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ANALYSIS AND SIMULATION OF NONLINEAR SELF-EXCITED VIBRATIONS IN TURNING

Commonly applied theories of self-excited vibrations in machine tools are prevailingly based on frequency domain analysis of linear models and are devoted in most cases only to evaluation of stability limits of strictly linear systems. In this way, the behaviour of more stable or more unstable systems cannot be studied and analyzed. Certain important nonlinear effects cannot be studied as well for limitations connected with classical representation of systems only in the frequency domain. Evaluation of stability by analysis of nonlinear systems in time domain has not been applied until now. The paper shows a new approach based on interpretation of the self-excited vibration systems as nonlinear servo-systems. With this approach, linear and nonlinear systems of any degree of stability can be studied by combination of frequency and time-domain methods. This can considerably contribute to better understanding of complex phenomena in various regimes and special situations.

1. INTRODUCTION

Self-excited vibrations in machine tools arise as a result of interaction between the elastic structure of machine and the metal cutting process. At certain conditions, typically at certain limiting width of chip, the interaction becomes too strong and the tool starts to vibrate relatively to the workpiece. The whole combined system becomes unstable and the amplitudes of vibration grow and may cause serious damage of the tool, spindle or the whole machine. Analysis of this phenomenon, performed already about fifty years ago by Prof. Tlusty, DrSc. and his colleagues [1], [2], [3] and [4] have shown that the stability of excited vibrations is influenced by the space orientation of the machine vibration system with respect to the normal to the machined surface and that the chatter occurs at smaller width of chip when the machined surface has already certain surface waviness produced at previous cut. Prof. Tobias [5] published the “lobe” diagrams, expressing how the change of spindle revolutions can influence critical width of cut on the limit of stability. Most of later publications like Main achievements of the classic theory are based on the following assumptions:

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- Only relative vibrations between the tool and workpiece in normal direction to the machined surface can influence the system stability
- A linear system is on the limit of stability, when the amplitude of vibrations Y(t) equals to the amplitude of the surface waviness Y_0 (t) caused by the previous cut
- Because the exciting cutting force F acting on the vibration system is for constant value of the width of cut b considered as proportional to the actual depth of cut (Y(t)) - Y_0 (t) by the specific cutting coefficient R, the depth of cut is always in phase with the cutting force F.
- As no phase shift. time delay or complex value of the coefficient R is considered, it follows that the real parts Re [Y(t)] and Re [Y_0(t)] will equal in size and will be opposite in sense.
- For a particular vibration system with the frequency response Φ, values of critical width of cut b_{lim} can be relatively easy calculated using the well known criterion:

$$b_{lim} = - 1/(2R \cdot \mathrm{Re}[\Phi]neg) \tag{1}$$

These assumptions are valid for linear systems on the limit of stability and are commonly applied in most of numerous later publications on the subject, like in [4],[6] and [7]. Nevertheless, for evaluation of stability by application of the criterion (1), the system uses only real part of the frequency characteristics of the machine structure as shown in the Fig. 1 and cannot be applied for a general analysis of its behaviour in both frequency and time domains.

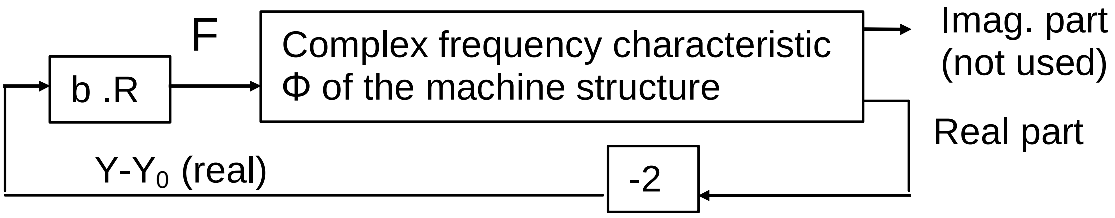

Fig. 1. Classical representation of the system for evaluation of the stability by application of the criterion (1)

2. INTERPRETATION OF SELF EXCITED VIBRATIONS AS FEEDBACK SYSTEMS

For more general analysis of system behaviour both in frequency and time domains, we have interpreted the waviness Y_o(t) as the vibration (Y(t), delayed in time. For turning processes, this time delay corresponds to the time T of one spindle revolution, for milling processes, it corresponds to the time interval T/n between two cutting edges of the rotating tool. Modern Matlab/Simuling methods allow us to apply this nonlinear feature in both frequency and time domains. Then, it is possible to introduce other nonlinearities and generate block diagrams of feedback systems for various cases and configurations and study behaviour of systems with different degree of stability. Instead of evaluating only the width

b_{lim} of the chip on the limit of stability, we may for instance study complete complex open loop characteristics and apply the Nyquist criterion of stability. The correspondence between the classic criterion (1) and new Nyquist diagram is depicted in the Fig. 2. The Nyquist critical point for the limit of stability coincides with the criterion (1), but the frequency response includes now the whole system, not only the mechanical structure.

