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# Grain-size distribution analysis of Quaternary sediments from the southern part of the Lodz region in Poland: a computational-methods approach

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**Abstract**

Eighteen samples of Quaternary unconsolidated sediments from the Piotrków Plateau and the Radomsko Hills in central Poland have been analysed for their average grain size, sorting, skewness and kurtosis. The analysis was carried out by seven computational methods of interpolation and nine extrapolation methods. It appears that linear interpolation, the traditional method (DOS), and the Josek and Gradistat Programs give comparable results, but that quadratic interpolation and the method of moments should not be applied since they yield unreliable results. The method of moments gives unduly high or unduly low parameter values because of the application of different, i.e. incomparable measures in the applied formulae. It should be stressed that only extrapolation provides, if performed under the right conditions, the possibility to determine some parameters, in particular skewness values.

**Keywords:** granulometric analysis, skewness, kurtosis, computational methods, Quaternary sediments, central Poland

## 1. Introduction

A graphical method to perform a granulometric analysis was presented more than half a century ago by Folk & Ward (1957). This allowed sedimentologists to calculate approximate grain-size parameters, obtained from graphs drawn by hand. The statistical parameters are calculated in phi or mm units. The accuracy of the results depended on the precision of the plot.

The development of computerized data analysis enabled much more precise calculations of statistical parameters such as the average size of grain, sorting, skewness and kurtosis by means of a computer. These parameters are considered by some earth scientists as essential for classifying sedimentary environments. They are designated by different methods (Folk, 1966; Grzegorczyk, 1970; Racinowski et al., 2001) and characterise the particle-size distribution in sediments. Environmental interpretation on the basis of such granulometric presentations deals mostly with Quaternary sediments (Passega, 1964; Visher, 1969; Grzegorczyk, 1970; Allen et al., 1972; McLaren, 1981; Brown, 1985; Merta, 1991; Mycielska-Dowgiałło, 1995, 2007; Asselman, 1999; Bravard & Peiry, 1999; Dade, 2000; Racinowski et al., 2001; Wachecka-Kotkowska, 2004; Flemming, 2007; Hartmann, 2007; Szmańda, 2007, 2010).

The constantly growing computing power of modern computers makes it possible to elaborate the same data by different mathematical and statistical methods (Blott & Pye, 2001). Selection of the most reliable computational method therefore now becomes a problem. A choice should take into account that mathematical methods chosen for the purpose should be optimal for palaeogeographic interpretation.

The present contribution is aimed at comparing the results of granulometric analysis by various computational methods, with the objective to find out which method is, or which methods are best applicable and give reliable results.

## 2. Methods of curve fitting

When one has a number of data points and tries to construct the function which most closely fits these data points, the procedure applied is called curve fitting. This can involve either interpolation, if an exact fit with the data is required, or smoothing, a procedure in which a 'smooth' function is constructed that approximately fits the data. Fitted curves can be used as an aid for data visualisation, to infer values of a function where no data are available. Extrapolation refers to the use of a fitted curve beyond the range of the data observed, and is subject to a greater degree of uncertainty since it may reflect the method used to construct the curve but also can reflect the observed data.

### 2.1. Interpolation

Two types of interpolation can be distinguished: (1) linear interpolation (commonly abbreviated as 'lerp') and (2) polynomial interpolation.

#### 2.1.1. Linear interpolation

This is a method of curve fitting using linear polynomials. It is the simple form of interpolation used by Folk & Ward (1957). If two known points are given by coordinates $(x_0, y_0)$ and $(x_1, y_1)$, the linear interpolant is the straight line between these points. For an $x$ in the $(x_0, x_1)$ interval, the $y$ value along the straight line is given from the equation

$$\frac{y - y_0}{x - x_0} = \frac{y_1 - y_0}{x_1 - x_0}$$

Solving this equation for $y$, which is the unknown value at $x$, gives:

$$y = y_0 + (x - x_0)\frac{y_1 - y_0}{x_1 - x_0} = \frac{(x - x_0)y_1 + (x_1 - x)y_0}{x_1 - x_0},$$

which is the formula for linear interpolation in the $(x_0, x_1)$ interval. Outside this interval, the formula is identical to linear extrapolation.

Linear interpolation on a set of data points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ is defined as the concatenation of linear interpolants between each pair of data points. This results in a continuous curve, with a discontinuous derivative.

Linear interpolation is fast and easy, but not very precise. The error is proportional to the square of the distance between the data points. The error in some other methods, including polynomial and spline interpolation, is proportional to higher powers of the distance between the data points, however, and is consequently larger. These other methods also produce smoother interpolants.

#### 2.1.2. Polynomial interpolation

This is the interpolation of a given data set by a polynomial: with some given points, a polynomial has to be found which goes exactly through them. It is a generalisation of linear interpolation. Note that the linear interpolant is a linear function. This interpolant is replaced by a polynomial of a higher degree.

Polynomials can be used to approximate more complex curves. Generally, if $n$ data points exist, there is exactly one polynomial of degree at most $n-1$ going through all these points (Fortuna et al., 2006). The interpolation error is proportional to the distance between the data points to the power $n$. Furthermore, the interpolant is a polynomial and thus infinitely differentiable. Polynomial interpolation thus solves all problems of linear interpolation. It has, however, also some disadvantages: calculating the interpolating polynomial is computationally more expensive than linear interpolation. It also may exhibit oscillatory artefacts, especially at the end points. These disadvantages can be avoided by using spline interpolation.

### 2.2. Extrapolation

As mentioned above, this method is similar to the process of interpolation (which constructs new points between known ones), but the results of extrapolations are often less meaningful, and they are subject to greater uncertainty. A reasonable choice for the extrapolation method can be made if one has *a priori* knowledge of the process that created the existing data points. Crucial is, for example, whether the data can be assumed to be continuous, smooth, possibly periodic, or something else.

#### 2.2.1. Linear extrapolation

Linear extrapolation means creating a tangent line at the end of the known data and extending it beyond that limit. This will provide good results only when used to extend the graph of an approximately linear function not too far beyond the known data. If the two data points nearest to the $x_0$ point to be extrapolated are $(x_{k-1}, y_{k-1})$ and $(x_k, y_k)$, linear extrapolation gives the function

$$y(x_0) = y_{k-1} + \frac{x_0 - x_{k-1}}{x_k - x_{k-1}}(y_k - y_{k-1})$$

(which is identical to lerp if $x_{k-1} < x_0 < x_k$). It is possible to include more than two points, and averaging the slope of the linear interpolant, by regression-like techniques, on the data points chosen to be included.

