division of the symmetrical part of the fin into sectors.

The fin efficiency, η, for the entire fin is the surface area weighted average of η_i for each sector

$$\eta = \frac{\sum\limits_{i=1}^{n} \eta_i A_i}{\sum\limits_{i=1}^{n} A_i} \ .$$ (5)

The fin efficiency determined by the sector method is lower than the actual fin efficiency since only radial heat conduction in the fin is assumed.

The circumferential heat flow between the sectors is neglected. Since the heat flow occurs in the direction of the least thermal resistance then the higher heat flow rate is dissipated from the real fin to the environment. Thus, Eq. (5) gives a little smaller fin efficiency in comparison with more sophisticated methods. An example of the application of the of equivalent annulus method and the sector method for determining the efficiency of the hexagonal fin is given in [9].
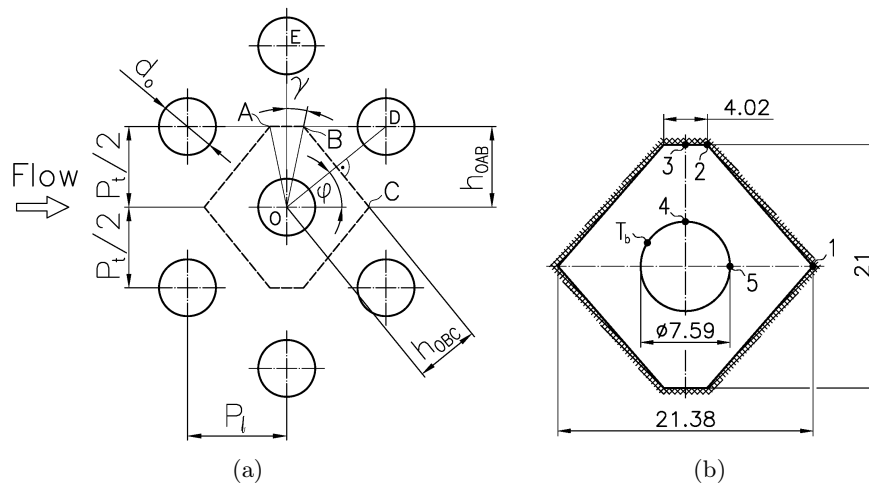

Figure 5: Division of plate fin into imaginary hexagonal fins (a) and dimensions of the fin (b).

A plate fin with staggered tube arrangement was divided into hexagonal imaginary fins (Fig. 5a). The dimensions of the analyzed fin are given in Fig. 5b. The efficiency of the fin shown in Fig. 5b was estimated by the sector method.

The obtained values of the fin efficiency: $\eta = 0.9373$ agrees quite well with the value obtained using the finite element method: $\eta = 0.9380$. Also

the value obtained by the method of the equivalent circular radius gave a satisfactory result: $\eta = 0.9394$, which is also close the FEM result.

3   Determining temperature distribution and fin efficiency using the CFD software

When the fin geometry is complex, then the temperature distribution and fin efficiency can be determined using the finite element method [9] or the finite control volume method [8,9]. Commercial software, like ANSYS Fluent or CFX, can be used for this purpose [5,6,9,19–21]. The fin efficiency is defined as a ratio of the heat flow transferred from the fin to the environment to the heat flow dissipated from the isothermal fin with the base temperature T_b to the environment

$$\eta = \frac{\dot{Q}}{\dot{Q}_{\max}} \ .$$
(6)

The heat flow $\dot{Q}$ is given by the expression

$$\dot{Q} = \int\limits_{A_l} h\,(T - T_f)\,dA \ ,$$
(7)

where the symbol A_l denotes the lateral surface area of the fin. When the heat transfer coefficient, h, and the environment temperature, T_f, are constant over the fin surface, then Eq. (7) can be transformed to

$$\dot{Q} = h\,A_l \left(\bar{T} - T_f \right)$$
(8)

where $\bar{T}$ is the area-averaged fin temperature, defined as

$$\bar{T} = \frac{\int\limits_{A_l} T\,dA}{\sum\limits_{i=1}^{N_e} A_{e,i}} = \frac{\sum\limits_{i=1}^{N_e} \bar{T}_{e,i}\,A_{e,i}}{A_l} \ .$$
(9)

