Vibration of the Oscillator Exchanging Mass with Surroundings

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Abstract

Vibration of two simple open systems (namely the linear mass-sprins oscillator and the mathematical pendulum) are investigated. During the motion, the body absorbs matter through its boundary. In both cases, mechanism of mass absorption is modeled as a perfectly 'inelastic' collision and constant rate of mass change is assumed. The paper is focused on the influence of mass change on the kinematic aspects of oscillations.

Keywords: vibration of open systems, mass variable, reactive force

1. Introduction

Mass is generally not conserved when a supply of mass is present, or when open systems with a flow of mass through their surface are to be considered. Mass of the mechanical system then is said to be variable. In such a situation, the general methodological approaches of mechanics have to be properly modified. In solid mechanics, the systems with a variable mass appear as the result of a problem-oriented modelling, e.g., when mass is expelled or captured by a structure or machine. The finite discontinual mass variation in a very short time was not of special interest for a long time and was not intensively discussed. Meshchersky was the first who considered the velocity change of a translatory moving body during step-like mass variation [1]. The motion of the continuously mass variable systems is much more investigated due to its application in rocket theory [2] and astronomy [3]. The motion is described with differential equations with variable parameters. For the case when the mass is varies continuously in time, the influence of the reactive force on the motion is investigated by Cvecitanin and Kovacic[4].

Mathematically the reactive force is the product of the mass variation function and the relative velocity of mass separated from or added to the particle. Usually, two special cases were considered: the first one for zero relative velocity and the second for zero absolute velocity of separated or added mass. In case of zero relative velocity, i.e. when the absolute velocity of the separated or added particle is equal to the velocity of the basic particle, the reactive force is also zero.

Based on the dynamics of the particle with time-varying variable mass and the basic laws of dynamics, the theoretical consideration of the dynamics of the body with time variable mass is presented in this paper. Two mechanical systems are considered: one dimensional oscillator and a mathematical pendulum.

The process of the mass increase is considered as the perfectly inelastic impact of a small mass on the main body. Based on the general equations of motion, the mathematical model for the oscillatory motion is formed.

2. Linear oscillator

In this section the open mechanical system is considered which absorbs matter from the surroundings. Its physical model is presented in Fig.1.

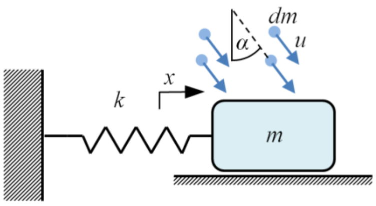

Figure 1. One-dimensional linear oscillator exchanging mass with the surrounding

Let us assume that the mass $m(t)$ of the body changes with time proportionally to the area of its surface with a constant rate – Γ. Mass change is described by the evolution equation

$$\dot{m}(t) = \Gamma \tag{1}$$

and at the beginning $m(0) = m_0$.

In this way the mass of the body changes in time linearly

$$m(t) = m_0 + \Gamma t\ ,\ \ t > 0\ . \tag{2}$$

The added mass *dm* drops at the body with the absolute velocity **u** (see Fig.1).

The momentum principle in the case of mass exchanging body is

$$m(t)\dot{\mathbf{v}} = \mathbf{F} + \dot{m}(t)(\mathbf{u} - \mathbf{v}), \tag{3}$$

where **F** is the resultant force, **v** velocity of the body and **u** velocity of the added particles of mass *dm*. In the case of free vibration, the mathematical model of the one-degree-of-freedom oscillator with time variable mass is

$$(m_0 + t\,\Gamma)\ddot{x}(t) = -k\,x(t) + \Gamma(u_x - \dot{x}(t)), \tag{4}$$

Where *x*(t) is the coordinate describing the position of the body, $u_x = |\mathbf{u}|\sin(\alpha)$ is the *x* – component of the velocity **u**, m_0 is the initial mass of the body and *k* denotes the stiffness coefficient.

