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VALUATION OF ROTODYNAMIC PUMPS OPERATION, BY MEANS OF, DIMENSIONAL ANALYSIS

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Abstract

The following article introduces determination method of rotodynamic pumps operation. The pumps have been installed in installations, basing on their work parameters, by means of dimensional analysis. Operation of rotodynamic pumps, as energetic devices, according to J. Girtler, has been treated as a new physical quantity of dimension, Joul multiplied by second [J·s]. It expresses transformation of energy supplied to liquid, being forced through by the pump. This quantity can be determined, on the basis of algebraic diagram of dimensional analysis, constructed by S. Drobot. This diagram allowed us to control correctness of inference rules, in relation to mathematical aspect, used in numerical functions of rotodynamic pumps operation, fitted in instalations.

Key words: *operation of rotodynamic pump, algebraic diagram of dimensional analysis, physical quantity of Joul multiplied by second dimension, dimensional analysis*

1. Introduction

Operation of rotodynamic pump consists in transportation of mechanical energy from any external source into the liquid which passes through the pump. This energy is used up on gain in angular momentary or liquid circulation and also overcoming of hydraulic resistance in pressure conduit.

It causes an increase of all energy components, such as: position pressure and kinetic energy in flowing liquid. An increase of liquid energy in rotodynamic pump is determined by constructional parameters of rotor and its rotational speed. On the other hand, pressure increase depends on delivery of rotodynamic pump.

According to the above, it makes sense to investigate the operation at the same time, by energy supplied to it, during its transfer to the liquid being just forced through. Rotodynamic pumps being passive working machines, belong to energetic devices. Operation valuation of energetic devices is proposed by J. Girtler in works [3,4,6]. He equates it to dimensional physical quantity of measure unit called Joul second. Such operation can be interpreted as transfer of mechanical energy to the liquid that is forced through by the pump, in a definite time.

Operation of rotodynamic pump, in which transfer of energy takes place, can be the carrier of information about its technical condition.

Parameters of rotodynamic operation depend on characteristic of installation in which the pump takes place.

In building of rotodynamic pumps, one can observe a tendency to increase rotational speed of the rotor and its higher speed, in order to diminish the size of the pump and of the electric motor and thus lower the cost of investment and exploitation [5,8].

It is the reason why in pumps of newer generation , one could observe destructive force of cavitation, worse than in pumps of older generation. Durability and reliability of pump operation, to a great extent, depend on cavitation character.

2. Rotodynamic pump operation in dimensional space

Rotodynamic pump operation makes the liquid molecules rotate together with the rotor, and at the same time move along the blades from the centre towards the circuit. It decreases the pressure at the rotor inlet, creating a phenomenon of liquid sucking into the pump's interior. Molecule movement is generated by the influence of circumferential forces of friction and radial centrifugal forces.

Water flow through the pump takes place in the field of gravity forces. Field of current changes according to energy position. During liquid flow through the rotor, its kinetic energy increases, which is caused by an increase of floating speed of liquid molecules in tis circular movement around the rotor axis. Further change of energy takes place under influence of decrease of relative speed of the liquid, owing to its flow through channels between stator blades of increasing transverse intersection or in diffuser. It means that revolving rotor of the pump generates energy increase in flowing liquid. This energy is equal to work done by the torque of pump's propulsion engine. The amount of energy transferred to liquid by the rotor, can be calculated, on the basis of second principle of dynamics, for rotary motion. According to this principle, the torque transferred by rotating rotor, causes the change of its angular momentum of liquid. The work done by the rotor of the pump causes an increase of pressure energy, overcoming of altitude difference, surmounting of pressure difference and friction resistance between inlet intersection and the outlet one of the pump [2,8]. Thus it causes energy increase of the liquid flowing through the rotor. So, the operation of rotodynamic pump can be defined by the following formula:

$$D = \int_0^t \rho \cdot Q \cdot \Delta E(\tau) \cdot \tau \cdot d\tau \qquad (1)$$

where:

ρ – density of pumped liquid in [kgm^{-3}],

Q - output of the pump (volumetric intensity of the flow) in [$m^3 s^{-1}$],

$\Delta E(\tau)$ – total increase of proper liquid energy in time function, during the flow through the pump, according to mass unit in [$m^2 s^{-2}$],

τ - time of liquid flow through the pump in [s],

D – operation of rotodynamic pump in [Js].