The symbol T stands for the temperature of the fin surface adjacent to the fluid, $\bar{T}_{e,i}$ denotes the average temperature of the finite element or finite volume surface with the area $A_{e,i}$ exposed to the environment. The fin surface neighboring with the fluid is divided into N_e elements. The maximum heat flow, $\dot{Q}_{\max}$, which can be transferred from the fin to the fluid is

$$\dot{Q}_{\max} = h\,A_l \left(T_b - T_f \ . \right)$$
(10)

The procedure developed above can easily be extended to account for the space dependent heat transfer coefficient. To demonstrate the effectiveness of the presented method the efficiency of the fin shown in Fig. 6 was computed using the commercial software.
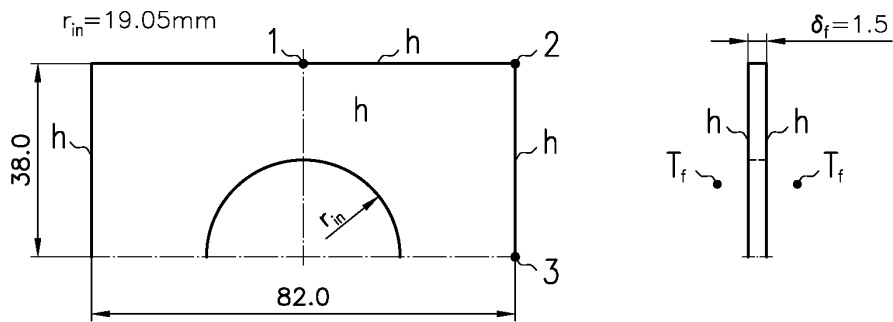
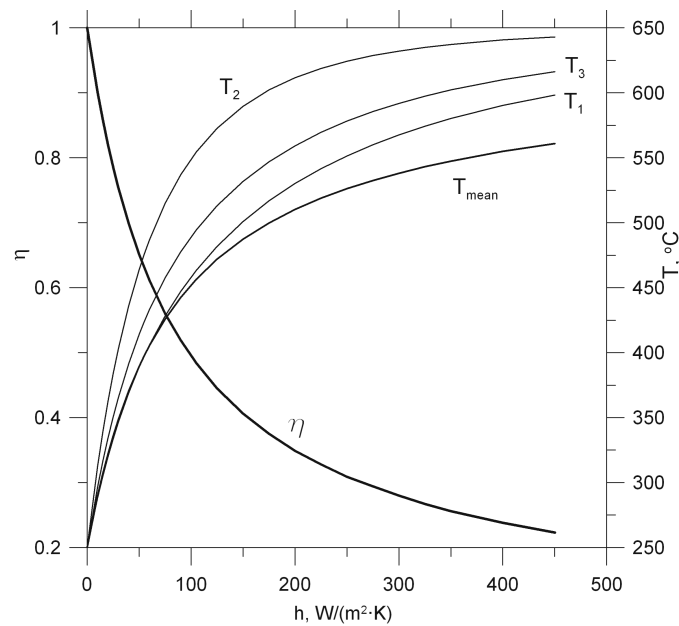Figure 6: Rectangular fin used in steam boiler economizers; dimensions are in millimeters.

Figure 7: Fin efficiency, η, temperatures T_1, T_2, and T_3 at the characteristic points 1, 2, and 3 marked in Fig. 6 and average temperature of the fin as a function of the heat transfer coefficient, h.

The fin base temperature T_b and flue gas temperature T_f (the environment temperature) are 250 ºC and 650 ºC, respectively. The fin is made from the low alloy steel 15Mo3 with the thermal conductivity $k(T)$ given by the expression

$$k = 42.773 + 4.42 \times 10^{-2}T - 9.59 \times 10^{-5}T^2 + 4.0 \times 10^{-8}T^3 \ ,$$

where k is expressed in W/(mK) and T in K. The quarter of the fin was divided into 20248 finite volumes. First, the three dimensional temperature field was computed, then the fin efficiency was determined. The computed temperature at the characteristic points (Fig. 6) and the fin efficiency are shown in Fig. 7 as a function of the heat transfer coefficient h.

