Finite-time integral sliding mode control for chaotic permanent magnet synchronous motor systems

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Abstract: In this paper, an integral finite-time sliding mode control scheme is presented for controlling a chaotic permanent magnet synchronous motor (PMSM). The controller can stabilize the system output tracking error to zero in a finite time. Using Lyapunov's stability theory, the stability of the proposed scheme is verified. Numerical simulation results are presented to present the effectiveness of the proposed approach.

Key words: chaos control, control chaotic Permanent Magnet Synchronous Motor, finite-time, sliding mode

1. Introduction

A permanent magnet synchronous motor (PMSM) plays an important role in industrial applications due to its simple structure, high power density, low maintenance cost, and high efficiency [1-4]. However, investigations show that PMSM displays chaotic behavior when motor parameters lie in a certain range [3, 4]. Chaos in PMSM, which decreases the system performance, is highly undesirable in most engineering applications.

Several researchers have investigated the problem of control chaos in PMSM [5-16]. Decoupling control [5], entrainment and migration control [6], optimal Lyapunov exponents placement [7], sensorless control [8-10], time delay feedback control [11], feedback control [12], backstepping control [13], passivity control [14], sliding mode control [15], and fuzzy control [16, 17]. During the last three decades, variable structure systems (VSS) and sliding mode control (SMC) have received significant interest and have become well-established research areas with great potential for practical applications. The theoretical development aspects of SMC are well documented in many books and articles [7, 16, 20, 25, 27, 29, 31].

The principle of the sliding mode control is to forcibly constrain the system [18, 19, 21], by suitable control strategy, to stay on the sliding surface on which the system will exhibit

desirable features [24, 26, 28]. The advantages of SMC are robustness, computation speed, compact implementation, controller order reduction, disturbance rejection, and insensitivity to parameter variations. The main disadvantage of the SMC strategy is the chattering phenomenon. SMC has been applied in many control fields which include robot control [30], motor control [5, 22], flight control [23], control of power systems [9, 30, 32, 33], and chemical process control [4].

In this paper, an integral sliding mode control law is designed to achieve the finite-time chaos suppression for the permanent magnet synchronous motor system. The finite-time stability of the closed-loop system is proved using the Lyapunov theory.

This article is organized as follows: the dynamic model of PMSM is presented in Section 2; this model is normalized so that PMSM exhibits chaotic behavior under certain conditions. Section 3 describes in detail the proposed approach design and verifies the stability of the controller according to Lyapunov stability. Effectiveness of the proposed schemes is demonstrated by simulation in Section 4. Finally, the conclusion is given in Section 5.

2. Modeling of permanent-magnet synchronous motor (PMSM)

The mathematical model of PMSM [3] is given by:

$$\begin{cases} \dfrac{d\omega}{dt} = \dfrac{1}{J}\left(n_d\,\psi\,i_q + n_p\left(L_d - L_q\right)i_d\,i_q - T_L - \beta\omega\right), \\[2mm] \dfrac{di_q}{dt} = \dfrac{1}{L_q}\left(u_q - R\,i_d - \omega L_d i_d - \omega\psi_r\right), \\[2mm] \dfrac{di_d}{dt} = \dfrac{1}{L_d}\left(u_d - R\,i_d + \omega L_q i_q\right), \end{cases} \tag{1}$$

where i_q, i_d and u_d, u_q are three d-axis and q-axis stator currents and voltage L_q, L_d are q-axis stator inductances, R corresponds to the stator resistor. J is the polar moment of inertia; β is the viscous friction constant, ψ_r is the rotor magnetic flux linking the stator and n_p is the number of pole-pairs, T_L represents the external load torque and ω corresponds to the rotor angular velocity.

By employing an affine transformation $(.) = \lambda^{-1}(.)$ and a time-scaling transformation,

$$t = \frac{R}{L_q}t',\ \text{where}\ \lambda = \begin{bmatrix} \tau k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & R/L_q \end{bmatrix},\quad \tau = \frac{L_d}{L_q},\quad k = \frac{\beta R}{L_q n_p \psi_r},\quad \gamma = \frac{\psi_r}{k L_q},\quad \sigma = \frac{\beta L_q}{LJ}.$$