#### 2.2.2. Polynomial extrapolation

A polynomial extrapolation curve can be created through the entire known data or just near the end. The resulting curve can then be extended beyond the end of the known data. Polynomial extrapolation is typically made by means of Lagrange interpolation or using Newton's method of finite differences. The resulting polynomial may be used to extrapolate the data.

Typically, the quality of a particular method of extrapolation is limited by the assumptions concerning the function obtained by the method. If the method assumes smooth data, a non-smooth function will be poorly extrapolated. Even for proper assumptions, the extrapolation can diverge strongly from the function.

## 3. Application to grain-size analysis

When we apply interpolation or extrapolation to the grain-size analysis of unconsolidated sediments, the curve to be created depends on the co-ordinates $x$ and $y$, where:
$x$ = the accumulated mass percentage,
$y = \varphi = log_2(d)$, where d = the grain diameter in mm (continuous function).

In order to get fractions using the Folk & Ward (1957) method, $\varphi$ was calculated for $x$ = 5, 16, 25, 50, 75, 84, and 95.

### 3.1. Interpolation

In the case of linear interpolation, the graph will be a broken line. In the case of polynomial interpolation with three points, the curve will be smooth (with a continuous derivative), similar to the one drawn by hand with plotting tools.

It is possible to conduct the interpolation with polynomials of higher degrees. However, carrying out trial runs of interpolation with polynomial of the third degree (4 points), the accuracy of calculations increases minimally, particularly considering that in the original Folk & Ward method the function is hand-drawn, but the calculation will take considerably more time; the cost increases with increasing degree of the polynomial.

The accuracy is thus comparable with the results obtained with the spline method. This method, however, could not be applied because of the requirement of fitting the curve exactly to all points, rather than passing in their proximity.

**Table 1.** Statistical formulae used in the calculation of grain size parameters. Source: GRADISTAT version 8.0 (November 2010), developed by Simon J. Blott. $f$ is the frequency in %; $m$ is the mid-point of each class interval in metric ($m_m$) or phi ($m_\varphi$) units; $P_x$ and $\varphi_x$ are grain diameters, in metric or phi units, respectively, at the cumulative percentile value of $x$.

(a) Arithmetic method of moments

| mean | standard deviation | skewness | kurtosis |
|---|---|---|---|
| $\bar{x}_a = \frac{\Sigma f m_m}{100}$ | $\sigma_a = \sqrt{\frac{\Sigma f (m_m - \bar{x}_a)^2}{100}}$ | $Sk_a = \frac{\Sigma f (m_m - \bar{x}_a)^3}{100\sigma_a{}^3}$ | $K_a = \frac{\Sigma f (m_m - \bar{x}_a)^4}{100\sigma_a{}^4}$ |

(b) Geometric method of moments

| mean | standard deviation | skewness | kurtosis |
|---|---|---|---|
| $\bar{x}_g = \exp\frac{\Sigma f \ln m_m}{100}$ | $\sigma_g = \exp\sqrt{\frac{\Sigma f (\ln m_m - \ln \bar{x}_g)^2}{100}}$ | $Sk_g = \frac{\Sigma f (\ln m_m - \ln \bar{x}_g)^3}{100 \ln \sigma_g{}^3}$ | $K_g = \frac{\Sigma f (\ln m_m - \ln \bar{x}_g)^4}{100 \ln \sigma_g{}^4}$ |

(c) Logarithmic method of moments

| mean | standard deviation | skewness | kurtosis |
|---|---|---|---|
| $\bar{x}_\phi = \frac{\Sigma f m_\phi}{100}$ | $\sigma_\phi = \sqrt{\frac{\Sigma f (m_\phi - \bar{x}_\phi)^2}{100}}$ | $Sk_\phi = \frac{\Sigma f (m_\phi - \bar{x}_\phi)^3}{100\sigma_\phi{}^3}$ | $K_\phi = \frac{\Sigma f (m_\phi - \bar{x}_\phi)^4}{100\sigma_\phi{}^4}$ |

A comparison of the results of grain-size analysis according to Folk & Ward (1957) is presented in the present contribution for the average grain diameter, standard deviation, skewness, and kurtosis. The data concern Quaternary sediments of various origin, typical for the Piotrków Plateau and the Radomsko Hills. The analysis was carried out for 18 samples (out of 1200 collected specimens) (Fig. 1) chosen after preliminary granulometric analyses. Different textural characteristics of samples was the main selection criterion. They were also chosen to represent sediments of different age and origin. The primary results from the analysis were used to make computer-aided calculations of the parameters with the following methods:

(1) linear interpolation (according to Lagrange formula – two points);
(2) quadratic (2nd degree polynomial) interpolation (Lagrange formula – three points);
(3) the computer method with a specific program in DOS (1990), here referred to as 'traditional' in the diagrams;
(4) with the help of a licensed program for Windows: JoSek SED InFor (Torun, 2002);
(5) the GRADISTAT version 8.0, November 2010 (A grain size distribution and statistics package for the analysis of unconsolidated sediments by sieving or laser granulometer), developed by Dr. Simon J. Blott, downloaded from Kenneth Pye Associates Ltd – http://www.kpal.co.uk/gradistat.html (Table 1).

## 3.2. Extrapolation

All methods of curve extrapolation have been applied to each of the eighteen samples, so that the results could be compared. The following nine, most practicable, methods, including the JoSek SED program (Torun, 2002), have been analysed in detail.

(1) Curve without extrapolation. A so-called 'hanging curve' is obtained. For example, if a certain percentage of the sample is held by the first sieve (the lower part of the graph) but if no 100% is obtained with the last sieve (the upper part of the graph). This creates problems in obtaining extreme values of percentiles and further calculations. To solve the problem, extrapolation must be applied.

(2) Prolongation (l). The curve connects points representing cumulative values for limiting sieves (on which plotting the curve with the above method was finished), with points determined by empty sieves and axes extremes (0% and 100% for the arithmetic grid and 0.01 and 99.99% for the probability grid). This extrapolation was proposed by the authors of the JoSek SED program (Torun, 2002) and tested with positive results by Mycielska-Dowgiałło (2002).
(3) Prolongation (II). The intermediate method between Prolongation (I) and Folk &Ward (see below). The user defines the inclination angle of the complementary line segment himself in accordance with the graph scale.
(4) Graph after Folk & Ward (1957). This involves extending the last segment of the cumulative curve until 0% (100%) on the arithmetic grid or 1% (99%) on the probability grid is reached.
(5) Geometric extrapolation**,** which is a simple prolongation of the vector sum of increments.
(6) Weighted extrapolation (related to the sieve). This method indicates the proportion of the angle for each curve segment relative to the sieves for these segments.
(7) Weighted extrapolation (related to the mass). This method indicates the proportion of the angle for each curve segment relative to the increase of the mass on the sieve for these segments.
(8) The phi displacement. The user defines by how many full phi units (1 to 20) the graph should be shifted to reach the borderline 0% (100%) on the arithmetic grid or 1% (99%) on the probability grid.
(9) Inclination angle. The user determines the inclination angle for the last segment of the graph within the 0–90° interval.