4   Mathematical formulation of the finite volume-finite element method for determining transient temperature in fins

Two-dimensional heat conduction equation for a fin in which temperature difference across the thickness is negligible has the following form [25]:

$$c(T) \, \rho(T) \, \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} - \frac{2\,h}{\delta_f} \, (T - T_f) \ , \qquad (11)$$

where the heat flux vector $\mathbf{q}$ is given by the Fourier law

$$\mathbf{q} = -k_x(T) \, \frac{\partial T}{\partial x} \mathbf{i} - k_y(T) \, \frac{\partial T}{\partial y} \mathbf{j} \, . \qquad (12)$$

The symbols in Eqs. (11) and (12) denote: c – specific heat, ρ – density, T – fin temperature, T_f – fluid temperature, t – time, h – heat transfer coefficient, δ_f – fin thickness, k_x and k_y – thermal conductivity in x and y direction, $\mathbf{i}, \mathbf{j}$ are the unit vectors un the positive x and y directions.

The surface integral of Eq. (11) over a given region Ω in 2D-space is

$$\int_\Omega c(T) \, \rho(T) \, \frac{\partial T}{\partial t} \, dA = -\int_\Omega \nabla \cdot \mathbf{q} \, dA - \int_\Omega \frac{2\,h}{\delta_f} \, (T - T_f) \, dA \, . \qquad (13)$$

Applying the divergence theorem to Eq. (13) gives

$$V_\Omega \, c(\bar{T}) \, \rho(\bar{T}) \, \frac{d\bar{T}}{dt} = -\int_\Gamma \mathbf{q} \cdot \mathbf{n} \, ds - \frac{2\,h}{\delta_f} \, (\bar{T} - T_f) \, A_\Omega \, , \qquad (14)$$

where $\bar{T}$ is the mean temperature, $\mathbf{n}$ – the outward unit normal on s, ds is the arch length along Γ, and A_Ω is the surface area of the region Ω.

Equation (14) can be expressed for the finite region Ω_{1aoc1} as

$$\frac{A_{123}}{3} \frac{\delta_f}{2} c(T_1) \rho(T_1) \frac{dT_1}{dt} =$$

$$= -\frac{\delta_f}{2} \int\limits_a^o \mathbf{q} \cdot \mathbf{n}\, ds - \frac{\delta_f}{2} \int\limits_o^c \mathbf{q} \cdot \mathbf{n}\, ds - \frac{A_{123}}{3} h \left(\bar{T} - T_f \right) , \qquad (15)$$

where the symbol A_{123} denotes the surface area of the triangle 1-2-3 (Fig. 8). The lateral surfaces 1-a and 1-c are assumed to be thermally insulated.

Equation (13) can be rewritten for nonlinear transient heat conduction equation in the medium with anisotropic and temperature dependent thermal conductivity

$$c(T) \rho(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[k_x(T) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[k_y(T) \frac{\partial T}{\partial y} \right] - \frac{2h}{\delta_f} (T - T_f) .$$

$$(16)$$

The temperature in the triangle 1-2-3 is approximated by a linear function

$$T = a_1 + a_2 x + a_3 y , \qquad (17)$$

where the constants a_1, a_2, and a_3 determined from the conditions

$$T(x_1, y_1) = T_1 , \quad T(x_2, y_2) = T_2 , \quad T(x_3, y_3) = T_3 \qquad (18)$$

are

$$a_1 = \frac{1}{2A_{123}} \left[(x_2 y_3 - x_3 y_2) T_1 + (x_3 y_1 - x_1 y_3) T_2 + (x_1 y_2 - x_2 y_1) T_3 \right] ,$$

$$a_2 = \frac{1}{2A_{123}} \left[(y_2 - y_3) T_1 + (y_3 - y_1) T_2 + (y_1 - y_2) T_3 \right] , \qquad (19)$$

$$a_3 = \frac{1}{2A_{123}} \left[(x_3 - x_2) T_1 + (x_1 - x_3) T_2 + (x_2 - x_1) T_3 \right] ,$$

where x_1, x_2, x_3 and y_1, y_2, y_3 are coordinates of triangle 1,2,3 vertex.