Then system (1) can be transformed to the following form, which is more suited for control-design purpose:

$$\begin{cases} \dfrac{d\omega}{dt} = \sigma(i_q - \omega) - T_L, \\[2mm] \dfrac{di_q}{dt} = -i_q - i_d\omega + \gamma\omega + u_q, \\[2mm] \dfrac{di_d}{dt} = -i_d + i_q\omega + u_d, \end{cases} \qquad (2)$$

where ω is the angle speed, i_q and i_d denote the quadrature and the direct-axis currents of the motor, respectively σ and γ are the system parameters, T_L is the load torque, u_q and u_d are the quadrature and direct-axis stator voltage components, respectively. In our paper, we only take the case $u_q = u_d = 0$ then, system (2) becomes:

$$\begin{cases} \dfrac{d\omega}{dt} = \sigma(i_q - \omega), \\[2mm] \dfrac{di_q}{dt} = -i_q - i_d\omega + \gamma\omega, \\[2mm] \dfrac{di_d}{dt} = -i_d + i_q\omega. \end{cases} \qquad (3)$$

Fig. 1 and Fig. 2 illustrate the typical chaotic attractor's behavior of PMSM for a certain range of its parameters. In our paper we choose the case of $(\sigma, \gamma) = (5.45, 20)$ and initial condition. $\omega(0) = 0.5$, $i_q(0) = 0.6$, $i_d(0) = 0.5$.
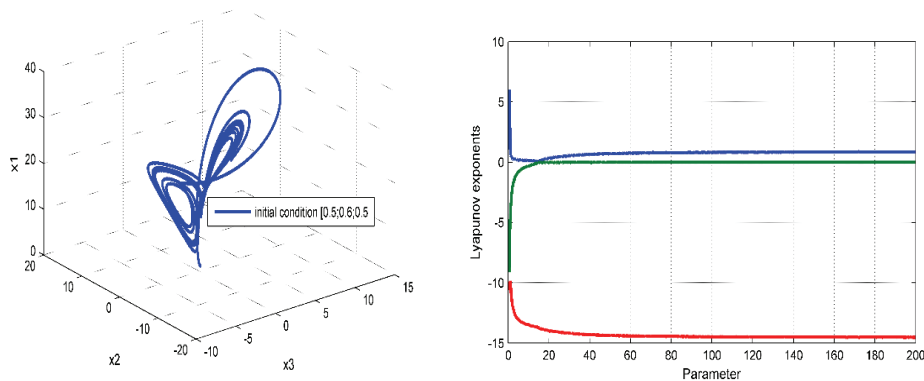

Fig. 1. Characterizations of chaos in PMSM: (a) chaotic attractor; (b) lyapunov exponent

3. Integral terminal sliding mode control for PMSM

Consider a nonlinear system as follows:

$$\dot{x} = f(x) + g(x)u, \qquad (4)$$

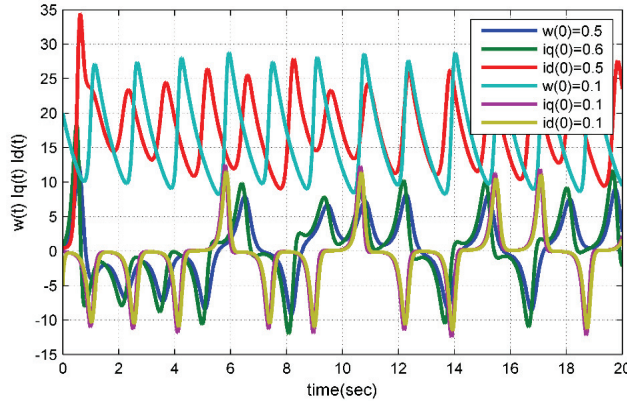where $x \in R$, $f(x) \in R$, $g(x) \in R$ and $g^{-1}(x) \neq 0 \in R$.

Fig. 2. Time response of the state of PMSM for two initial conditions (0.5, 0.6, 0.5)-(0.1, 0.1, 0.1)

Let us introduce a sliding variable:

$$S = x + \theta x_1 , \tag{5}$$

$$\dot{x}_1 = \text{sgn}(x) \text{ with } x_1(0) = -\frac{x(0)}{\theta} , \text{ where } \theta > 0 .$$

According to [20], x will be always kept on S. If we guarantee that the surface S is always zero then x will converge to zero in a finite time.