Operation of the pump is characterized by dimensional quantities which possess number and dimension. They create dimensional space, in which there are product and involution of dimensional numbers of real exponent. Results of these operations, together with real exponent belong to dimensional space, which means that its elements are dimensional and non-dimensional quantities, i.e. numbers. Therefore pump operation can be expressed as dimensional function of many variables. Such operation of the pump presented by the above formula (2) is in dimensional space π, whose arguments are also elements of this space. It is not a numerical function and therefore it must fulfil additional conditions of invariance and dimensional homogeneity. These conditions do not limit numerical functions, i.e.: such, whose both, arguments and the function value are non-dimensional quantities [1,3,6,7,9].

In this way we can obtain general physical equation of liquid flow through the pump rotor of the following form:

$$D = m \cdot g \cdot H \cdot \tau = \Phi(\rho, n, d, Q, \nu, \tau) \tag{2}$$

where:
m – liquid mass flowing through the pump in [kg],
g – acceleration of gravity in [ms^{-2}],
H – height of pump hoisting in [m],
d- nominal diameter of pump rotor in [m],
n - rotational speed of the rotor in [s^{-1}],
ν - coefficient of kinematic viscosity of the liquid in [m^2s^{-1}],
- remaining denotations as in formula (1).

It is worth mentioning that condition of invariance results from a description possibility of physical dimensional quantities in different unit arrangements. In an accepted system of basic units of measure SI, matrix of involution exponents of function arguments (2) is of the third order. It means that in case of dimensional function (2) three quantities out of six arguments are dimensionally dependent on these three.

Therefore, from dimensional function (2) one can choose arguments dimensionally independent and separate them from the remaining ones, in other words, one can accept dimensional base of this function in twenty different ways (combination $\binom{6}{3}$=20).

Not all accidentally accepted dimensional bases will be proper. This fact results from dimensional independence of units system: kg, m, s. Out of twenty random ways of choosing dimensional function bases of rotodynamic pump operation, only 8 of them are mathematically correct, which are given in table 1.

According to works [1,6] one can accept that the best base of dimensional function (2) will be the following dimensionally independent quantities: ***ρ*, d, n.**

Applying Buckingham theorem [1,6] to function (1), after accepting the above dimensional base, we obtain the following equation:

$$D = m \cdot g \cdot H \cdot \tau = f_1(\phi_Q, \phi_\nu, \phi_\tau) \cdot \rho \cdot n \cdot d^5 \tag{3}$$

where:

f_1 - numerical function,

$\phi_Q = \dfrac{Q}{n \cdot d^3}$ - efficiency discriminant (dimensionless argument of numerical function f_1),

$\phi_\nu = \dfrac{\nu}{n \cdot d^2}$ - discriminant of viscosity resistance (interior friction),

$\phi_\tau = n \cdot \tau$ - discriminant of pump operation time,

-the remaining denotations as above.

Formula (3) can be rearranged to the following form:

$$\Phi_D = \frac{m \cdot g \cdot H \cdot \tau}{\rho \cdot n \cdot d^5} = f_1\left(\frac{Q}{n \cdot d^3}, \frac{\nu}{n \cdot d^2}, n \cdot \tau\right) \tag{4}$$

where:
Φ_D - dimensionless discriminant of rotodynamic pump operation,
-remaining denotation as above.