The last step during the application of each of the above nine methods was identifying the best possible extrapolation from the curve, to obtain a final, most probable interpretation.

Samples contain, as a rule, particles of unspecified size per fraction, as is the case for the finest material retained in the pan after sieving. Ideally, the whole size range in a sample should be analysed, and this may require further analysis of the finest sediment remaining after sieving. According to Blott & Pye (2001), the larger the quantity of sediment remaining in the pan, the less accurate the calculation of grain-size parameters, with statistics calculated by the method of moments being the most susceptible. Errors in Folk & Ward (1957) parameters become significant only when more than 5% of the sample is undetermined. Samples containing more than 5% of sediment in the pan should therefore ideally be analysed using a different technique, such as sedimentation or laser granulometry. However, care must be taken when merging data obtained by different methods.

The next problem is the interpretation of the final segments (below $\varphi_5$ and above $\varphi_{95}$) of the grain curve when extrapolation is applied. These final curve segments are essential because information is least there. The interpretation regarding, for instance, the transportation process (B and C segments according to Mycielska-Dowgiałło, 1995) depends on the method of extrapolation and the course of the resulting graph.

### 3.3. The method of moments

A moment is, in the present context, loosely speaking, a quantitative measure of the shape of a set of points. Any distribution can be characterised by a number of features such as the mean, the variance, and the skewness, and the moments of a function describe the nature of its distribution. The first moment of the distribution of a random variable, X, is the expectation operator, i.e., the population mean. The second moment, the variance (the positive square root of which is the standard deviation), is widely used and is a measure of the ’width’ of a set of points.

Other moments describe other aspects of a distribution such as how the distribution is skewed from its mean, or peaked. The third moment is a measure of the lopsidedness of the distribution. The normalised third central moment is called the ‘skewness’. A distribution that is skewed to the left (which means that the tail of the distribution is heavier on the left) has a negative skewness. A distribution that is skewed to the right (the tail of the distribution is heavier on the right) has a positive skewness. The fourth moment indicates whether the distribution is tall and skinny or short and squat, if compared to the normal distribution of the same variance. Since it is the expectation operator of a fourth power. The fourth moment is, where defined, always non-negative; except for a point distribution, it is always positive. The kurtosis is defined to be the normalised fourth central moment. If a distribution has a peak at the mean and long tails, the fourth moment will be high and the kurtosis is positive (leptokurtic); in contrast, bounded distributions tend to have low kurtosis (platykurtic).

In statistics, the method of moments is a method that can be used to estimate population parameters (such as mean, variance, and median) by equating sample moments with unobservable population moments and then solving those equations for the quantities to be estimated.

Because of the widespread use of the Fritsch instrumentation in laboratories that perform grain-size analyses, and where only the method of moments is applied (the 'Fritsch Autosieve' evaluation computer program), the results of this method were also included into our comparison due to the possibilities offered by the Gradistat and Josek Sed programs. These two applications can calculate grain parameters according to both Folk & Ward (1957) and the moments method. We thus can show if the results obtained from the same initial data yield the same results when interpreted.

## 4. Geological and geographical setting of the sample area

The locations where samples were collected for the present study are situated in the borderland of the Middle-Polish Lowlands (Bełchatów Plateau, Piotrków Plateau) and the Polish Uplands (Radomsko Hills, Przedbórz-Małogoszcz Range), in the region of the Middle Polish ice sheet (Fig. 1). The Quaternary deposits are here 0–110 m thick (Wachecka-Kotkowska, 2004). In the southern, upland part, Mesozoic monadnocks occur. The Luciąża river valley begins in this area, and most samples for grain analysis were gathered here (Table 2). This valley is situated near to the maximum extent of the

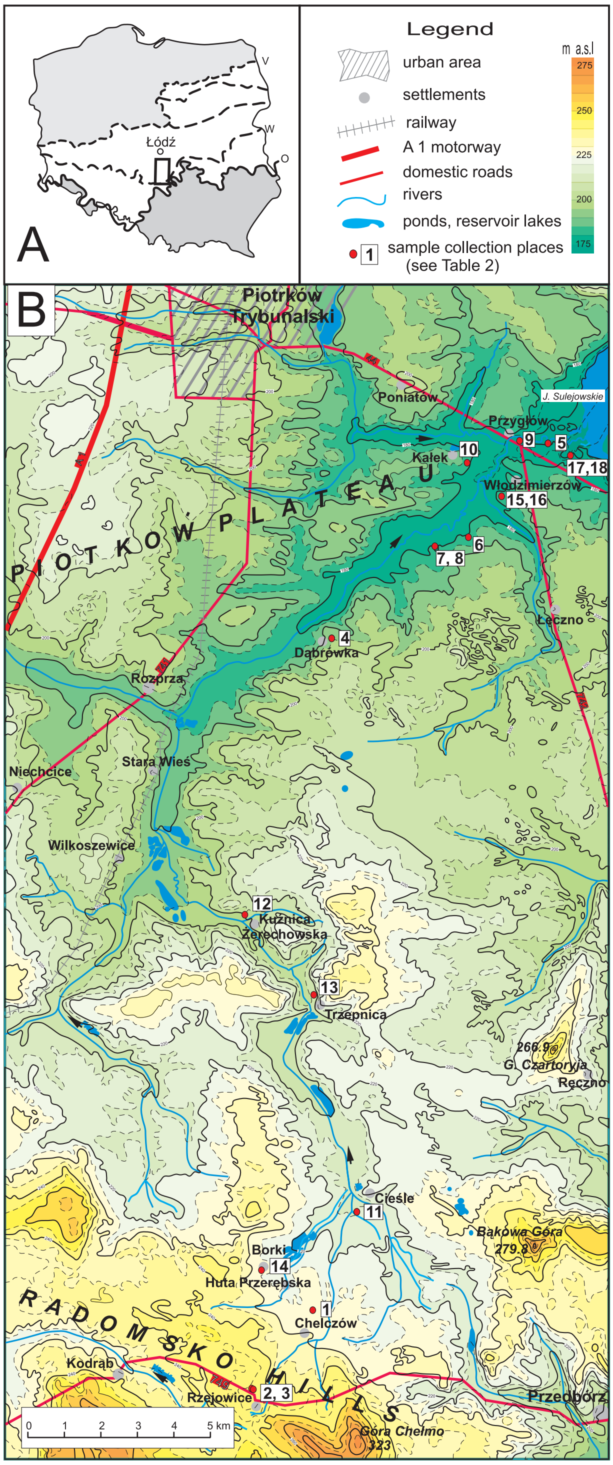

**Fig. 1.** Study area.
**A:** Location with extent of the Pleistocene glaciations;
**B:** Locations where samples were collected.