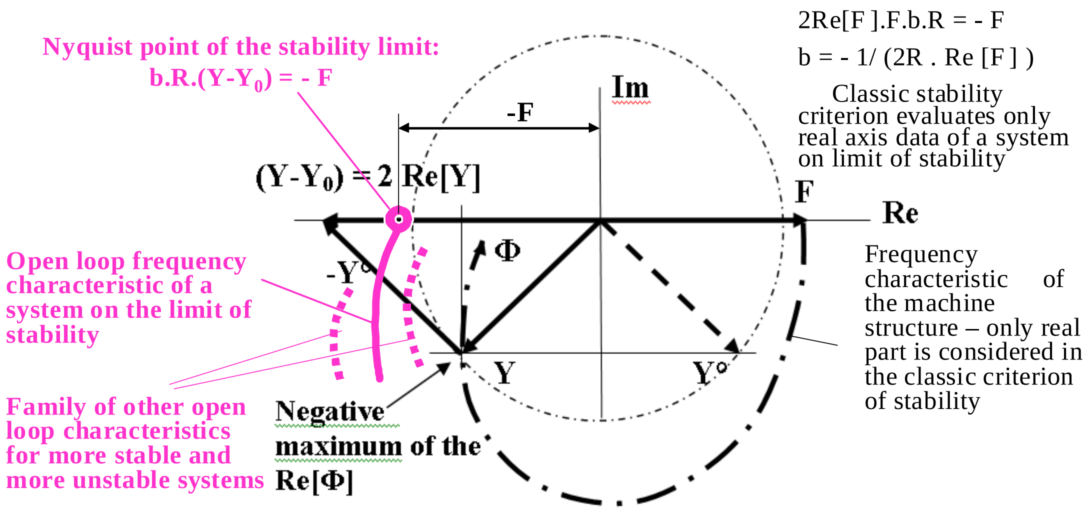

Fig. 2. Correspondence between the classic and Nyquist stability criterions

The programmed depth of cut H(t) represents here the input function of the system. Coloured areas represent nonlinear blocks. Corresponding general block diagram for case of turning processes is shown in the Fig.3. The depth of cut $(Y(t)– Y_0\,(t))$ is here represented in a full complex form, not only as the parts Re $[Y_0\,(t)]$ and Re $[Y(t)]$. The programmed depth of cut H(t) represents here the input function of the system. Coloured areas represent nonlinear blocks. Medium value of the input H(t) causes certain medium value of the cutting force and a corresponding medium displacement of the machine structure, which doesn't theoretically influence the stability of a linear system. Actual depth of cut h(t) is expressed by the equation:

$$h(t) = H(t) – (Y\,(t) – Y_0\,(t)) \tag{2}$$

When the value of $(Y_0\,(t) - Y(t))$ is bigger than the value of H(t), the tool jumps out of the material and the cutting force F(t) stops to be generated. This is in the diagram in Figure 3 represented by a nonlinear block which lets to pass unchanged only positive values of h(t) and blocks the negative ones. Jumping of tool out of material is a nonlinear effect limiting the amplitude of vibrations of unstable systems. Positive part of h(t) represents the actual depth of cut. Multiplication by the width of cut b produces the cross section of the chip and next coming multiplication by the specific cutting resistance R produces the cutting force F(t).

Classic stability criterion in the equation (1) supposes that the value of R is real, but we may also deal with a certain time delay Z between the chip cross section b . ($Y(t) – Y_0(t)$) and the cutting force F. Then, the depth of cut (Y_0 (t) – Y(t)) will be here represented in a

