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SYMMETRY PERCEPTION IN PROSPECTIVE PRESCHOOL AND PRIMARY SCHOOL TEACHERS: A PILOT STUDY

s. 169-177

ABSTRACT

Children learn basic geometry through observing and manipulating objects and shapes and recognizing their essential properties. One of the basic omnipresent geometric concepts is symmetry. Children's knowledge in many matters depends on their guardians and teachers. Therefore, they should possess appropriate content-related knowledge. The pilot study examines prospective preschool and primary school teachers' ability to recognize axially symmetrical figures. The methodology is based on a designed test and analysis of obtained results gathered from a cohort of 56 prospective preschool and primary school teachers in the second year of university study in Slovakia. The results confirmed previous findings about vertical symmetry being the easiest recognizable form of symmetry, followed by horizontal symmetry. Importantly, prospective teachers tended to lack knowledge of axial symmetry as they rarely referred to it in given tasks. Further, the study points out important factors in designing a discrimination test for symmetry.

KEYWORDS

rhomboid, axial symmetry, analogy-based learning, visual perception, pre-service elementary teachers.

Introduction

Symmetry is a mathematical concept usually formally introduced to primary or early secondary education pupils. Internationally accepted and cited The Common Core Standards (2010) introduce a concept of symmetry in the 4th grade by asking pupils to recognize a line of symmetry for a two-dimensional figure. However, some standards require pupils to work with the concept of symmetry even earlier. The National Educational Program in Slovakia (2015) mentions the concept of symmetry for the first time in the 1st grade by demanding pupils complete an axially symmetrical figure in a lattice. Furthermore, the Principles and Standards for School Mathematics (NCTM 2000) suggest pupils should become familiar with the concept of symmetry as early as pre-school and continue through to the 2nd grade by recognizing and creating symmetrical shapes. Deducing from the tendency to incorporate basic symmetry activities in the early educational stage, one can consider the concept simple and easily graspable. This can be based on its geometric nature and connection to pupils' real-life experiences.

A concept of symmetry falls into a group of concepts that visibly binds pupils' knowledge obtained from math class to their real-world experience (NCTM 2000, p. 15-16). This statement can be underpinned by omnipresent symmetrical physical objects that pupils interact with daily. For instance:

- Shoes lying next to each other (if stowed properly) are plane symmetrical with an imaginary plane of symmetry between them,
- A rectangular bed without a headboard, mattress, pillow, and blanket is plane symmetrical; that is why it does not make a huge difference what side of these items you use unless you have a specific preference,
- Toy cars (apart from the steering wheel) are plane symmetrical with a plane of symmetry dividing the car into its left and right halves,
- Dolls are plane symmetrical with their sagittal plane,
- Balls have multiple symmetry-related properties that can be even more visualized by their seams, etc.

Symmetry can be observed not only in the physical appearance of objects but also in a melody, rhythm, and pace. Moreover, some educational approaches highlight the connection between music and mathematics, where symmetry plays an important role (Prídavková, 2021; Azaryahu & Adi-Japha, 2022). As one can notice, symmetry is a part of our lives. Based on its omnipresence, it is relevant to question whether we must go through a formal educational process to learn and distinguish between symmetrical and asymmetrical objects.

Identification of symmetrical figures

A myriad of literature concerns the human ability to discriminate or express preferences between symmetrical and asymmetrical visual patterns, starting with early childhood to adulthood. Fisher, Ferdinandsen, and Bornstein (1981) found that even 4-month-old infants can discriminate vertically symmetrical forms from asymmetrical ones but cannot discriminate horizontally symmetrical ones from asymmetrical ones. Another study on 4-month-old infants revealed they process vertically symmetrical figures more efficiently than vertically translated or obliquely symmetrical figures (Bornstein, Krinsky 1985).

In infancy, children process vertically symmetrical patterns more efficiently than horizontally symmetrical patterns or patterns created by translating a single pattern, which might be caused by particular stimuli they are often exposed to (Bornstein, Ferdinandsen, Gross 1981). Another suggestion comes from the bilateral symmetry of the brain itself (Julesz 1971).

Even among children older than four years old, symmetrical figures are preferred and processed more efficiently compared to asymmetrical ones (Boswell, 1976; Chipman & Mendelson, 1975; Paraskevopoulos, 1968), which does not seem to be culturally determined (Bentley, 1977; Deregowski, 1972). This may be because, overall, there is less information in symmetrical figures than in asymmetrical figures (Cattaneo et al. 2010). That may be why symmetrical figures are remembered better than asymmetrical figures (e.g., Attneave 1955; Boswell 1976; Garner & Clement 1963; Koffka 1935; Paraskevopoulos 1968). Moreover, research suggests that vertically symmetrical figures may be remembered better than horizontally symmetrical and diagonally symmetrical (Rossi-Arnaud et al. 2006; Rossi-Arnaud et al. 2012).