**Table 2.** Samples selected for the grain-size analyses.

| Stratigraphy (oxygene stages) | Sample number | Site name | Geomorphology | Lithology | Origin | Location (region/details) |
|---|---|---|---|---|---|---|
| Mesoholocene (MIS 1) | 18 | Murowaniec II | valley floor | gravelly sand | fluvial | Piotrków Plateau<br>1 km W from the mouth of the Luciąża river to the Pilica river |
| | 17 | Murowaniec I | | slighty gravelly sand | | |
| Eoholocene (MIS 1) | 16 | Włodzimierzów I | lowest river terrace | gravelly sand | | Piotrków Plateau<br>4 km NW of Sulejów,<br>lower part of the Luciąża river valley |
| | 15 | Włodzimierzów II | | gravelly sand | | |
| Late Vistulian (MIS 2) | 14 | Borki | parabolic dune | sand | aeolian | Radomsko Hills<br>between Przedbórz and Kodrąb |
| | 13 | Borowiec | slope of valley | slighty gravelly sand | slope wash | Radomsko Hills, 1 km N of Trzepnica,<br>middle part of the Luciąża river valley |
| | 12 | Kuźnica | | sand | | Piotrków Plateau, 3 km N of Trzepnica,<br>lower part of the Luciąża river valley |
| Plenivistulian (MIS 3-4) | 11 | Cieśle-Piła | | sand | | Radomsko Hills<br>1.5 km N of Przerąb,<br>higher part of the Luciąża river valley |
| | 10 | Kałek | high river terrace IInd | sand | periglacial fluvial | Piotrków Plateau, lower part of the Luciąża river valley<br>6 km NW of Sulejów |
| | 9 | Przygłów | fluvioperiglacial cover | slighty gravelly sand | | 3 km NW of Sulejów |
| | 8 | Kłudzice I | | slighty gravelly sand | | 7 km NW of Sulejów |
| | 7 | Kłudzice II | | sand | | |
| | 6 | Kłudzice Nowe | terrace of marginal valley | gravelly sand | proglacial/ marginal fluvial | 8 km NW of Sulejów |
| | 5 | Bunkier | | gravelly sand | | 3.5 km N of Sulejów |
| Early Saalian, Wartanian (MIS 6) | 4 | Dąbrówka | fluvioglacial plain in valley | slighty gravelly sand | | 5 km N of Rozprza |
| | 3 | Rzejowice II | outwash plain | gravelly sand | fluvioglacial | Radomsko Hills<br>between Przedbórz and Kodrąb |
| | 2 | Rzejowice I | | slighty gravelly sand | | |
| | 1 | Chelczów | kame | slighty gravelly muddy sand | | |

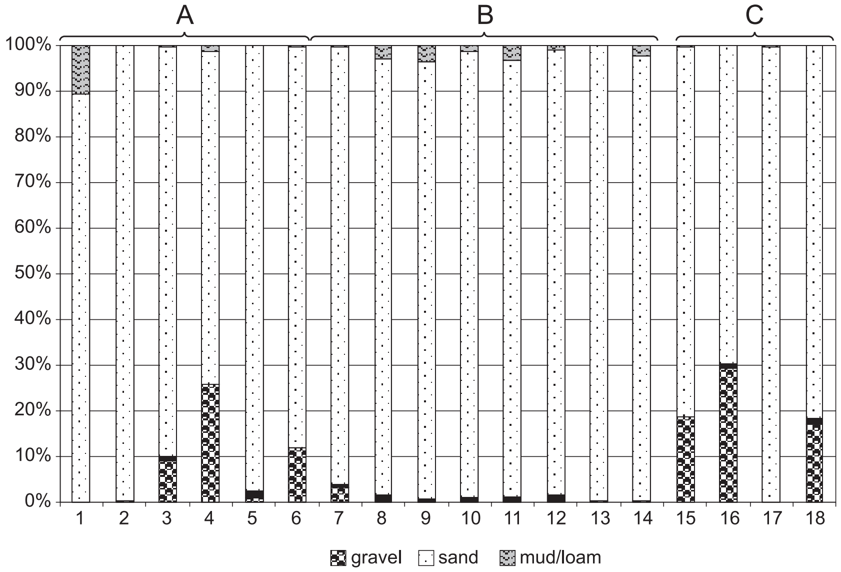

**Fig. 2.** Granulometry of the sediments under study. Sample locations are listed in Table 2.
**A:** Glacial (Wartanian, MIS 6) sediments; **B:** Periglacial (Vistulian, MIS 4–2) sediments; **C:** Holocene sediments.

Warta stage (MIS 6, Late Saalian) (Wachecka-Kotkowska & Górska-Zabielska, 2011). A terminal moraine and hummocky dead-ice moraine occur on the Bełchatów Plateau, in the western part of the investigated area; the major watershed between the Vistula and Odra rivers runs over their highest parts (Fig. 1). In the North, a flat glacial plain occurs which is built of tills, clays and sands (Wachecka-Kotkowska & Olszak, 2010). This plain contains sandy and gravelly surfaces of ice-marginal and proglacial valley tracts and includes outwash plains of the Piotrków Plateau, cut by small valleys and the Pilica and Luciąża river valleys. The area of the Pilica-Luciąża river system is covered with dunes.

## 5. Selection of samples

For the analysis of the granulometry of the Pleistocene and Holocene unconsolidated sediments under study, 18 samples have been chosen out of 1,200 collected specimens. Their characteristics are shown in Table 2 and Figure 2. The 18 samples were collected from all types of sediments of various age and origin, representing all morphogenetic conditions (glacial, periglacial and Holocene). The criterion of selection was their specific grain-size distribution. Samples were selected in the way to present normal, Gaussian distribution. For the full presentation of grain-size parameters some samples from the field were added, in which the upper (>95%) or lower (<5%) curve segments should be determined by extrapolation.