Let (ω^r, i_q^r, i_d^r) denotes an equilibrium point (ω^r, i_q^r, i_d^r). The aim of this paper is to design a controller that stabilizes the system (3) to the equilibrium point and guarantees chaos suppression. We add the single control $u(t)$ to system (3) and then the controlled PMSM system can be expressed by:

$$\begin{cases} \dfrac{d\omega}{dt} = \sigma(i_q - \omega), \\[2mm] \dfrac{di_q}{dt} = -i_q - i_d\omega + \gamma\omega + u, \\[2mm] \dfrac{di_d}{dt} = -i_d + i_q\omega. \end{cases} \tag{6}$$

Let the error states of system be:

$$e_1 = \omega^r - \omega, \ e_2 = i_q^r - i_q \text{ and } e_3 = i_d^r - i_d .$$

Then dynamical system can be expressed as:

$$\begin{cases} \dot{e}_1 = \sigma(e_2 - e_1), \\[2mm] \dot{e}_2 = -(e_1e_2 + \omega e_3 + i_d e_1) \ - e_2 + \gamma e_1 + u, \\[2mm] \dot{e}_3 = e_1e_3 + \omega e_2 + i_q e_1 - e_3 . \end{cases} \tag{7}$$

A suitable sliding surface can be chosen as:

$$S = e_2 + \theta e_{2I} , \tag{8}$$

$\dot{e}_{2I} = \text{sgn}(e_2)$ with $e_{2I}(0) = -\dfrac{e_2(0)}{\theta}$.

If e_2 reaches S it will converge to zero in a finite time

$T_S = \dfrac{|e_2(0)|}{\theta}$.

The time derivative of S is

$$\dot{S} = \dot{e}_2 + \theta\,\text{sgn}(e_2). \tag{9}$$

By letting $S = 0$, one can get the equivalent control as:

$$u_{eq} = e_1 e_3 + \omega e_3 + i_d e_1 + e_2 - \gamma e_1 - \theta\,\text{sgn}(e_2). \tag{10}$$

The reaching control is selected as follows:

$$u_{rech} = -\mu\,\text{sgn}(S), \tag{11}$$

where $\mu > 0$ is a positive number.

The sliding mode controller law is designed as:

$$u = u_{eq} + u_{rech} . \tag{12}$$

Theorem

Consider dynamic error system (7). The integral sliding mode controller (12) yields convergence of e_2 and $\dot{e}_2$ to 0 in a finite time, and the errors e_1 and e_3 are globally asymptotically stable at (0, 0).

Proof

Let the Lyapunov function $V(S)$ be defined by:

$$V(S) = \frac{1}{2}S^2 . \tag{13}$$

The time derivative of this function with respect to time along (7):

$$\dot{V}(S) = S\dot{S}, \tag{14}$$

$$\dot{V}(S) = S(-(e_1 e_3 + \omega e_3 + i_d e_1) - e_2 + \gamma e_1 + \theta\,\text{sgn}(e_2) + u). \tag{15}$$

Substituting (12) into (15) gives:

$$\dot{V}(S) = -\mu|S|. \tag{16}$$

According to the sliding condition [20], e_2 and $\dot{e}_2$ converge to zero in a finite time. After e_2 has converged to 0, the dynamical errors e_1 and e_3 can be written as:

$$\begin{cases} \dot{e}_1 = -\sigma\, e_1, \\ \dot{e}_3 = i_q e_1 - e_3. \end{cases} \tag{17}$$

If the candidate Lyapunov function is defined as:

$$V(e) = \frac{1}{2}(e_1^2 + e_3^2), \tag{18}$$

then the time derivative of $V(e)$ is:

$$\dot{V}(e) = e_1\dot{e}_1 + e_3\dot{e}_3, \tag{19}$$

$$\dot{V}(e) = -\sigma e_1^2 - e_3^2 + i_q e_1 e_3. \tag{20}$$

Inequality (21) can be rewritten as:

$$\dot{V}(e) = -e^T \Gamma e \ < 0, \tag{21}$$

with $e = [e_1\ \ e_3]^T$ and $\Gamma = \begin{bmatrix} \sigma & 0 \\ -i_q & 1 \end{bmatrix} > 0\ \ (\sigma > 0)$.

4. Simulation results

Simulations, using MATLAB Software Package, are carried out to verify the effectiveness of the proposed method. The parametric values of PMSM are the same as those in Section 3. Without loss of generality, we select $(\omega^r,\, i_q^r,\, i_d^r) = (0, 0, 0)$ as the desired equilibrium point. The control method takes effect after $t = 4.5$ s. Fig. 3 shows the dynamic error system states of PMSM and the manipulated signals u.

From the results, it can be seen that the proposed integral sliding mode controller structure shows good performance in achieving the output regulation.