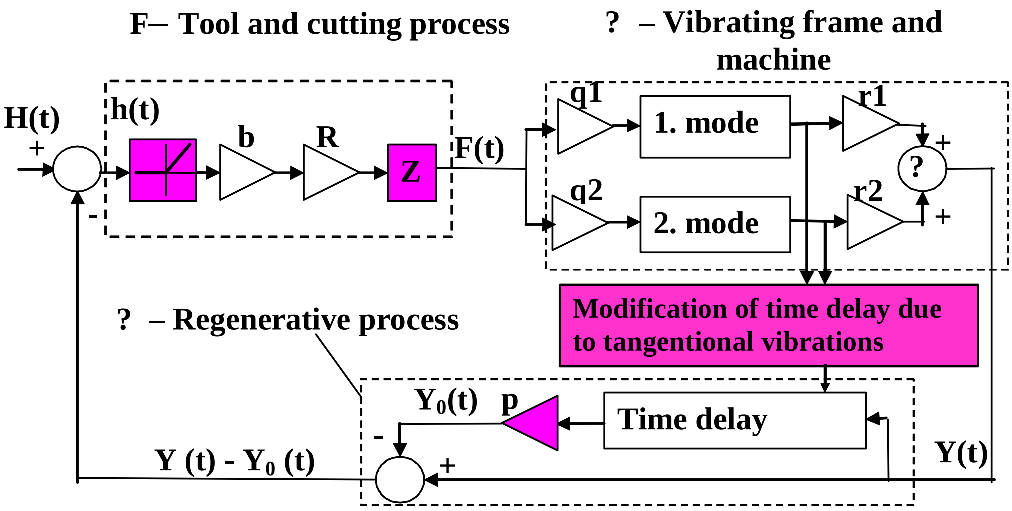

Fig. 3. General block diagram for analysis of self-excited vibrations in a non-linear two-mode system

full complex form, not only as the difference between real parts Re [Y_0 (t)] and Re [Y(t)]. Constant value of the input H(t) causes certain medium value of the cutting force and a corresponding medium displacement of the machine structure, which doesn`t influence the stability.

When the value of (Y(t) - Y_0 (t)) is bigger than the value of H(t), the tool jumps out of the material and the cutting force F(t) stops to be generated. This is represented in the Fig. 3 by a nonlinear block which lets to pass unchanged only positive values of h(t) and blocks the negative ones. Jumping of tool out of the material is a nonlinear effect limiting the amplitude of vibrations of unstable systems. Positive part of h(t) represents the actual depth of cut. Multiplication by the width of cut b produces the cross section of the chip and multiplication by the specific cutting resistance R produces the cutting force F(t). Possible complexity of the specific cutting coefficient R is in the Fig. 2 represented by the time delay Z.

Let us call the whole part of the diagram with the input h(t) and output F(t), in which the force F(t) is generated, as the group F. The block diagram in the Fig. 3 has, besides of the group F, two other groups these being the group Φ and group Ψ. Group Φ represents a two-mode vibration structure of the machine and tool and includes direction factors q1 and r for getting the output vibration component in the normal direction to the machined surface. Vibration in tangential direction can be considered as a modification of the time delay T in the next block group Ψ, which solves the waviness effects of previously machined surfaces and performs the conversion of the vibration Y(t) into the actual depth of cut ($Y(t)– Y_0(t)$).