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Furthermore, data suggest successive development of the symmetry concept as a product of maturity. Hu and Zhang (2019) focused on translational symmetry, bilateral (vertical) symmetry, and rotational symmetry, resulting in developmental succession taking place in 4-6-year-old children, where a concept of general symmetry is progressively developed and enriched by translational symmetry, followed by bilateral symmetry, with rotational symmetry coming at last. The specific types of symmetry are initially simply a part of the general symmetry, further differentiating from each other in the same order as they were conceptualized as a part of general symmetry.

Despite the immense amount of literature showing that humans implicitly recognize or discriminate symmetrical figures, it is important to focus on teachers as they foster children's mathematical and cognitive development. They must be competent and fully aware of mathematical concepts children should acquire as they learn. For example, Mafakheri (2022) showed that pre-service primary teachers in Iran may have a weak knowledge of the symmetry axis. The presented pilot study aims to examine pre-service teachers' knowledge of symmetry in Slovakia. The study results may help build a more vigorous curriculum for prospective teachers' professional preparation.

Methodology

The goal of the study is to examine the ability of prospective preschool and primary school teachers to discriminate asymmetrical figures from symmetrical figures. Participants have not attended any courses in geometry or teaching methods up to date. Therefore, the study can be seen as an entry-level inquiry that may provide valuable information for university course developers and lecturers. For that purpose, a four-question test was developed, with each question focused on selecting the odd one (see Table 1). Specifically, the questions were focused on discriminating:

1. An asymmetrical figure from vertically symmetrical figures,
2. An asymmetrical figure from horizontally symmetrical figures,
3. An asymmetrical figure from oblique (diagonally) symmetrical figures,
4. An asymmetrical figure from symmetrical (vertically, horizontally, diagonally) figures (see attachment).

Additionally, another question was included where participants were given a square, rectangle, rhombus, and rhomboid. The first three mentioned are axially symmetrical (vertically, horizontally, or diagonally), whereas the rhomboid is only translationally symmetrical (a line that divides the rhomboid into two identical shapes can be drawn)[1]. Thus, the question focused on discriminating an axially asymmetrical figure from axially symmetrical figures.

In the discrimination-based test, the following requirements in designing individual questions to avoid any distortion from the actual topic were applied:
- Figures had the same number of line segments,
- Figures were equivalent in perimeter, contour, and area,
- Stimuli were constructed of identical components.

[1] This also holds for previously mentioned shapes (square, rectangle, and rhombus). Additionally, translational symmetry in mathematics refers to a specific function rather than an object's property.

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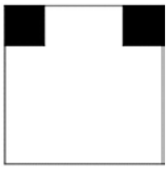
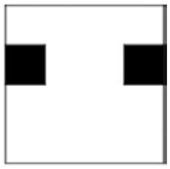
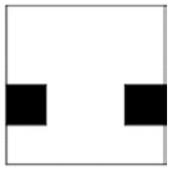
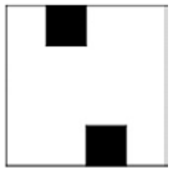
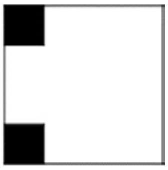
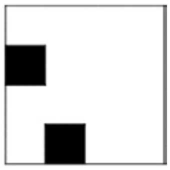
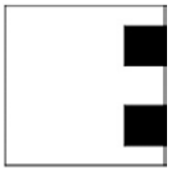
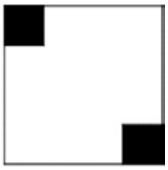
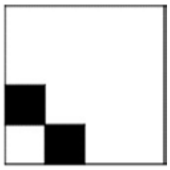
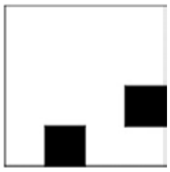
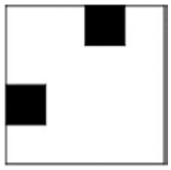
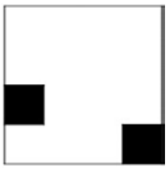
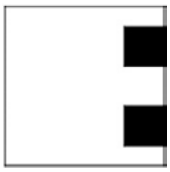
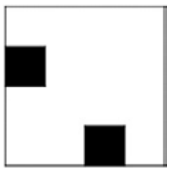
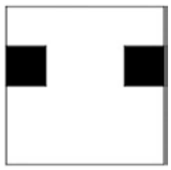
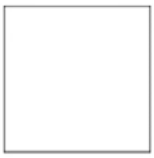
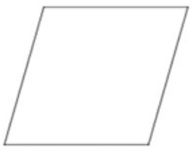
Table 1. Discrimination-based test figures