Next, we examine the robustness of the proposed controllers in the presence of the model uncertainty and external disturbances. Fig. 4 shows the performances of controllers for 40% increase in the parameters and external disturbance were added to PMSM: omiga 1 = $= 0.2\cos(0.3t)$, $omiga\ 2 = 0.3\cos(0.4t)$, $omiga\ 3 = 0.6\cos(0.6t)$. Therefore, it can be concluded that the proposed control schemes are robust to changes in the parameters and to disturbances acting on the system.

Finally, we examine the robustness of the proposed controllers in the presence of the measurement noise. In this case, white Gaussian noises with variances of 3% are simultaneously added to the outputs measurements. The system transient's responses for the controller and the control action are shown in Fig. 5.
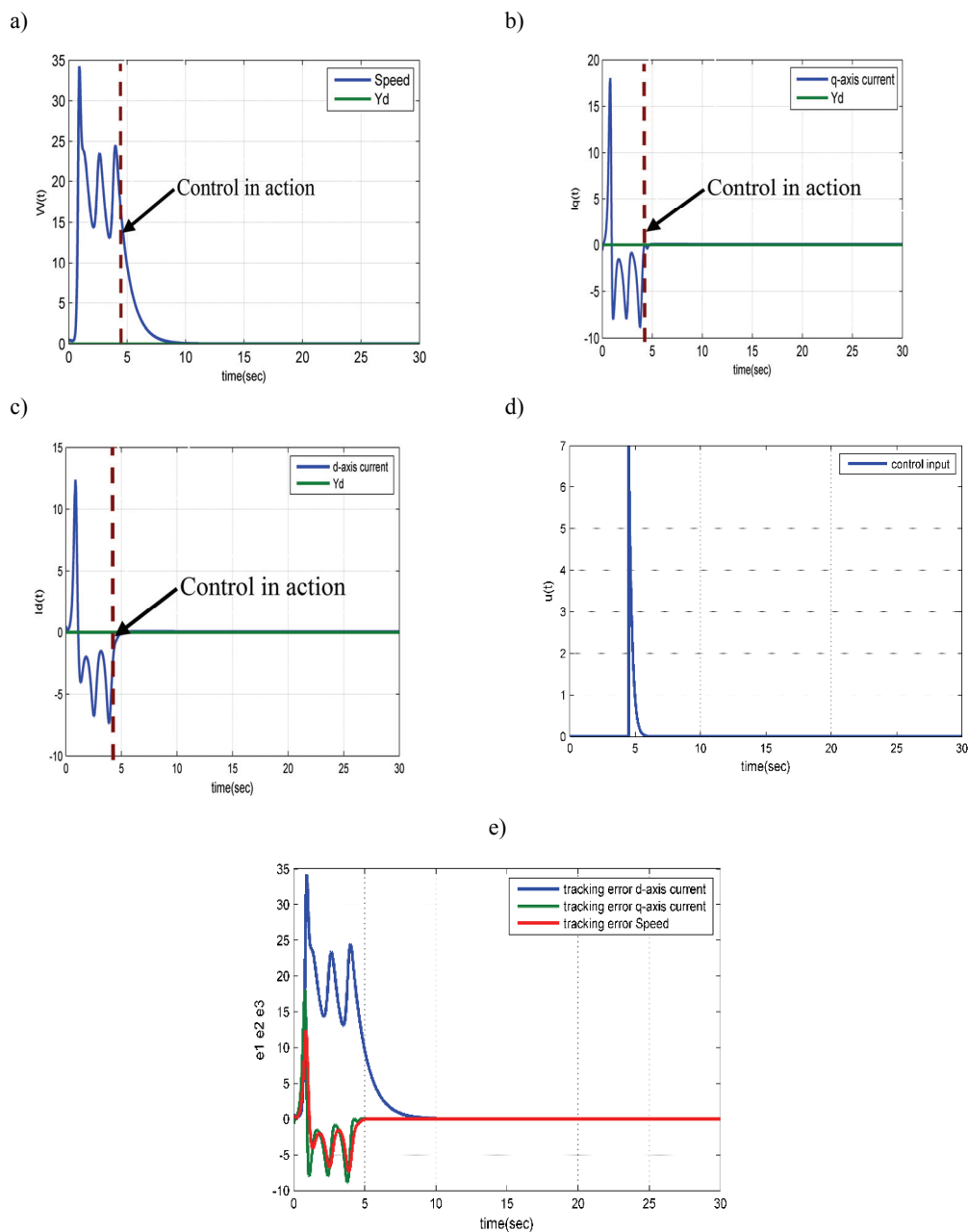
a)

b)

c)

d)

e)