The integrals in Eq. (15) can be evaluated using the Fourier law (12) and approximate temperature distribution (17)

$$-\int\limits_a^o \mathbf{q} \cdot \mathbf{n}\, ds = -k_x(T_o) \tfrac{y_a - y_o}{2A_{123}} \left[(y_2 - y_3) T_1 + (y_3 - y_1) T_2 + (y_1 - y_2) T_3 \right] +$$

$$+ k_y(T_o) \tfrac{x_a - x_o}{2A_{123}} \left[(x_3 - x_2) T_1 + (x_1 - x_3) T_2 + (x_2 - x_1) T_3 \right] ,$$
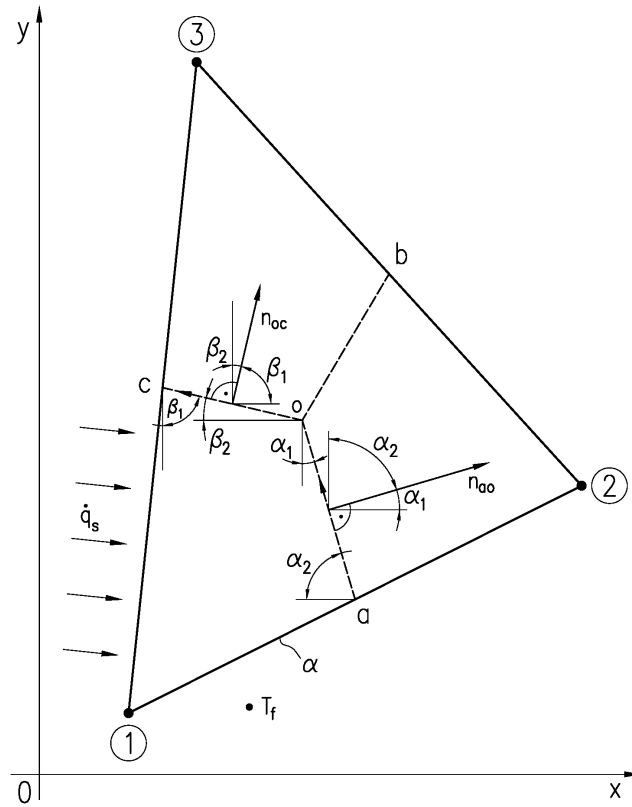
$$(20)$$

Figure 8: Control volume for energy conservation equation in the region $1-a-o-c-1$, which is a part of the finite volume associated with the node 1.

$$-\int\limits_o^c \mathbf{q} \cdot \mathbf{n}\, ds = k_x\left(T_o\right) \frac{y_c-y_o}{2A_{123}}\left[\left(y_2-y_3\right)T_1+\left(y_3-y_1\right)T_2+\left(y_1-y_2\right)T_3\right]-$$
$$-k_y\left(T_o\right) \frac{x_c-x_o}{2A_{123}}\left[\left(x_3-x_2\right)T_1+\left(x_1-x_3\right)T_2+\left(x_2-x_1\right)T_3\right]\ ,$$

$$(21)$$

where $x_o, y_o,\ x_a, y_a,$ and x_c, y_c are the coordinates of the points o, a and c (Fig. 8). If the lateral surface 1-3 is heated by the heat flux q_s and the surface 1-2 is heated by convection, then additional integrals must be taken into account in Eq. (17):

$$-\int\limits_1^c \mathbf{q} \cdot \mathbf{n}\, ds = q_s s_{1c} \qquad\qquad (22)$$

and

$$-\int\limits_{1}^{a} \mathbf{q} \cdot \mathbf{n}\, ds = h\left(T_f - \frac{T_1 + T_a}{2}\right) s_{1a} =$$

$$= h\left(T_f - \frac{T_1 + \frac{T_1+T_2}{2}}{2}\right) s_{1a} = h\left[T_f - \left(\frac{3T_1}{4} + \frac{T_2}{4}\right)\right] s_{1a} \quad (23)$$

where the side lengths s_{1c} and s_{1a} are given by (Fig. 8)

$$s_{1c} = \sqrt{(x_c - x_1)^2 + (y_c - y_1)^2}\,, \qquad (24)$$

$$s_{1a} = \sqrt{(x_a - x_1)^2 + (y_a - y_1)^2}\,. \qquad (25)$$

Substituting integrals (22) and (23) into Eq. (17) and considering additional integrals (24) and (25) which account heat transfer on the sides 1-c and 1-a gives

$$c(T_1)\rho(T_1)\frac{A_{123}}{3}\frac{\delta_f}{2}\frac{dT_1}{dt} =$$

$$= k_x\left(T_o\right)\frac{y_c - y_a}{2A_{123}}\frac{\delta_f}{2}\left[(y_2 - y_3)T_1 + (y_3 - y_1)T_2 + (y_1 - y_2)T_3\right] +$$