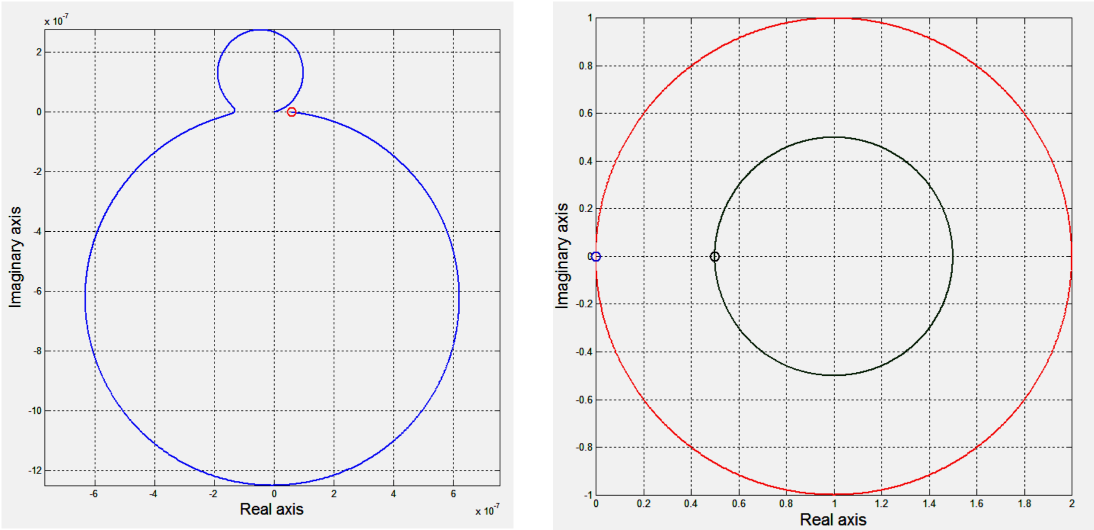

Fig. 4. Frequency response of the groups Φ and Ψ

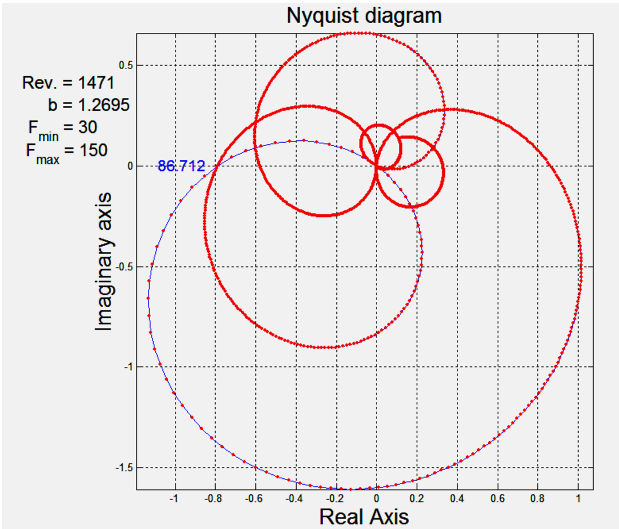

Fig. 5. Resultant open loop frequency response of the system with all three block groups F,Φ and Ψ interconnected

Parameter p varies between one and zero and respects the fact that in turning only a part of the machined surface may enter into repeating turning cut. Fig. 3 shows examples of frequency characteristics of groups Φ and Ψ for a two-mode vibration system with natural frequencies 72 Hz and 120 Hz. Tangential vibrations are not considered here. Fig. 5 shows the resultant open loop frequency response of the system with all three interconnected groups F, Φ and Ψ. Values of parameters are b = 1. 10^{-3} m, R = 2.10^{9} Pa, Z = 0 sec, T = 0,1 sec. and mode frequencies 72 and 120 Hz. When a slight time delay Z of 0.002 sec is considered, corresponding to certain reasonable phase shift at the frequency on the limit of stability, characteristics from the Fig. 5 modifies to shapes shown in the Fig. 6. For the given value of b, the system is no more stable.

3. SPECIAL PHENOMENA NEAR THE LIMIT OF STABILITY

Changes of the spindle revolutions influence directly the values of the time delay T and the form of open loop Nyquist characteristics shown in the Fig. 5 and 6. Application of developed procedures makes it possible to plot “lobe” diagrams showing changes of critical b_{lim} for different spindle revolutions as shown in the Fig. 7 for both just shown systems. System with time delay Z shows much lower values of b_{lim} in its “lobe” diagram.

Analysis has shown that the critical width of cut b_{lim} depends strictly only on the phase shift between the vibration Y(t) and surface waviness $Y_0(t)$. Corresponding graphical proof is shown in the Fig. 8 and in a 200 times magnified detail shown in the Fig. 9. Self excited vibrations occur only in certain range of phase shift and in other areas cannot exist even at most extreme values of b. The situation is depicted as 3D diagrams in the Fig. 10 and 11. The relation between the phase and $b_{lim.}$ keeps unchanged, but the occurrence of stability limits varies quite unexpectedly and jumps among different spindle revolutions.