After rearranging, the equation of motion (4) takes the form

$$(m_0 + t\,\Gamma)\ddot{x}(t) + \Gamma\dot{x}(t) + k\,x(t) = \Gamma u_x\ , \tag{5}$$

The second term on the left hand side of (6) can be recognized as a damping of viscous type, and a constant force occurs on the right side.
The equation of motion (5) is supplemented by the initial conditions

$$x(0) = x_0, \;\; \dot{x}(0) = v_0 . \tag{6}$$

The analytical solution of the problem (5) – (6) is as follows

$$x(t) = AJ_0\left(2\sqrt{\frac{k}{\Gamma}}\sqrt{\frac{m_0}{\Gamma}+t}\right) + BY_0\left(2\sqrt{\frac{k}{\Gamma}}\sqrt{\frac{m_0}{\Gamma}+t}\right) + \frac{\Gamma}{k}u_x , \tag{7}$$

where $J_0(.)$ and $Y_0(.)$ are the Bessel functions of the first and second kind, respectively, A and B are the unknown constants which fulfil the following equations resulting from initial conditions (6)

$$AJ_0\left(2\sqrt{\frac{k\,m_0}{\Gamma^2}}\right) + BY_0\left(2\sqrt{\frac{k\,m_0}{\Gamma^2}}\right) + \frac{\Gamma}{k}u_x = x_0 \tag{8}$$

$$-\sqrt{\frac{k}{m_0}}AJ_1\left(2\sqrt{\frac{k\,m_0}{\Gamma^2}}\right) - \sqrt{\frac{k}{m_0}}BY_1\left(2\sqrt{\frac{k\,m_0}{\Gamma^2}}\right) = v_0 . \tag{9}$$

Some results of calculations are presented for the chosen values of parameters m_0=1 kg, Γ=0.01kg/s, k=10N/m, u_x=2m/s, and initial values: x_0=0.1 m, v_0=0m/s.
The solution (7) is presented in Fig.2.

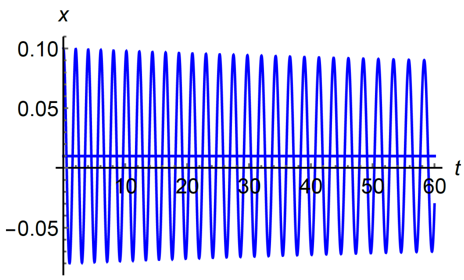

Figure 2. Time history of the body motion

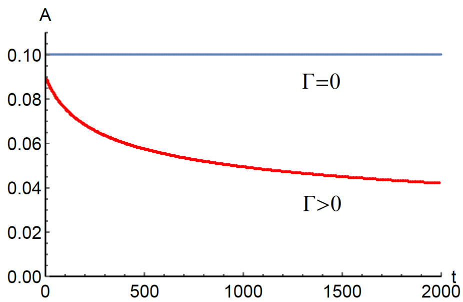

Figure 3. Amplitude vs. time for constant and varying mass oscillator

The system oscillates around the equilibrium state given by the particular solution $u_x \Gamma / k$. The shift, in the time history in Fig. 2, appears due to the spring extension caused by the constant momentum supply of the added mass. The amplitude decreases in time while the mass of the oscillator grows, which is illustrated in Fig. 3.

The amplitude – frequency spectra, obtained using the discrete Fourier transform for the system with constant and varying mass are shown in Fig. 4.

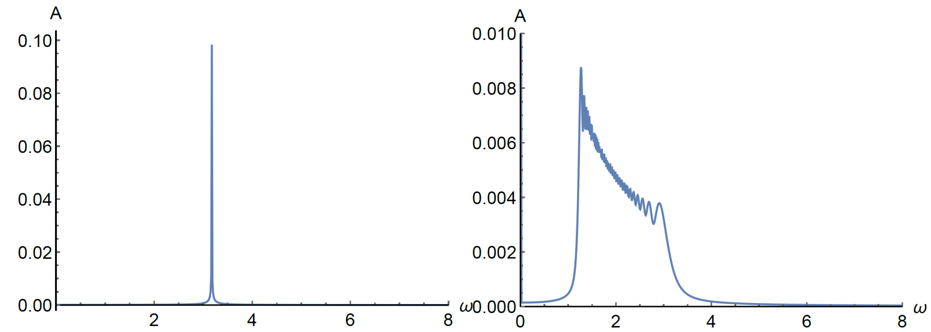

Figure 4. The amplitude spectra of the oscillations with constant and variable mass

The amplitude spectra in the case of mass exchange and those with constant mass are quite different. This effect is connected with variation of the self-frequency of the system in time.

3. Pendulum exchanging mass with surroundings

The problem of motion of the pendulum which exchanges mass with surroundings is investigated in this section. The process of mass exchanging is the same as described above. In this case, the governing equation of pendulum motion is

$$(m_0 + t\Gamma)L\ddot{\varphi}(t) + \Gamma L\dot{\varphi}(t) + u\Gamma\sin(\alpha - \varphi(t)) + g(m_0 + t\Gamma)\sin\varphi(t) = 0 \tag{10}$$

with the initial conditions

$$\varphi(0) = \varphi_0, \ \ \dot{\varphi}(0) = \omega_0 , \tag{11}$$

where L and m_0 are length of the pendulum and its initial mass respectively.