The analysed samples represent grain-size distributions that are unimodal (locations: Chelczów, Rzejowice I, Dąbrówka, Kłudzice I and II, Kałek, Kuźnica, Borki, Cieśle-Piła, Borowiec and Murowaniec I), bimodal (locations: Rzejowice II, Przygłów, Włodzimierzów I and II and Murowaniec II) or trimodal (locations: Bunkier and Kłudzice I) (Fig. 3). Most of the unimodal samples are moderately or moderately well sorted. Only one fluvial sample, Kałek (Table 2), is well sorted (over 71% fine sand). Two proglacial/ice-marginal meltwater-deposited samples (Chelczów and Kłudzice Nowe) are poorly sorted. The bimodal samples are generally poorly sorted, except that from Przygłów, where the deposit has a mixed periglacial fluvial/aeolian origin. The trimodal gravely sands, which are by definition poorly sorted, formed under varying sedimentary conditions.

## 6. Nature of the sampled sediments

The samples were taken from (1) glacial (Wartanian, MIS 6) sediments, (2) periglacial (Vistulian, MIS 4–2) sediments and (3) interglacial (Holocene) sediments.

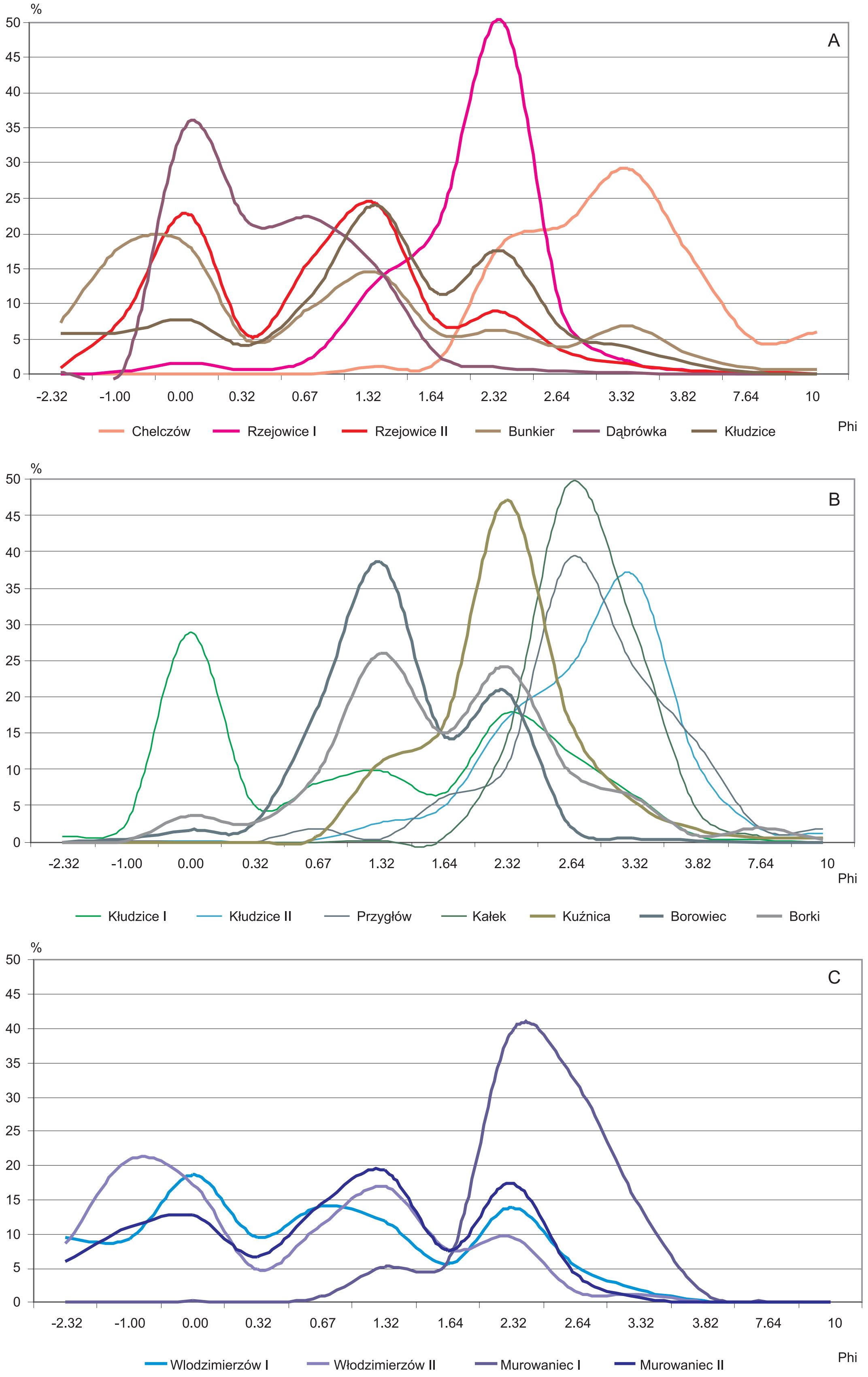

**Fig. 3.** Grain-size distribution. Sample numbers are listed in Table 2.
**A:** Glacial (Wartanian, MIS 6) sediments; **B:** Periglacial (Vistulian, MIS 4–2) sediments; **C:** Holocene sediments.

### 6.1. Wartanian sediments

Wartanian tills crop out only in the western and northern part of the study area. They form locally a massive cover, building a flat or undulating plain between the Bełchatów Plateau and the Piotrków Plateau. Elsewhere glaciofluvial sediments, represented morphologically by kames built from slightly gravelly muddy sand (e.g. Chelczów), gravelly and sandy outwash plains (e.g. Rzejowice) and fluvioglacial/marginal plains in valleys (e.g. Dąbrówka) occur in between the till plateaus.

### 6.2. Vistulian sediments

In the Vistulian periglacial climate, local depressions played the role of local denudation bases. The middle Plenivistulian alluvial sands and silts became about 20–22 ka ago mostly covered in the Luciąża and Pilica river valleys (the Kłudzice, Przygłów, Kałek and Cieśle-Piła sites) by periglacial fluvial sands and gravels of medium thickness. Material washed down from slopes became also reworked by the meltwater streams (e.g. Kuźnica and Borowiec). A phase of sand sedimentation in a more severe periglacial climate occurred in the late Plenivistulian. Deposition of slope and delta sediments then took place in the lower parts of the area.

During the Late Vistulian (MIS 2), scarce organic deposits accumulated in depressions. The Late Vistulian deposits are slightly coarser and less sorted than the Plenivistulian sediments. Afterwards dunes started to form all over the study area (Borki site); this went on until the Atlantic (Holocene). The coversand areas in the eastern part of the Luciąża river basin and the dunes in its valley are morphological effects of this process.

### 6.3. Holocene sediments

The Holocene sediments are most often sandy. At some locations, peatbogs developed on Early Holocene sands (Włodzimierzów site). Rejuvenated valleys became filled with gravels and sands. Organic and mixed organic/mineral sediments in the form of peats and peaty alluvial deposits alternating with sands originated in depressions with stagnant water (Wachecka-Kotkowska, 2004). Infillings of fluvial channels are most often represented by point bars consisting of cross-stratified sands (Murowaniec site).