Table1. *Choice possibilities of arguments dimensionally independent , so called, dimensional bases in function of rotodynamic pump operation* $D = \Phi(\rho, n, d, Q, \nu, \tau)$.

o number	Form of dimensional function	Dimensional base	Remarks
1	$D = f_1(\phi_Q, \phi_\nu, \phi_\tau) \cdot \rho \cdot n \cdot d^5$ $\phi_Q = \frac{Q}{n \cdot d^3}, \cdots \phi_\nu = \frac{\nu}{n \cdot d^2}, \cdots \phi_\tau = n \cdot \tau$	ρ, n ,d	
2	$D = f_2(\phi_d, \phi_\nu, \phi_\tau) \cdot \rho \cdot \sqrt[3]{\frac{Q^5}{n^2}}$ $\phi_d = d \cdot \sqrt[3]{\frac{n}{Q}}, \cdots \phi_\nu = \frac{\nu}{\sqrt[3]{n \cdot Q^2}}, \cdots \phi_\tau = n \cdot \tau$	ρ, n, Q	
3	$D = f_3(\phi_d, \phi_Q, \phi_\tau) \cdot \rho \cdot \sqrt{\frac{\nu^5}{n^3}}$ $\phi_d = d \cdot \sqrt{\frac{n}{\nu}}, \cdots \phi_Q = Q \cdot \sqrt{\frac{n}{\nu^3}}, \cdots \phi_\tau = n \cdot \tau$	ρ, n, ν	
4	$D = f_4(\phi_n, \phi_\nu, \phi_\tau) \cdot \rho_\nu \cdot Q \cdot d^5$ $\phi_n = \frac{n \cdot d^3}{Q}, \cdots \phi_\nu = \frac{\nu \cdot d}{Q}, \cdots \phi_\tau = \frac{\tau \cdot Q}{d^3}$	ρ, Q, d	
5	$D = f_5(\phi_n, \phi_d, \phi_\tau) \cdot \rho \cdot \frac{Q^9}{\nu^{11}}$ $\phi_n = \frac{n \cdot Q^2}{\nu^3}, \cdots \phi_d = \frac{d \cdot \nu}{Q}, \cdots \phi_\tau = \frac{\tau \cdot \nu^3}{Q^2}$	ρ, Q, ν	
6	$D = f_6(\phi_n, \phi_d, \phi_\nu) \cdot \rho \cdot \sqrt[3]{Q^5 \cdot \tau^2}$ $\phi_n = n \cdot \tau, \cdots \phi_d = \frac{d}{\sqrt[3]{Q \cdot \tau}}, \cdots \phi_\nu = \nu \cdot \sqrt[3]{\frac{\tau}{Q^2}}$	ρ, Q, τ	
7	$D = f_7(\phi_n, \phi_d, \phi_Q) \cdot \rho \cdot \sqrt{\nu^5 \cdot \tau^3}$ $\phi_n = n \cdot \tau, \cdots \phi_d = \frac{d}{\sqrt{\nu \cdot \tau}}, \cdots \phi_Q = \frac{Q}{\sqrt{\nu^3 \cdot \tau}}$	ρ, ν, τ	
8	$D = f_8(\phi_n, \phi_Q, \phi_\nu) \cdot \frac{\rho \cdot d^5}{\tau}$ $\phi_n = n \cdot \tau, \cdots \phi_Q = \frac{Q \cdot \tau}{d^3}, \cdots \phi_\nu = \frac{\nu \cdot \tau}{d^2}$	ρ, τ, d	

Discriminant of rotodynamic pump operation (4) can be expressed as product of dimensionless discriminants of hoisting pump height and the volume of flowing liquid through its rotor, in the following form:

$$\Phi_D = \Phi_V \cdot \Phi_H \cdot \Phi_\tau = f_1\left(\frac{Q}{n\cdot d^3}, \frac{\nu}{n\cdot d^2}, n\cdot\tau\right) \quad (5)$$

where:

$\Phi_V = \frac{m}{\rho\cdot d^3} = \frac{m}{\frac{m}{V}\cdot d^3} = \frac{V}{d^3}$ - dimensionless discriminant of liquid volume flowing through the rotor of rotodynamic pump,

$\Phi_H = \frac{g\cdot H}{n^2\cdot d^2}$ -discriminant of pump hoisting height ,

- the remaining denotation as above.

It is worth noticing that in discriminant of pump hoisting height Φ_H in the formula (5) liquid density does not appear, therefore, one can draw conclusion that the height of pump hoisting does not depend on liquid density.

