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# STUDY OF CURRENT SPACE PHASOR TRAJECTORY OF THE THREE-PHASE ASYNCHRONOUS MOTOR WITH ONE PHASE OPEN CIRCUIT FAULT

## BADANIE TRAJEKTORII PRZESTRZENNEGO FAZORA PRĄDU SILNIKA ASYNCHRONICZNEGO TRÓJFAZOWEGO PRZY USZKODZENIU JEDNEJ FAZY

**Abstract:** The presented paper deals with the calculation of space phasor trajectory of the magnetization current of induction machine supplied by a converter under open-phase fault. To explore possibilities to reduce the impact of failure on the function of the machine is a winding node connected to zero point. Under open phase fault it is another possibility to improve behavior of the two-phase operation. There is still to apply a circular rotating magnetic field to the machine by imposing a 60° phase shift between the stator currents of the two supplied stator windings.

Keywords: Fourier series, asynchronous motor, two-phase operation, fault-tolerance

#### 1. Introduction

Reliability of an electrical drive has been always a primary requirement concern in many industrial application. The requirement of reliability is related to the concept of fault tolerance drive system. The electrical drive usually consists of electrical motor supplied by a semiconductor inverter and a suitable control strategy which is able to menage the drive system during a fault.



Fig. 1. Inverter supplied three-phase AM drive

The most frequently used drive of recent times is frequency-controlled three-phase asynchronous motor supplied from a voltage source inverter. A frequent failure state is a failure of one phase of inverter as the result of current overload of power transistors. Minimize disruption of the inverter is achieved by short-circuit transistors protection in each of transistors leg.

The feature is achieved by means of the reconfiguration of the inverter structure during the fault of inverter. The inverter fault tolerant capability is related to the drive coast and the desired system reliability level.

#### 2. Inverter mathematical model

For inverter's operation study at steady state we consider following idealized conditions:

- Power switch, that means the switch can handle unlimited current and blocks unlimited voltage.
- The voltage drop across the switch and leakage current through switch are zero.
- The switch is turned on and off with no rise and fall times.
- Sufficiently good size capacity of the input voltage capacitors divider, so we can assume that converter input DC voltage is constant for any output currents.

This assumptions help us to analyze a power circuit and help us to build a mathematical model for the inverter at steady state. If the desired reference voltage is sine-wave, two parameters define the control:

- *Coefficient of the modulation m* equal to the ratio of the modulation and reference frequency.
- *Voltage control coefficient r* equal to the ratio of the desired voltage amplitude and the DC supply voltage.

The leg transistors are controlled to create the output leg voltages (phase-to negative rail of the DC bus 0) of the form

$$u_{01} = \frac{U_e}{2} + r \frac{U_e}{2} \sin \theta;$$
  

$$u_{02} = \frac{U_e}{2} + r \frac{U_e}{2} \sin \left( \theta - \frac{2\pi}{3} \right);$$
 (1)  

$$u_{03} = \frac{U_e}{2} + r \frac{U_e}{2} \sin \left( \theta + \frac{2\pi}{3} \right)$$

The coefficient of the modulation divides desired period of output voltage to *m* intervals with  $\frac{2\pi}{m}$  width. In each of intervals it is necessary to create a voltage pulse whose area is equal to the area bounded by the desired waveform of the output voltage within the interval.