Vertical symmetry				
Horizontal symmetry				
Diagonal symmetry				
General symmetry				
Axis of symmetry				

Together, 56 pre-service preschool and primary school teachers in the second year of university study took the test that was distributed online among randomly selected groups of students containing half of the students enrolled in the program in the particular year. The test was designed, and the results were gathered using the Google Forms platform. Collected data were analysed quantitatively and qualitatively as each task was complemented by a question asking participants to explain their answers in words. Quantitative data analysis was based on descriptive statistics, particularly taking the percentage of correctly answered questions. Qualitative data analysis was based on the rationale behind each submitted response. Presented qualitative analysis results are answers with the highest frequency.

Results

Data analysis was conducted individually for each question. The first step was to determine the degree of correctly answered questions expressed by taking percentages. The correctness is based on our expectations and partially relates to measuring tool validity. The following was found out:

- In discriminating asymmetrical from vertically symmetrical figures, 100 % of respondents chose the asymmetrical figure.
- In discriminating asymmetrical from horizontally symmetrical figures, 86 % of respondents chose the asymmetrical figure.
- In discriminating asymmetrical from diagonally symmetrical figures, only 43 % of respondents chose the asymmetrical figure.
- In discriminating asymmetrical from generally symmetrical figures (vertically, horizontally, or diagonally), only 30 % of respondents chose the asymmetrical figure.
- In discriminating asymmetrical from generally symmetrical figures (rhomboid from square, rectangle, and rhombus), 55 % of respondents chose rhomboid.

A qualitative analysis of participants' reasons for selecting figures was conducted as part of the test results examination.

- In question 1, most participants explained choosing an asymmetrical figure from a set of vertically symmetrical figures by its property not being aligned. The second most frequent reason was that the figure was optically-oriented differently from the other figures.
- In question 2, most participants explained choosing an asymmetrical figure from a set of horizontally symmetrical figures by its property not being aligned.
- In question 3, most participants explained choosing an asymmetrical figure from the set of figures by its property of not being diagonally symmetrical. However, most participants selected a different figure and reasoned adequately, although not according to our expectations. This can be assigned as a defect of the used testing tool, as the black squares touched.
- In question 4, most participants explained selecting an asymmetrical figure from the set of figures by its property of not being symmetrical. Other reasons for choosing this figure were either not described or related to another aspect of the presented figure, particularly that a black square touched more than one side of the outer square enclosure. However, most participants selected a different figure explaining it by the distance and position aspects.
- In question 5, most responders selected an asymmetrical figure (rhomboid) but did not provide relevant explanations. Choosing other figures was substantiated by their properties related to equal lengths of sides or angle size.

Discussion and conclusions

a) Recapitulation

As research suggests, the ability to discriminate symmetrical figures from asymmetrical ones seems to develop in each human naturally. There are many types of symmetry, such as vertical, horizontal, diagonal (all referred to as axial symmetry or reflectional symmetry), rotational, translational, scale, glide, and so on. The ability to sense these types seems to develop at different rates, with vertical symmetry appearing first. Following already conducted research on symmetry perception among children, the study aimed to investigate symmetry perception among prospective preschool and primary teachers. Symmetrical figures are among the first children encounter in preschool and school facilities. Therefore, it is important to study teachers' perceptions as they foster students' development. This study was focused on prospective teachers in their second year of university study in order to determine their entry-level symmetry perception as they have not encountered specialized lessons on symmetry yet. During their study, they must take a course in geometry in the fourth year of university study. Obtained results may have practical applications as they can be used to increase the quality of geometry courses.

b) Interpretation of the results

The results of the presented study showed that vertical symmetry is the easiest to identify by the tested cohort. More precisely, an asymmetrical figure stands out among vertically symmetrical figures more than among horizontally and diagonally symmetrical figures. The results also suggest horizontal symmetry may be easier to identify than diagonal symmetry. Participants' explanations for selecting a particular figure were more related to symmetry with the diagonal symmetry and general symmetry questions. Almost no one used term „symmetry" in vertical in vertical symmetry and horizontal symmetry-related questions. This suggests it is easier to identify symmetry in figures with many apparent details than simple ones, such as common planar geometric shapes. This may be based on the amount of information within the figure and corresponding mirror-wise patterns. However, the abovementioned interpretations are based on results obtained from a relatively small cohort using a non-standardised test that needs further improvement. Thus, generalisations can be drawn only after more thorough research.

c) Drawing on previous research

Except for symmetry-related conclusions, the qualitative analysis uncovered a vast deficiency in participants' terminology. Mostly, participants referred to squares as cubes. Many similar misconceptions among preschool and elementary pre-service teachers were identified in the past. Mokriš and Scholtzová (2016) identified a misconception in primary pre-service teachers as some misidentified a rhombus as a square and a rhomboid as a rectangle, where the rhomboid was misidentified more frequently. Also, some of them misidentified a squircle as a square. The presence of such misconceptions may be a result of poor definition knowledge of geometric shapes.