Also, from the above dependence it appears that the height of pump hoisting of rotor diameter d , changes proportionally to rotational velocity square. Besides, the inverse of height discriminant of pump hoisting, constitutes one of the forms of Froud number, and the inverse of discriminant is defined by the following formula:

$$F_r = \frac{1}{\Phi_H} = \frac{d^2\cdot u^2}{g\cdot H} \approx \frac{u^2}{g\cdot H_{th}} \quad (6)$$

where:-

u - circumferential velocity of pump in [rad · s^{-1}],

H_{th} - theoretical height of pump hoist in,

- remaining denotations as above.

The formula (6) shows that Froud's number determines the condition of forces similarity necessary to hoist the liquid to height H_{th} .

The resistance of liquid flow caused by its viscosity is determined by Reynold's number, expressed by the following formula:

$$R_e = \frac{1}{\Phi_\nu} = \frac{d^2\cdot n}{\nu} \approx \frac{d\cdot n}{\nu} \quad (7)$$

where:-

- denotations as above.

The value of Reynold's number, in relation to a characteristic dimension d and to circumferential speed u prevailing on the circumference of the rotor with a diameter d , determines character of liquid flow through the pump. It should be mentioned here, it was experimentally proved that by heavy flows of liquid through the pump's rotor, Reynold's numbers are characterized by great values, which means that the influence of liquid viscosity on flow character is minimal [7,9].

Therefore in equation (3) we can ignore the discriminant of viscosity resistance ϕ_ν , and thus dimensional function of pump operation is expressed by means of numerical function of two variables presented in the following form:

$$D = m\cdot g\cdot H\cdot\tau = f_1(\phi_Q, \phi_\tau)\cdot\rho\cdot n\cdot d^5 \quad (8)$$

where:

- denotation as in formula (3).

Operation of the pump can also be expressed , by means of power absorbed by it, in the form of general physical equation of the following dimensional function:

$$D = N \cdot \tau^2 = \Psi(\rho, d, n, Q, \nu, \tau) \qquad (9)$$

where:-
N –power absorber by pump in [m ^{2} kg s $^{-3}$],
- denotation as in formulas (1) and (2).

Dimensional function Ψ, determined by formula (9) can be in similar way, as in formula (1) and adopting the same dimensional base, transformed into numerical function of the following form:

$$D = N \cdot \tau^2 = f_2(\xi_Q, \xi_\nu, \xi_\tau) \cdot \rho \cdot n \cdot d^5 \qquad (10)$$

where:-
$\xi_Q = \frac{Q}{n \cdot d^3}$, $\xi_\nu = \frac{\nu}{n \cdot d^2}$, $\xi_\tau = n \cdot \tau$ - independent dimensionless arguments of numerical function f_2, so called, similarity invariants,
- denotations of the remaining quantities, as in previous formulas.

From the above formula it appears that if in dimensional function, the same dimensional quantities independent of the remaining ones were accepted, then adequate variable arguments of their numerical functions will be equal, i.e. $\phi_\nu = \xi_\nu$. Hence their name of similarity invariants.

In the simplest case, the forms of dimensional numerical functions, under discussion, can be approximated by linear functions. Such approximations have been given in table 2.

Table 2. *Linear estimations of numerical functions of rotodynamic pump in three – dimensional space of dimensional base ρ, d, n*

On	Linear estimations of dimensional function	Dimensional base
1.	$D = A_0 \cdot \rho \cdot Q \cdot d^2 + B_0 \cdot \rho \cdot \nu \cdot d^3 + C_0 \cdot \rho \cdot n^2 \cdot d^5 \cdot \tau + D_0 \cdot \rho \cdot n \cdot d^5$	ρ, d, n
2.	$D = A_1 \cdot \rho \cdot Q \cdot d^2 + C_1 \cdot \rho \cdot n^2 \cdot d^5 \cdot \tau + D_1 \cdot \rho \cdot n \cdot d^5$	ρ, d, n
Dimensionless numbers, so called, similarity invariants $\varphi_Q = \frac{Q}{d^3 n}$; $\varphi_\nu = \frac{\nu}{d^2 n}$; $\phi_\tau = n \cdot \tau$		
A_i; B_i; C_i; D_i – real numbers determined on the basis of measurements (i = 0,1).		