## 7. Representation of the grain-size analyses

### 7.1. Interpolation

The results obtained for the various Folk &Ward (1957) indices and the method of moments are shown at Figure 4.

#### 7.1.1. Mean grain size

The main index – the mean grain size – has been calculated in various ways. This index cannot always be calculated using quadratic interpolation, because this kind of interpolation is based on one more point than lerp. The same holds for interpolation with higher polynomials. Oscillatory artifacts appear (sample 5, Table 2), as mentioned above. For the DOS and JoSek methods, the results are practically the same. The Gradistat outcome is very similar to that of DOS. The method of moments, from both JoSek and Gradistat, is calculated in different way. JoSek makes the results higher, except for the Chelczów sample, where Gradistat gives a higher value. The largest differences are found for the bi- and trimodal glacial sediments.

#### 7.1.2. Sorting

For well sorted deposits, the differences in the spread around the average are unimportant. Apart from the method of moments from Gradistat, the differences are even negligible. Linear and quadratic interpolations give the highest values for poorly sorted deposits, whereas the moments method gives the lowest values. The method of moments from Gradistat indicates too high degree of poor sorting and too low degree of good sorting (e.g. Chelczów, Bunkier and Włodzimierzów I; Table 2).

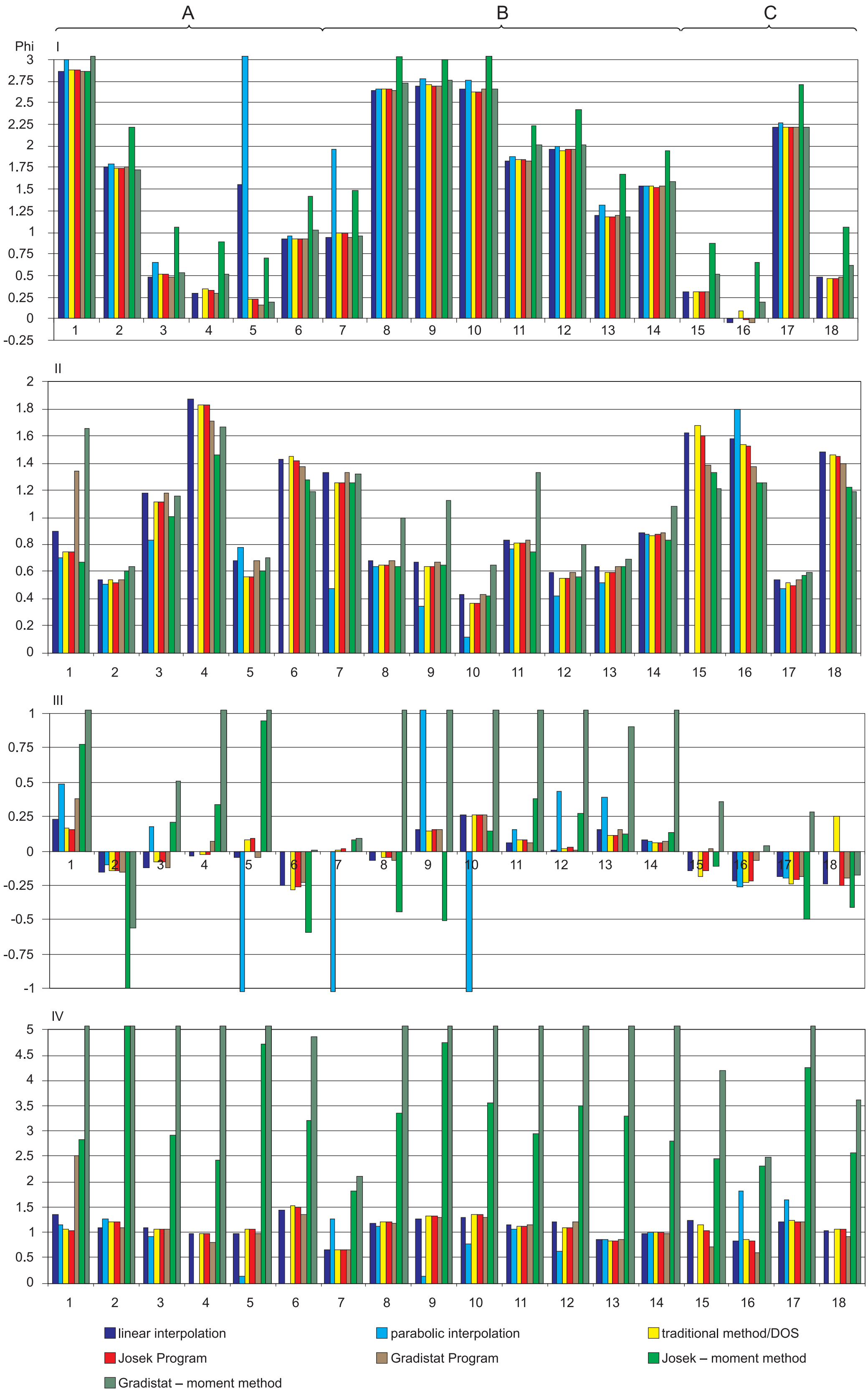

**Fig. 4.** Results of interpolation with various methods for 18 samples. Sample locations are listed in Table 2. **A:** Glacial (Wartanian, MIS 6) sediments; **B:** Periglacial (Vistulian, MIS 4–2) sediments; **C:** Holocene sediments. I = mean; II = sorting; III = skewness; IV = kurtosis.

#### 7.1.3. Skewness

Skewness is an index for which the various interpolation methods yield contradictory results for one sample (Fig. 4). For the same sample, a positive skewness can be indicated with one method and a negative skewness can be found with another method (e.g. Bunkier, Kałek and Murowaniec I, II). Moreover, some methods gives large values for the skewness while other methods indicate small values (e.g. Rzejowice, Dąbrówka, Bunkier, Kłudzice II, Przygłów, Kuźnica and Borki).The least reliable methods are the quadratic interpolation, Gradistat and JoSek methods of moments. Consequently, the method of moments is the least reliable as it gives the most diversified results. Only lerp, the traditional method (DOS), JoSek and Gradistat give sufficiently reliable results.

#### 7.1.4. Kurtosis

With regard to kurtosis, both methods of moments significantly increase this parameter for all samples. Apart from the three-point interpolation, that sometimes gives no results, the outcomes are similar. The linear interpolation, the traditional method (DOS), Josek and Gradistat are useful.