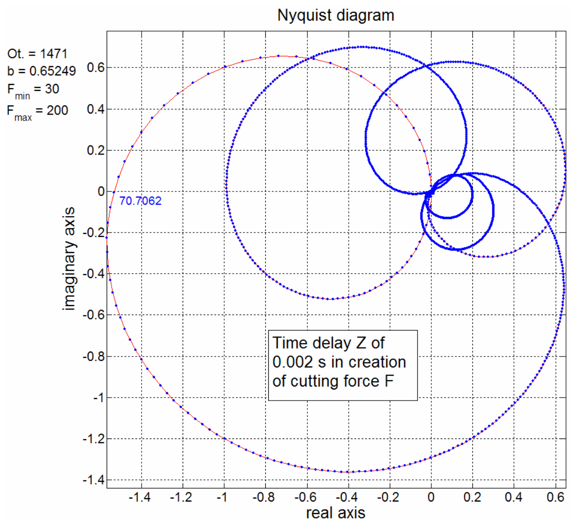

Fig. 6. Frequency characteristic modified due to Influence of time delay Z=0.002

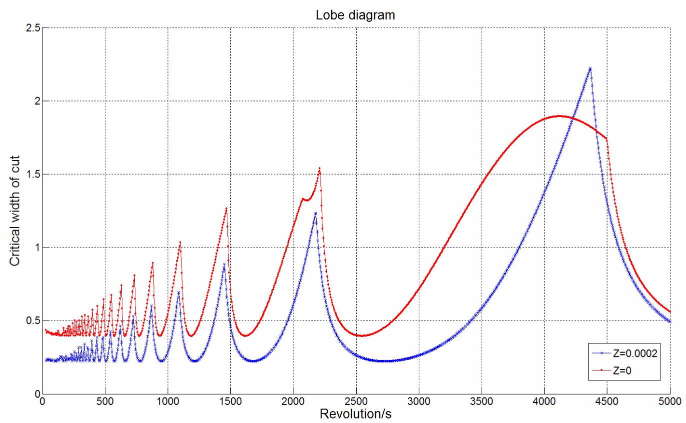

Fig. 7. Lobe characteristics for Z=0 and Z= 0.002 for a range of 30 to 5000 rev/min

This can be still better seen in the Fig. 11, which shows the behaviour of the system at continuous change of spindle revolutions. The most interesting of these phenomena is that for any spindle revolutions exist always only one critical phase shift between the vibration and surface waviness and only one corresponding critical width of cut.

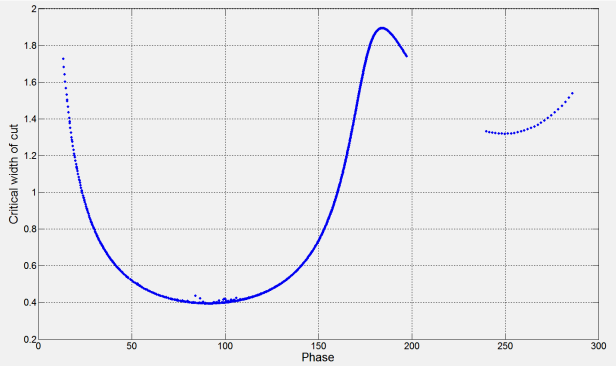

Fig. 8. At any spindle revolutions, the critical width of cut depends strictly only on the phase shift between the vibration and surface waviness

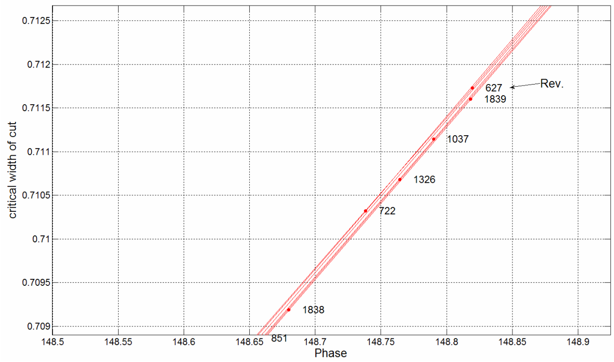

Fig. 9. A 200 times magnified detail of a portion of the Fig. 8

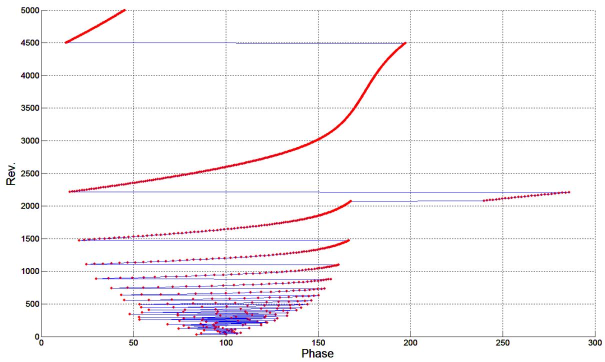

Fig. 10. A 3D representation of stability limits occurrence within chosen range of spindle revolutions

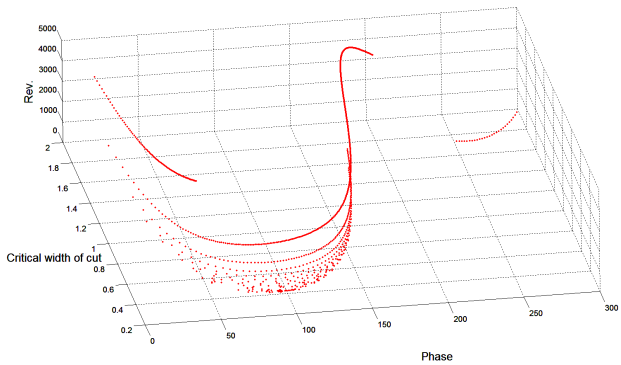

Fig. 11. Jumping of phase and corresponding critical width of cut when changing the spindle revolutions

4. TIME DOMAIN SIMULATIONS

Criteria of stability used in classical linear theory of self–excited vibrations is based on the comparison of two successive vibration amplitudes, these being the amplitude of actual vibration Y and amplitude Y_0 of surface waviness. This approach supposes that the system on the limit of stability founds itself in a special state of steady vibrations, after dying off all previous transient phenomena. We may designate this criterion as a “steady-vibration” criterion. Unfortunately, this presumption may not occur in real situations. Vibration and surface waviness may interfere and be combined with various process shocks. Then, the simulation in time-domain could be preferably applied for better understanding what is really happening. General block diagram shown in the Fig. 2 can be used without change for this purpose making it possible to simulate even the influence of specified nonlinear phenomena.