Fig. 3. System responses with the control in action at $t = 4.5$ s: (a) speed; (b) q-axis current; (c) d-axis current; (d) control input; (e) tracking error

It can be seen that the set-point tracking behavior is very satisfactory. Note that the proposed controller maintains the PMSM state in a small neighborhood of the desired value despite the noise on the measurement.
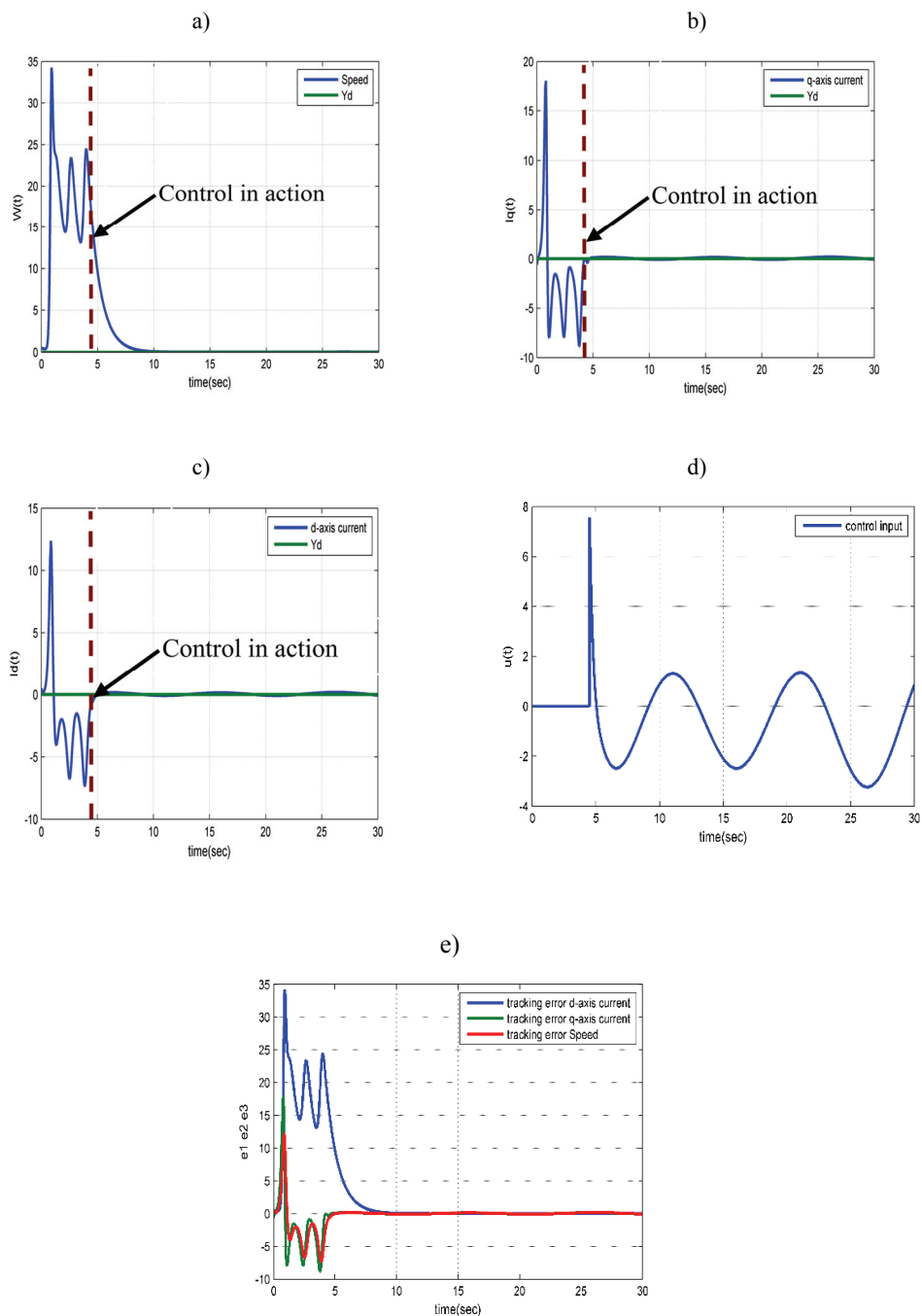
a)

b)

c)

d)

e)

Fig. 4. System responses in the presence of the model uncertainty and external disturbances with the control in action at t = 4.5 s: (a) speed; (b) q-axis current; (c) d-axis current; (d) control input; (e) tracking error

a)

b)

c)

d)

e)