We assume that the front pulse is defined by angle  $\alpha_n$  and back pulse by angle  $\beta_n$ . Comparing the areas we receive for the angles of the n-th voltage interval following formulas

$$\alpha_n = \frac{\pi}{m} \left( 2n - \frac{1}{2} \right) + \frac{r}{2} \left[ \cos n \frac{2\pi}{m} - \cos \frac{\pi}{m} (2n - 1) \right]$$
$$\beta_n = \frac{\pi}{m} \left( 2n + \frac{1}{2} \right) + \frac{r}{2} \left[ \cos n \frac{2\pi}{m} - \cos \frac{\pi}{m} (2n + 1) \right]$$
(2)

We can output voltage of the first inverter leg to express as a complex Fourier series of the form

$$u_{01} = \frac{U_e}{2} + 2U_e \sum_{k=1}^{\infty} \sum_{n=1}^{m} c_n e^{jk\theta};$$

$$\begin{cases} c_n = \frac{1}{j2k\pi} \left( e^{-jk\alpha_n} - e^{-jk\beta_n} \right) \\ valid \text{ for } k \neq 0 \end{cases}$$
(3)

Where:  $U_e$  is inverter input DC voltage

Output voltages of the other two transistor legs can be expressed by the relations

$$u_{02} = u_{01} a; \quad u_{03} = u_{01} a^2;$$
 (4)

where: *a* is shift coefficient defined as  $a = e^{-j\frac{2k\pi}{3}}$ 

Assume the wye connection of the stator winging, the phase voltages can by expressed as

$$u_{1} = \frac{2}{3}U_{e}\sum_{k=1}^{\infty}\sum_{n=1}^{m}c_{n}\left(2-a-a^{2}\right)e^{jk\theta}$$

$$u_{2} = \frac{2}{3}U_{e}\sum_{k=1}^{\infty}\sum_{n=1}^{m}c_{n}\left(2a-a^{2}-1\right)e^{jk\theta}$$

$$u_{2} = \frac{2}{3}U_{e}\sum_{k=1}^{\infty}\sum_{n=1}^{m}c_{n}\left(2a^{2}-1-a\right)e^{jk\theta}$$
(5)



Fig. 2. Phase voltage waveform

The Fig.2 depicts the phase voltage waveform calculated on the base of equation (5). The waveform was calculated for DC input voltage  $U_e = 540V$ , output frequency f = 50Hz and modulation frequency  $f_m = 2kHz$  (m = 40).

# 3. Magnetization current space phasor trajectory

A space phasor of magnetization current is closely related to the size and shape of the rotating magnetic field of the machine.

Spatial phasor is calculated from the phase currents and follow by Clark transformation. The phase current will be calculated from phase voltages

$$i_{1} = \frac{2}{3}U_{e}\sum_{k=1}^{\infty}\sum_{n=1}^{m}c_{n}\left(2-a-a^{2}\right)\frac{e^{jk\theta}}{R_{1}+jk\omega L_{1}}$$

$$i_{2} = \frac{2}{3}U_{e}\sum_{k=1}^{\infty}\sum_{n=1}^{m}c_{n}\left(2a-a^{2}-1\right)\frac{e^{jk\theta}}{R_{1}+jk\omega L_{1}}$$

$$i_{3} = \frac{2}{3}U_{e}\sum_{k=1}^{\infty}\sum_{n=1}^{m}c_{n}\left(2a^{2}-1-a\right)\frac{e^{jk\theta}}{R_{1}+jk\omega L_{1}}$$
(6)

Where:  $R_1$  is stator winging resistance and  $L_1$  is stator winding inductance for no loaded machine

$$\begin{bmatrix} i_{\alpha} \\ i_{\beta} \\ i_{0} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} i_{1} \\ i_{2} \\ i_{3} \end{bmatrix};$$

$$i_{\alpha} = \frac{1}{3} (2i_{1} - i_{2} - i_{3}); \quad i_{\beta} = \frac{1}{\sqrt{3}} (i_{2} - i_{3}); \quad (7)$$

$$i_{0} = \frac{1}{3} (i_{1} + i_{2} + i_{3});$$

The Fig.3 shows the current space phasor trajectory for no faulted machine. For calculus were taken a parameters 4,4kW asynchronous machine with stator resistance  $R_1 = 1\Omega$  and stator inductance  $L_1 = 15 \, mH$ .