$$+ k_y\left(T_o\right)\frac{x_c - x_a}{2A_{123}}\frac{\delta_f}{2}\left[(x_2 - x_3)T_1 + (x_3 - x_1)T_2 + (x_1 - x_2)T_3\right] +$$

$$+ \frac{A_{123}}{3}h\left(T_f - T_o\right) + q_s s_{1c}\frac{\delta_f}{2} + h\left[T_f - \left(\frac{3T_1}{4} + \frac{T_2}{4}\right)\right]s_{1a}\frac{\delta_f}{2}\,,$$

$$(26)$$

where the symbol T_o stands for the temperature at the gravity center of the triangle 1-2-3 (Fig. 8):

$$T_o = \frac{T_1 + T_2 + T_3}{3}\,, \qquad (27)$$

The sides 1-a and 1-c are insulated in internal finite volumes, thus the last two terms in Eq. (28) should be omitted. A temperature distribution in the rectangular fin attached to the oval tube was calculated using the developed method. The dimensions of the analyzed fin, expressed in millimeters, are shown in Fig. 9a. The fin was divided into 19 finite volumes (Fig. 9b). The lateral surfaces: 4-19, 1-13, 2-3 are thermally insulated, while on the surfaces: 4-6-3 and 1-5-2 convection heat transfer occurs. The computations were carried out for the following data: $c = 896$ J/(kg K), $\rho = 2707$ W/m K), $k = 207$ W/(m K), $\delta_f = 0.08$ mm, $T_f = 0$ °C, $T_b = 100$ °C, $T_0 = 0$ °C, $h = 25$ W/(m² K). The temperature distribution was also calculated using commercial software ANSYS, v.11.0 [26].
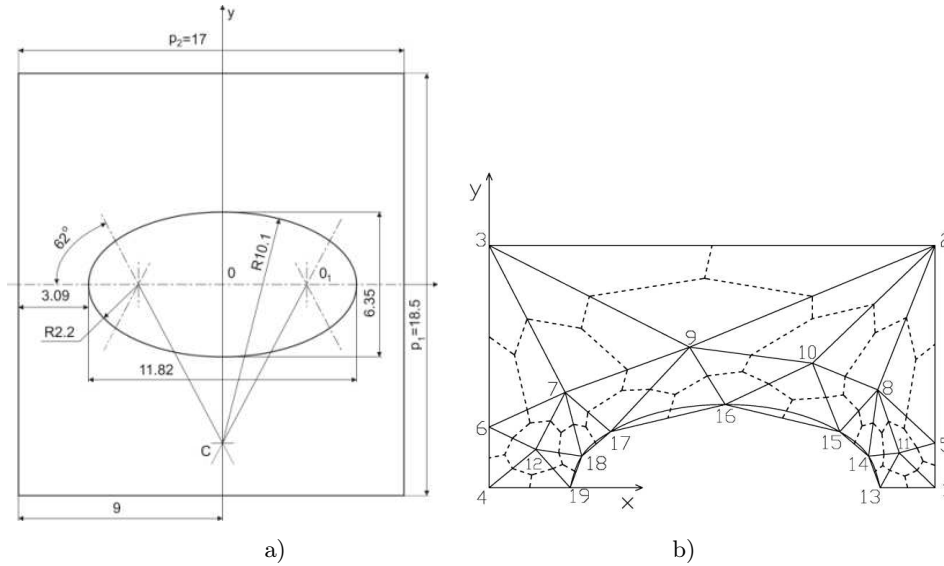
Figure 9: Rectangular fin attached to oval tube (a) and division of the fin model into finite volumes for the FVM-FEM (b).
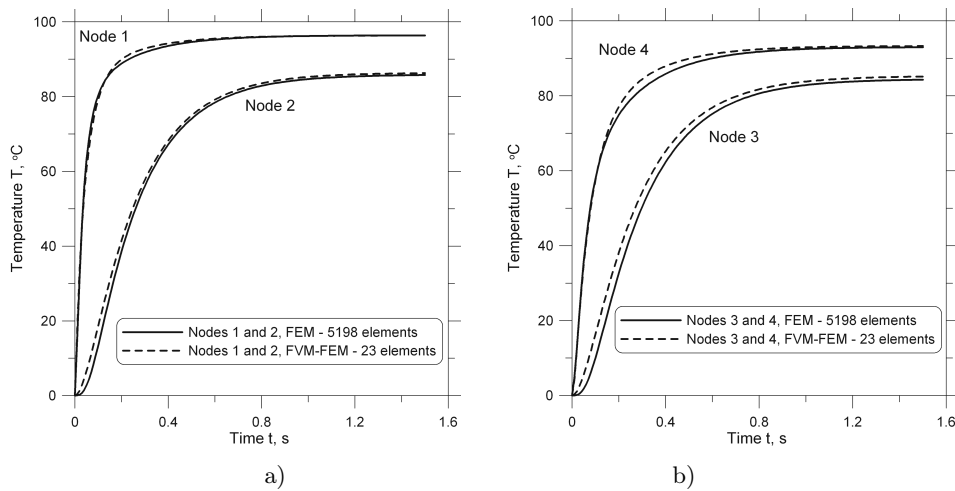