#### 7.1.5. Conclusion regarding interpolation

It must be concluded on the basis of the above data that, regarding interpolation, Folk &Ward (1957) indices are more accurate than moments.

### 7.2. Extrapolation

#### 7.2.1. Mean grain size

When calculating the average grain-size, all methods of extrapolation give the same results. So, parameters $\varphi_{16}$ and $\varphi_{84}$, far from both ends of distribution, can be determined for each sample. The mean grain size equals –0.041 phi at the Włodzimierzów II site, so it is not visible in Figure 5.

#### 7.2.2. Sorting

The same remarks can be made for sorting. There are cases where no extrapolation is possible (e.g. Kłudzice Nowe, Włodzimierzów I, II and Murowaniec II). Two groups of results, obtained with two groups of methods, can be distinguished: a first group consisting of prolongation (I), prolongation (II), and graphical after Folk & Ward (1957), and a second group consisting of geometric extrapolation, weighted extrapolation related to the sieve, weighted extrapolation related to the mass, the phi displacement, and the inclination angle.

#### 7.2.3. Skewness

Considering skewness, one must reject the ‘curve without extrapolation’ method, because it gives abnormal positive skewness. All values obtained are identical or very similar; at the Włodzimierzów II site (sample 15), the largest deviations occur.

#### 7.2.4. Kurtosis

The ‘curve without extrapolation’ method decreases the value of kurtosis parameter. The deviations obtained resemble those obtained for the skewness and concern the same samples, where $\varphi_5$ and/or $\varphi_{95}$ do not exist (open-end distributions). There are, again, two groups of methods, giving slightly different results.

#### 7.2.5. Conclusion regarding extrapolation

A good presentation of the results of grain-size analysis requires extrapolation, because the values of all indices, except the mean, are inaccurately assessed. Methods of prolongation, no matter which one, give the same results. These methods yield relatively high values. The other extrapolation methods give lower values, but the differences are minimal.

## 8. Conclusions

The comparative analysis of the results obtained by computer data processing indicates a certain regularity. In the first place, the results obtained with linear interpolation, the traditional DOS, Gradistat and JoSek Programs are, under certain conditions, comparable: mutual differences usually do not exceed 5%. In the second place, the results from quadratic (and higher–polynomial) interpolation are not accurate if the mass of sediment in the extreme

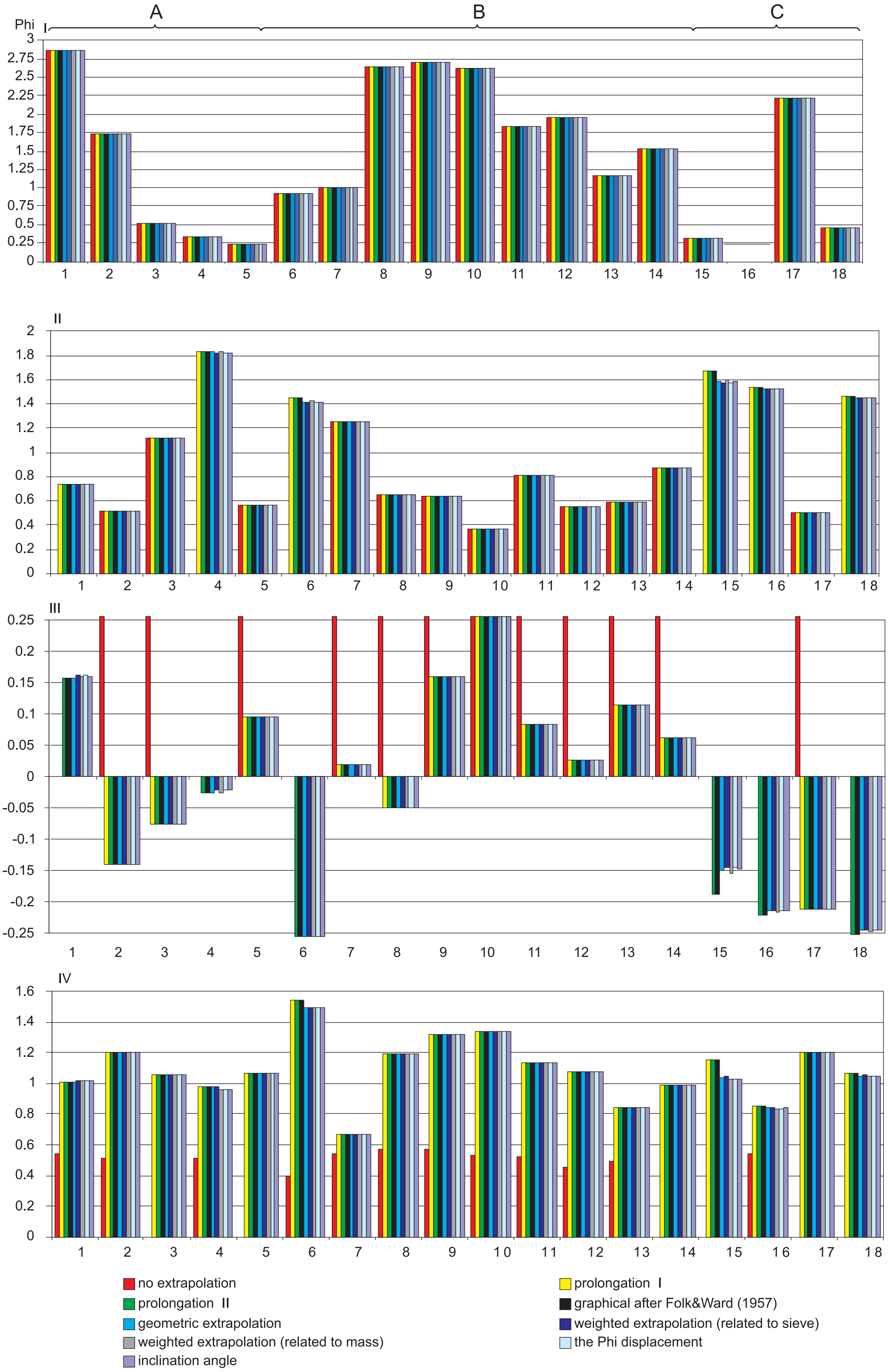

**Fig. 5.** Results of extrapolation with various methods for 18 samples. Sample numbers are listed in Table 2. **A:** Glacial (Wartanian, MIS 6) sediments; **B:** Periglacial (Vistulian, MIS 4–2) sediments; **C:** Holocene sediments. I = mean; II = sorting; III = skewness; IV = kurtosis.

grain-size fractions is more than 5% of the total sample weight. Such an interpolation gives a negative results, and therefore not every parameter (grain size, sorting, skewness, kurtosis) can be calculated with sufficient accuracy. In the third place, the method of moments significantly over- or underestimates the values of the various parameters because of the use of other mathematical formulae. Moreover, the results of the method of moments obtained from the Gradistad and JoSek Programs for the same indicators are different. Therefore, the results obtained with the Fritsch graphic-computational programs and the Folk & Ward (1957) method should be compared with caution. For the calculation of higher (third and fourth) moments, the lower (first and second) ones are employed. Any inaccuracies in the lower moments accumulate in the higher ones, which results in a multiplication effect.