The problem (10) – (11) is solved only numerically due to geometric nonlinearities in considered problem described by Eq. (10).

Results of two simulations concerning small and large oscillations are presented hereafter. Calculations have been made for the following values of parameters: m_0=1kg, Γ=0.01kg/s, u=2m/s, L=0.7m and $\alpha=\pi/3$.

In Fig. 5 time histories of two regimes of motion are presented. One of them, caused by the initial values φ_0=0.1, ω_0=0, is related to the small oscillations and the second one, caused by φ_0=1.3, ω_0=0, concerns the large oscillations.

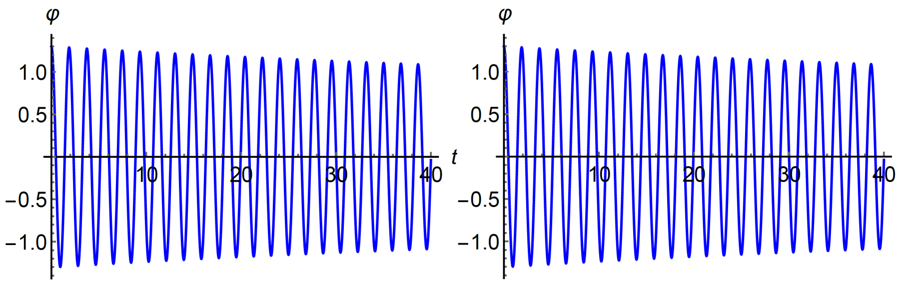

Figure 5. Time histories of small (left) and large (right) oscillations

The character of vanishing amplitude of the pendulum which absorbs mass in comparison to constant amplitude in the case of pendulum with constant mass is presented in Fig. 6.

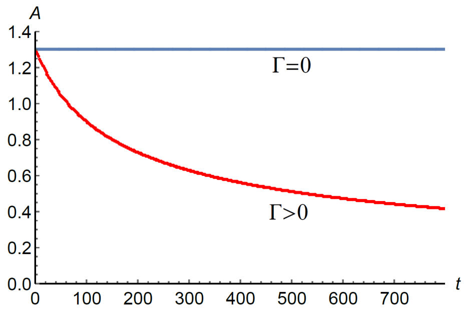

Figure 6. Amplitude vs. time for constant and varying mass pendulum

In Fig. 7 the amplitude-frequency spectra for the case of small and large oscillations are presented.

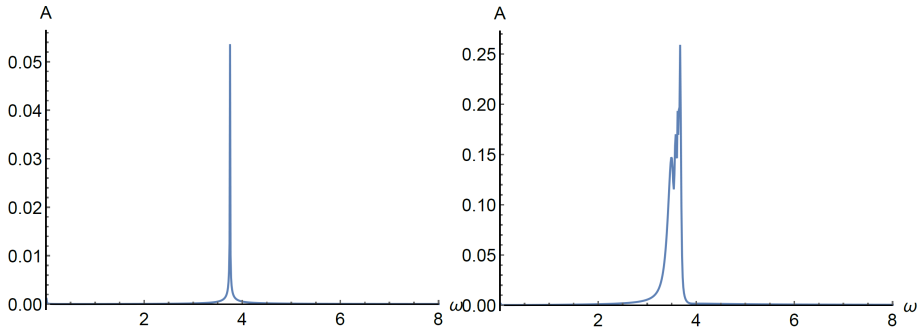

Figure 7. Amplitude spectra for the case of small and large oscillations

Similarly as for the linear oscillator the amplitude spectrum is strongly affected by the effect of mass variation.

4. Conclusions

Two open systems with one degree of freedom have been investigated. One of them described by the linear differential equation and the second one described by the nonlinear equation. Nonlinearities in the pendulum equation are of geometrical type. In the governing equations some time depending coefficients appear due to changing mass of the system. One additional term has the same form as viscous damping, and appears in the both discussed structures. Other additional term can be recognized in the linear oscillator as a constant force, whereas in the pendulum its counterpart term is time dependent and nonlinear.

The analytical solution of the initial value problem describing motion of the linear oscillator of variable mass has been achieved. The pendulum oscillation might have been analyzed only numerically due to nonlinearities.

The mass increase causes decreasing amplitude of oscillation in both tested structures. The mass exchange with surroundings affects the amplitude-frequency spectra both for the linear oscillator and pendulum.

Acknowledgments

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