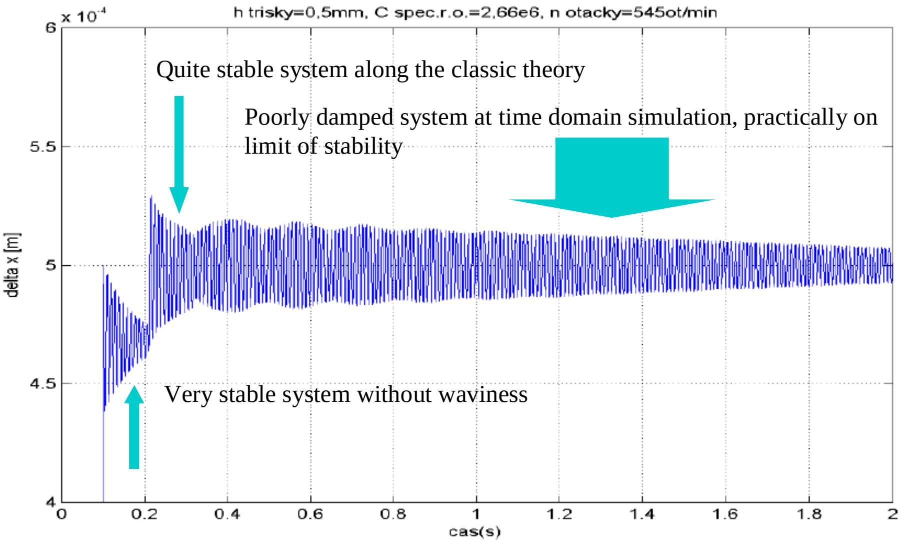

Fig. 12. Simulation in time-domain showing effects of shocks from incoming surface waviness

Fig. 12 shows an example of a transient process simulation for a case, when the tool starts to machine a clean material. At the beginning, the vibrations are well damped. After the time period T, the system gets the first shock from incoming surface waviness. The time domain simulation shows a very poor damping of vibrations due to periodically repeating shocks from incoming waviness.

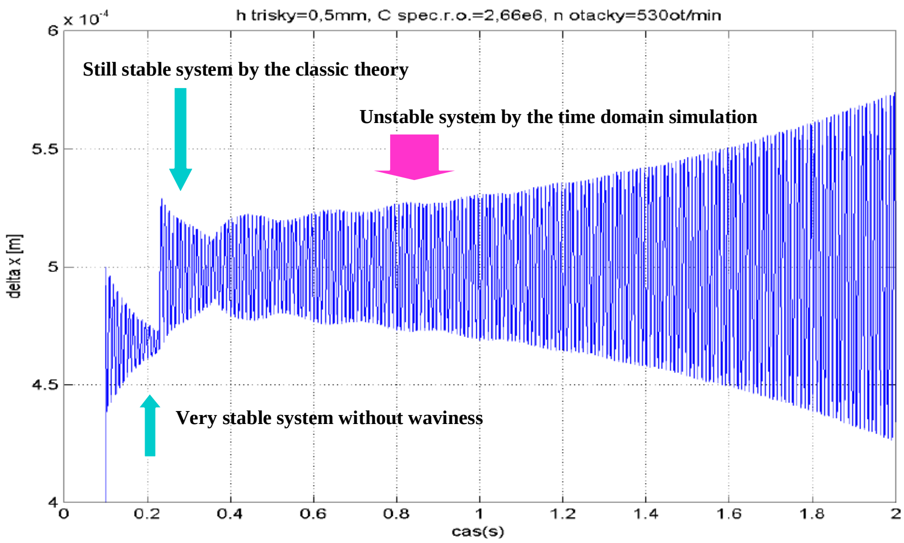

Fig. 13. Simulation in time-domain showing unstable behavior due to shocks from waviness

Fig. 13 shows the situation with slightly higher width of cut b. Due to repeating shocks in time periods of T, the system is periodically excited and becomes unstable. For longer T, the system has more time to damp before next shocks arrive and may have a better chance to keep stability.