Fig. 3. Current space phasor trajectory for no faulted machine

#### 4. One-phase open circuit fault

In case of loss one of phase will be two of uninterrupted phases connected in series and supplied by the line voltage of the inverter.

$$u_{12} = 2U_e \sum_{k=1}^{\infty} \sum_{n=1}^{m} c_n (1-a) e^{jk\theta}$$
(8)

In Fig.4 is shown the waveform of an inverter output line voltage.

The current is given by



Fig. 2. Line voltage waveform

Space phasor of magnetizing current is pulsed, just as the magnetic field of the stator. Fig. 5 shows the trajectory of the space current phasor calculated on the base of equations (9).



*Fig. 5. Current space phasor trajectory for faulted machine* 

Motor has lost starting torque, continued operation is not possible.

#### 5. Two-phase operation

Despite the failure of one phase, there is still possibility to create on the stator elliptic rotating magnetic field. The solution is to connect the neutral point of the stator winding to the middle point of the DC bus as can be seen in the Fig.1. After the node is connected to the zero point of the DC bus, the phase voltages are given by equations

$$u_1 = 2U_e \sum_{k=1}^{\infty} \sum_{n=1}^{m} c_n e^{jk\theta}; \quad u_2 = a u_1 \quad (10)$$



*Fig. 7. Phase voltage waveform with connected node to zero point* 

For the currents following equations are valid

$$i_1 = 2U_e \sum_{k=1}^{\infty} \sum_{n=1}^{m} c_n \frac{e^{jk\theta}}{R_1 + jk\omega L_1}; \quad i_2 = a i_1$$
(11)

After Clark transformation

$$i_{\alpha} = \frac{2}{3} U_{e} \sum_{k=1}^{\infty} \sum_{n=1}^{m} c_{n} (2-a) \frac{e^{jk\theta}}{R_{1} + jk\omega L_{1}}$$

$$i_{\beta} = \frac{2}{\sqrt{3}} U_{e} \sum_{k=1}^{\infty} \sum_{n=1}^{m} c_{n} a \frac{e^{jk\theta}}{R_{1} + jk\omega L_{1}} \qquad (12)$$

$$i_{0} = \frac{2}{3} U_{e} \sum_{k=1}^{\infty} \sum_{n=1}^{m} c_{n} (1-a) \frac{e^{jk\theta}}{R_{1} + jk\omega L_{1}}$$

Trajectory of the current space phasor is elliptical, as shown in the Fig. 8.



Fig. 8 Current space phasor trajectory with connected node to zero point

Variable size of current space phasor cause varying the magnetic field around the perimeter of the stator. But resulting is higher electromagnetic torque ripple.

#### 6. Circular rotating magnetic field

There is an important conclusion, that under open phase fault it is still possible to apply a circular rotating magnetic field to the machine by imposing a 60° phase shift between the stator currents of the two supplied stator windings.

The equation (11) and (12) for calculating the stator currents are still valid. The shift coeffi- $\frac{2k\pi}{2k\pi}$ 

cient is now defined as  $a = e^{-j\frac{2\pi n}{6}}$ 



Fig. 9. Current space phasor trajectory for 60° phase shift

Fig. 9 depicts the trajectory of the current space phasor for 60° phase shift between the stator current.



Fig. 10. Current space phasor trajectory for  $60^{\circ}$  phase shift f = 30Hz and r = 0.9

By currents 60° shift was compensated backward current component which caused the elliptical shape of the trajectory of spatial phasor.

Weakening of the magnetic field can be partially compensated by an increase of supply voltage in area of the low frequencies. In any case, the machine performance drops below 50% of nominal power.

In Fig. 10 is shown the current space phasor for output fervency of f = 30Hz and suply voltage control coefficient of r = 0.9.

#### Conclusion

In a presented contribution the trajectory of stator current space vector in a case one phases fault is discussed. There is shown is the possibility of two-phase operating with rotating magnetic flux in reduced machine performance.

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