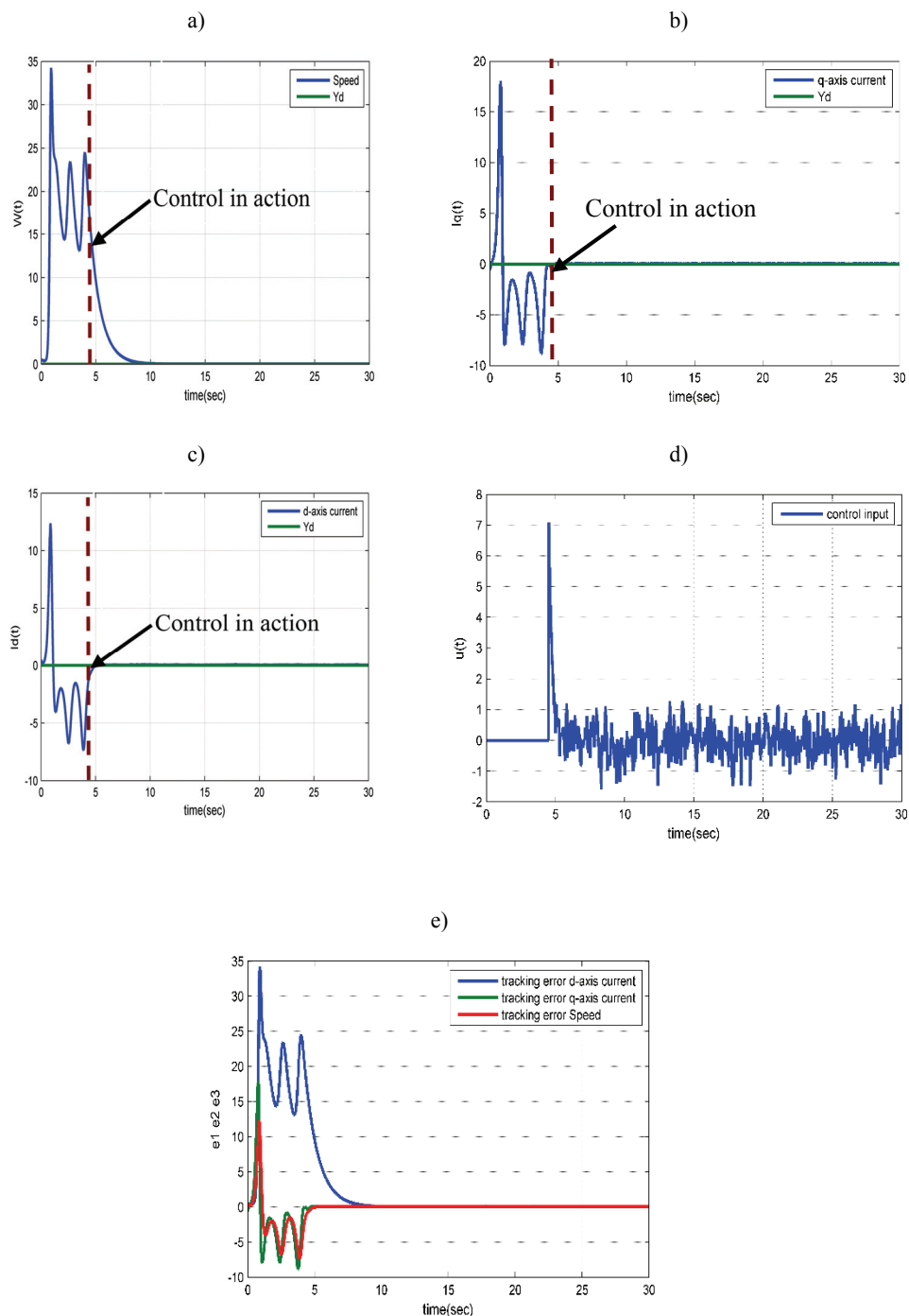Fig. 5. System responses with the presence of the white noise with the control in action at $t = 4.5$ s:
(a) speed; (b) q-axis current; (c) d-axis current; (d) control input; (e) tracking error

5. Conclusion

In this paper, we presented an integral finite-time sliding mode control scheme for a permanent-magnet synchronous motor. The closed-loop stability is proved. Through numerical simulations, we illustrated the feasibility of the designed control system. Moreover, the proposed control exhibits a satisfactory performance when used with disturbance and dynamics uncertainty.

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