They should be treated as a rough definition of dependences between quantities which describe the influence of rotodynamic pump on the liquid flowing through the pump.

3. Diagnostic properties of dimensional functions

From dimensional analysis it appears: delivery of a pump Q, operation of rotodynamic pump D and rotational speed n , they constitute basic hydraulic parameters of the pump.

They determine each state of pump operation and for this reason, they can be used as diagnostic symptoms of technical condition.

Besides, its technical state can be determined by similarity invariants, called as discriminants: delivery of a pump φ_Q , operation of rotodynamic pump Φ_D and time of rotodynamic pump operation ϕ_τ **.** These discriminants have constant values, determined by the values of geometric parameters, kinematic and dynamic ones of the pump.

It means, that on the basis of these values deviations of the pump under investigation, and comparing it with a model one, completely efficient, it will be possible to make a diagnosis about its technical condition.

Taking into consideration the expression (2) and ignoring viscosity of the liquid flowing through rotodynamic pump, we can determine the hydraulic performance of rotodynamic pump, by means of the following formula:-

$$\eta_h = \frac{f_1(\varphi_Q;\varphi_v;\varphi_\tau)}{f_0(\varphi_Q)} \tag{11}$$

where:
- denotations as in previous formulas.

From the formula (11) it follows directly that hydraulic efficiency of the pump depends on the discriminant of working capacity, Reynold's number and time of rotodynamic pump operation.

On the basis of the size of relative energetic losses, taking place in the pump, one can evaluate its general technical condition. Therefore operation of rotodynamic pump is a diagnostic symptom [4]. Total energetic losses in rotodynamic pump can be determined on the basis of the definition in the following way:-

$$\frac{\Delta H}{H_{th}} = 1 - \frac{H_e}{H_{th}} = 1 - \eta_h = 1 - \frac{f_1(\varphi_Q;\varphi_v;\varphi_\tau)}{f_0(\varphi_Q)} = K + f_3\left(\varphi_Q;\frac{1}{\varphi_v};\varphi_\tau\right) \tag{12}$$

where:
ΔH – height corresponding with the loss of mechanical energy during flow of the liquid through the pump,
K – experimentally fixed constant value for the pump of a given type,
H_e – effective hoisting value the pump,
H_{th} – theoretical hoisting value,
$\Delta H/ H_{th}$ –relative energetic loss in the pump,
- the remaining denotations as in previous formulas.

Variability testing of relative energetic losses or hydraulic efficiency η_h leads to determining of the function f_3.

In order to determine analytical form of numerical function (12) one should carry out diagnostic testing of rotodynamic pump in specified work conditions. Basing on these measurements, one can assume some conditions concerning the form of numerical function.

4. Recapitulation

Analyzing procedures of creating dimensional functions and their transformation into numerical functions, one can state:

1. Diagnostic testing is carried out in order to make a diagnosis about technical state of the pump;
2. It is necessary for a dimensional function to fulfil interpretation rules accepted together with notions determining working processes of the pump;
3. Selected dimensional quantities interfere in specification of technical condition of the pump, in

an essential way, and limit considerably a possibility of its specification;

4. Technical condition of the rotodynamic pump can be made a diagnosis of, by means of such parameters of its work , as:
   - operation of rotodynamic pump D,
   - capacity (volumetric capacity of the flow) Q ,
   - diameter of pump's rotor d ,
   - density of flowing liquid through the pump ρ ,
   - rotational speed of rotodynamic pump rotor n,
   - time of pump operation τ ,
   - coefficient of kinematic viscosity of the liquid flowing through the pump ν .
5. On the basis of the knowledge of linear estimations of numerical functions, presented in table 1, it will be possible to make a diagnosis of technical condition of the pump;
6. Measurements results in the form of dimensional functions of rotodynamic pump operation, in its important technical states, can be collected, in so called information bank.

5. Bibliography

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