Marchis (2012) found that some pre-service preschool and primary school teachers cannot correctly define basic geometric shapes because of their poor ability to recognize a shape, have little knowledge of its properties, and cannot identify essential properties of specific geometric shapes that distinguish them from others. Certainly, poor knowledge of shapes definition is what corresponds with the existence of misconceptions. Fujita and Jones (2006) asked elementary pre-service teachers, "What is the quadrilateral described as a parallelogram which has a right angle?" to which more than one-third of students did not respond correctly. Couto and Vale (2014) found that only a few prospective mathematics teachers of basic education could explain why a right triangle cannot be equilateral. Another study revealed that elementary pre-service teachers struggle with defining the rhombus (Pickreign, 2007) if asked to without presenting any model of a rhombus.

Symmetry may not be mentioned among other essential properties of planar shapes at first, although it is discussed with students at some point. The study by Moravcová, Robová, Hromadová, and Halas (2021) revealed that young people who are shortly out of high school struggle to identify the axis of symmetry in planar shapes. Particularly, a significant number of respondents claimed that a rhomboid has some lines of symmetry.

The last question in our test consisted of four planar shapes: rectangle, square, rhomboid, and rhombus. All previous questions were concerned with discriminating asymmetrical shapes where those symmetrical held properties of axial symmetry (vertical, horizontal, or slant). We expected participants to notice a symmetry pattern in the first four questions and apply analogy in answering the fifth question, selecting rhomboid as it has no axis of symmetry. Although most respondents chose the rhomboid, no explanations were provided. This suggests that respondents have the intrinsic ability to discriminate rhomboid from a set of axially symmetrical shapes; however, they are unaware of the axis of symmetry in planar shapes.

d) Limitations and Future Studies

The proposed research and its results discussed the perception of symmetry among people. Particularly, the research was conducted on a sample of university students enrolled in preschool and elementary education studies. The research tool was developed using specific requirements mentioned in the methodology part. Despite the effort to avoid distortions from the researched topic, a couple of factors were identified during qualitative data analysis that might have limited the validity of the research tool. Thus, the conducted study should be considered an initial investigation dedicated to learning about symmetry perception among pre-service preschool and primary school teachers and a valid testing tool creation process. Therefore, the obtained results inform how respondents perceive figures and how a better testing tool could be developed. In future studies, more aspects will be implemented when improving the testing tool, which should help reveal more valid data.

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POSTRZEGANIE SYMETRII U PRZYSZŁYCH NAUCZYCIELI PRZEDSZKOLI I SZKÓŁ PODSTAWOWYCH: BADANIE PILOTAŻOWE

STRESZCZENIE

Dzieci uczą się podstawowej geometrii poprzez obserwację i manipulowanie przedmiotami i kształtami oraz rozpoznawanie ich podstawowych właściwości. Jednym z podstawowych, wszechobecnych pojęć geometrycznych jest symetria. Rozwój wiedzy dzieci w dużej mierze zależy od wsparcia i nauki ze strony opiekunów oraz nauczycieli. Powinni więc posiadać odpowiednią wiedzę merytoryczną. Badanie pilotażowe sprawdza zdolność przyszłych nauczycieli przedszkoli i szkół podstawowych do rozpoznawania figur osiowo symetrycznych. Metodologia opiera się na zaprojektowanym teście i analizie uzyskanych wyników zebranych od grupy 56 przyszłych nauczycieli przedszkoli i szkół podstawowych, studiujących na drugim roku studiów uniwersyteckich na Słowacji. Wyniki potwierdziły wcześniejsze ustalenia dotyczące symetrii pionowej jako najłatwiejszej rozpoznawalnej formy symetrii, a następnie symetrii poziomej. Co ważne, przyszłym nauczycielom brakowało wiedzy na temat symetrii osiowej, gdyż rzadko odwoływali się do niej w zadanych zadaniach. Ponadto badanie wskazuje na ważne czynniki przy projektowaniu testu dyskryminacji pod kątem symetrii.

SŁOWA KLUCZOWE

romb, symetria osiowa, uczenie się przez analogię, percepcja wzrokowa, nauczyciele szkół podstawowych