5. STABILITY EVALUATION AT TIME-DOMAIN SIMULATION

Analysis and simulation of transient phenomena of self-excited vibrations can as well be applied for quantitative and evaluation of stability, if a suitable stability criterion would be found. Various criteria have been unsuccessfully tested and all the time found cases where they failed. Finally, an automatic iteration procedure called "vibration integral" criterion has been developed which until now works in all tested linear and nonlinear cases. The procedure is depicted in the Fig. 12 and works in the following way: The transient process is simulated for a time longer than about 200 vibration waves First period, during which the tool cuts clean material without waviness is dropped out from further analysis. Then, the simulation time T_s is divided into two equal time periods. During the first period, the integral of absolute values of vibrations is collected in one register where it creates the sum I_1 and during the second period in another register where it creates the sum I_2. By iterative changing of the width of cut b, we find the $b_{lim\ y}$for which $I_1 = I_2$ and take it as the limit of stability. For other cases, it is also possible to evaluate the ratio I_1 / I_2 for quantitative evaluation of the stability degree. The time domain simulation and the "vibration integral" criterion works well also for nonlinear phenomena.

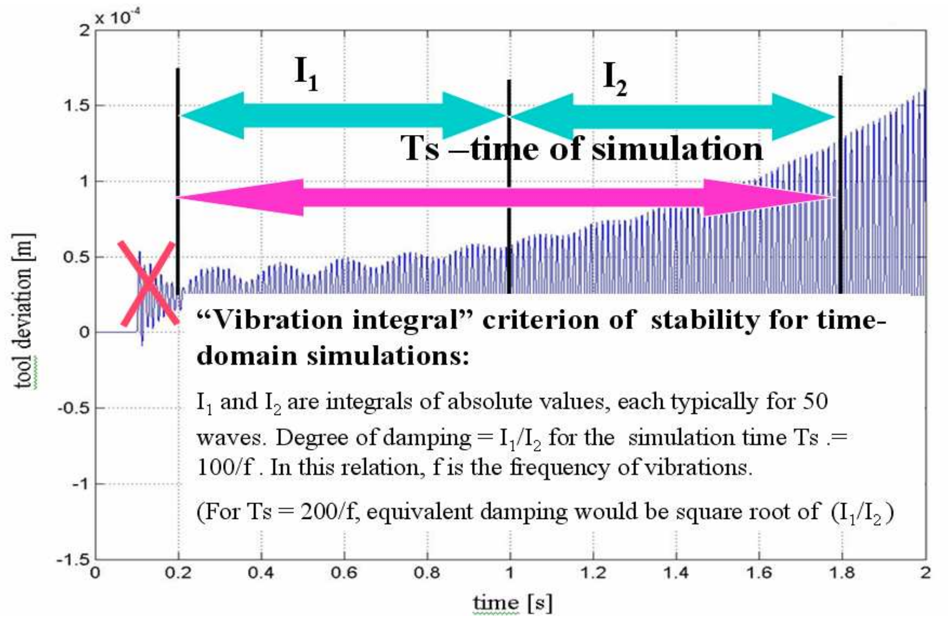

Fig. 14. „Vibration integral" criterion for quantitative evaluatin of stability from time/domain simulation

the integral of absolute values of vibrations is collected in one register where it creates the sum I_1 and during the second period in another register where it creates the sum I_2. By iterative changing of the width of cut b, we find the $b_{lim\ y}$ for which $I_1 = I_2$ and take it as the limit of stability. For other cases, it is also possible to evaluate the ratio I_1 / I_2 for quantitative evaluation of the stability degree. The "vibration integral" criterion worked well also for all until tested nonlinearities.

6. LOBE DIAGRAMS FOR SYSTEMS WITH DIFFERENT DEGREE OF STABILITY

Being able to determine limit of stability by evaluating of b_{lim} from simulated transient processes, we may repeat the same procedure for different spindle revolutions and draw the lobe diagrams from the simulated transient processes of linear or nonlinear systems. Example of such a lobe diagram is shown in the Fig. 15, which corresponds well with the diagram in Fig. 7. More over, with the quantitative "vibration integral" criterion, we may