Figure 10: Time changes of fin temperature at nodes 1 and 2 (a) and at nodes 3 and 4 (b) for $h = 25 \text{ W}/(\text{M}^2 \text{ K})$.

The steady-state temperatures for $h = 25 \text{ W}/(\text{m}^2 \text{ K})$ are: $T_1 = 98.11\,^\circ\text{C}$, $T_2 = 92.80\,^\circ\text{C}$, $T_3 = 92.21\,^\circ\text{C}$, $T_4 = 96.51\,^\circ\text{C}$ at the nodes 1, 2, 3, and 4 are in good agreement with the results obtained using the commercial

software for very fine mesh: $T_1 = 98.13\,^\circ\mathrm{C}$, $T_2 = 92.50\,^\circ\mathrm{C}$, $T_3 = 96.32\,^\circ\mathrm{C}$, $T_4 = 93.021\,^\circ\mathrm{C}$. The transient response of the fin illustrates Fig. 10. The agreement of the results obtained by the developed method and are good despite the coarse finite volume mesh used in the FVM-FEM.

Using the computed temperatures, the steady-state fin efficiency, η_{fin}, can be evaluated:

$$\eta_{fin} = \frac{\int\limits_{A_{fin}} h\,(T - T_f)\,dA}{h\,A_{fin}\,(T_b - T_f)} = \frac{\bar{T}_{fin} - T_f}{T_b - T_f}\,, \tag{28}$$

where the symbol $\bar{T}_{fin}$ stands for the mean temperature of the fin and A_{fin} is the total fin area on which the heat transfer occurs. The fin efficiency was also determined based on the temperature distribution obtained by the present method

$$\eta_{fin} = \frac{\sum\limits_{i=1}^{N} A_{123,\,i}\,(T_{o,\,i} - T_f) + \sum\limits_{j=1}^{N_l} A_{l,\,j}\,(\bar{T}_{l,\,j} - T_f)}{A_{fin}\,h\,(T_b - T_f)}\,, \tag{29}$$

where the symbols denote: $A_{123,i}$ – surface area of the ith triangle, $T_{o,i}$ – fin temperature at the gravity center of the ith triangle, N – number of triangles, $A_{l,j}$ – area of the jth lateral surface with the thickness δ_f , $\bar{T}_{l,\,j}$ – mean temperature of the j-th lateral surface with the thickness δ_f, N_l – number of lateral surfaces with the thickness δ_f.

The fin efficiency calculated from the expressions (28) and (29) for $h = 25\ \mathrm{W/(m^2\,K)}$ are $\eta_{fin} = 0.9502$ and $\eta_{fin} = 0.9491$, respectively. The accuracy of the present method is very satisfactory. In spite of the coarse finite volume mesh used in present method (Fig. 9) the agreement of the calculated efficiencies is very good.

5   Conclusions

The developed method for the analysis of the transient response of the rectangular plate fins is very effective. It can be used for determining the transient response and steady state efficiency of fins attached to the oval as well as to the circular tubes. The agreement between results obtained by the present and the FEM method is very satisfactory, despite a coarse mesh used in the presented FVM-FEM. The developed method can be applied

for transient analysis of compact heat exchangers to calculate correctly the heat flow rate transferred from the hot to cold fluid.

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