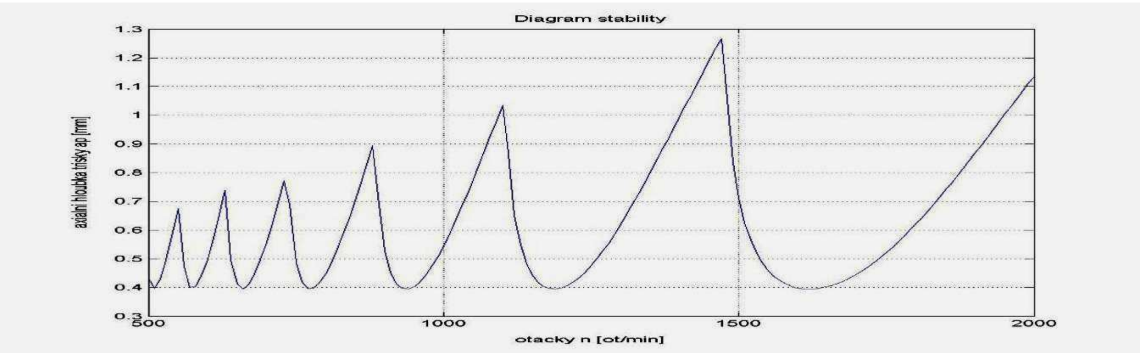

Fig. 15. Example of a lobe diagram made by an iteration procedure from time-domain simulation data

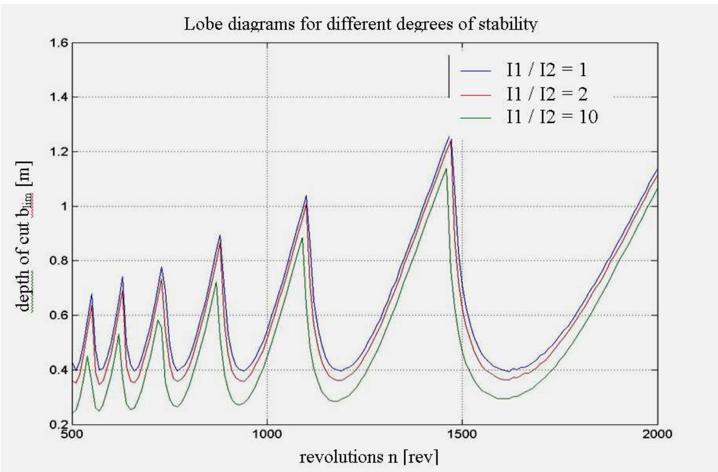

Fig. 16. Lobe diagrams for systems with various degrees of stability

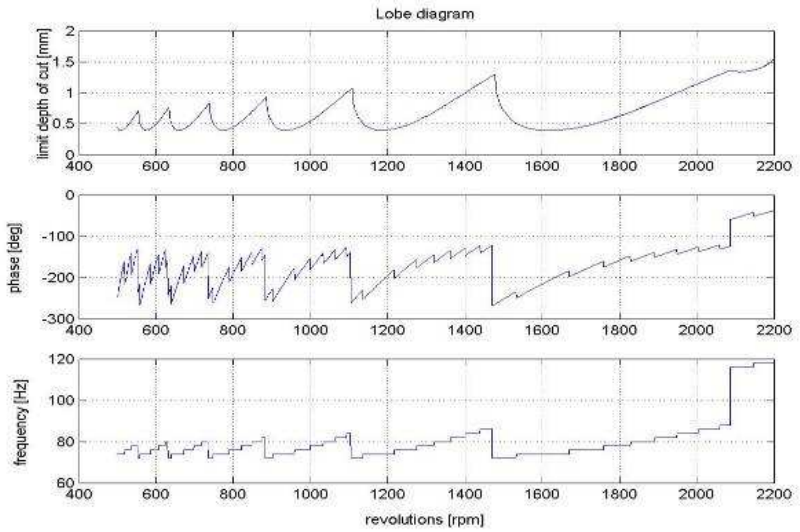

Fig. 17. Changes of depth of cut, phase and critical frequency for systems on the limit of stability

choose the demanded degree of system stability by choosing the ratio (I_1/I_2) and draw lobe diagrams for various “safety” against instability. Fig. 16 shows three lobe curves for ratios (I_1/I_2) equal to 1, 2 and 10. Internal system parameters for systems with different degree of stability can be registered during time-domain simulation. Fig. 17 shows changes of depth of cut, phase and critical frequency related to various spindle revolutions in systems on the limit of stability.

7. SUMMARY

A new approach to analysis of self- excited vibrations involving non-linear elements has been applied in the RCMT Research Centre of Manufacturing Technology in Prague. Interpretation of self-excited vibrations as non-linear feedback systems makes it possible to apply highly developed Matlab/Simulink methods and study frequency and transient phenomena in systems with various degree of stability. Simulation and analysis of transient responses can contribute considerably to better understanding of complex, nonlinear dynamic systems and of their behaviour in various regimes and special situations.

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