The final results from nine methods of extrapolation show that there are no clear differences between them, except the ′curve without extrapolation′ method. Refraining from extrapolation prevents, in many cases, to determine the last or first percentiles, so that the skewness cannot be calculated. The simplest extrapolation methods, called (I), (II) and (FW) (in the programs Prolongation (I), Prolongation (II) and Prolongation of FW, respectively) should rather be used, providing that identical methods are consequently employed in all analyses. They simply extend the first and last segment of the curve, both on the arithmetic grid (0% and 100%) and the probability grid (0.01% and 99.99%), respectively (see Torun, 2002).

Distribution data are more uncertain for extrapolation than for interpolation but, the results of extrapolation, although more difficult to calculate, proved to be more accurate than the results of interpolation for all 18 samples. Furthermore, the results of interpolation greatly depend on the proper interpolation method. This dependence is less significant for extrapolation.

## References

Allen, G.P., Castaining P., Klingebiel, A., 1972. Distinction of elementary sand population in the Gironge estuary (France) by r-mode factor analysis of grain size data. *Sedimentology* 19, 21–35.

Asselman, N.E.M., 1999. Grain-size trends used to assess the effective discharge for floodplain sedimentation, river Waal, The Netherlands. *Journal of Sedimentary Research* 69, 51–61.

Blott, S.J., Pye, K., 2001. Gradistat: A grain size distribution and statistics package for the analysis of unconsolidated sediments. *Earth Surface Processes and Landforms* 26, 1237–1248.

Bravard, J.P., Peiry, J.L., 1999. The CM pattern as a tool for the classification of alluvial suites and floodplains along the river continuum. [In:] S.B. Marriott & J. Alexander (Eds): *Ancient floodplain evolution and techniques for analysis.* Geological Society, London, Special Publications 163, 259–268.

Brown, A.G., 1985. Traditional and multivariate techniques in the interpretation of floodplain sediment grain size variations. *Earth Surface Processes and Landforms* 10, 281–291.

Dade, W.B., 2000. Grain size, sediment transport and alluvial channel pattern. *Geomorphology* 35, 119–126.

Flemming, B.W., 2007. The influence of grain-size analysis methods and sediment mixing on curve shapes and textural parameters: Implications for sediment trend analysis. *Sedimentary Geology*, 202, 425–435.

Folk, R.L., 1966. A review of grain size parameters. *Sedimentology* 6, 73–93.

Folk, R.L., Ward, W.C., 1957. Brazos River bar: a study in the significance of grain size parameters. *Journal of Sedimentary Petrology* 27, 3–26.

Fortuna, Z., Macukow, B, Wąsowski, J., 2006. *Metody numeryczne* [Numerical methods], Wydawnictwo Naukowo-Techniczne, Warszawa, 384 pp.

Grzegorczyk, M., 1970. Metody przedstawiania uziarnienia osadów [Methods of presentation of grain size distribution of sediments] *Prace Komisji Geograficzno-Geologicznej* 10 (2), Poznań, 38–40.

McLaren, P., 1981. An interpretation of trends in grain size measures. *Journal of Sedimentary Petrology* 51, 611–624.

Merta, T., 1991. A new, universal method of thin-section to sieve transformation of granulometric data. *Acta Geologia Polonica* 41, 117–148.

Mycielska-Dowgiałło, E., 1995. Wybrane cechy teksturalne osadów i ich wartość interpretacyjna. [Some textural features of deposits and their interpretation value] [In:] E. Mycielska-Dowgiałło & L. Rutkowski (Eds): *Badania osadów czwartorzędowych.* [Investigation of Quaternary sediments]. Warsaw University, Warszawa, 29–57.

Mycielska-Dowgiałło, E., 2007. Metody badań cech teksturalnych osadów klastycznych i wartość interpretacyjna wyników [Research methods for textural features of clastic deposits and their significance for interpretation]. [In:] E. Mycielska-Dowgiałło & J. Rutkowski (Eds): *Badania cech teksturalnych osadów czwartorzędowych i wybrane metody oznaczania ich wieku*. [Investigation of textural features of Quaternary sediments and some methods for determining their age]. SWPR Press, Warszawa, 95–180.

Passega, R., 1964. Grain size representation by CM patterns as a geological tool. *Journal of Sedimentary Petrology* 34, 830–847.

Racinowski, R., Szczypek, T. & Wach, T., 2001. Prezentacja i interpretacja wyników badań uziarnienia. [Presentation and interpretation of the results of grain-size analysis]. Silesian University, Katowice, 146 pp.

Szmańda, J.B., 2007. Porównanie interpretacji warunków transportu osadów na diagramie C/M i analizy krzywych kumulacyjnych aluwiów pozakorytowych Wisły w Toruniu. [Grain-transport conditions – comparison of interpretations on the basis of a C/M diagram and analysis of a cumulative curve diagram. Case study – overbank alluvial deposits of the Vistula River, Toruń, Poland]. [In:] E. Smolska E. & D. Giriat (Eds): Rekonstrukcja dynamiki procesów geomorfologicznych – formy rzeźby i osady [Reconstruction of geomorphological process dynamics – landscape forms and sediments]. Warsaw University & Polish Academy of Sciences, Warsaw, 367–376.

Szmańda, J.B., 2010. Litodynamiczna interpretacja środowiska fluwialnego na podstawie wskaźników uziarnienia – przegląd wybranych metod [Lithodynamic interpretation of the fluvial environment based on grain-size distribution parameters – a review of some methods]. *Landform Analysis*. 12, 109–125.

Torun, 2002. *JoSek Sed – program obliczeń i analiz. Wprowadzenie do programu* [JoSek Sed – a program for calculations and analysis. Introduction to the program]. InFor, Toruń, 5–7.

Visher, G.S., 1969. Grain size distribution and depositional processes. *Journal of Sedimentary Petrology* 39, 1074–1106.

Wachecka-Kotkowska, L., 2004. Ewolucja doliny Luciąży – uwarunkowania klimatyczne a lokalne [Evolution of the Luciąża river valley – local and climatic conditions]. *Acta Geographica Lodziensia* 86, 161 pp.

Wachecka-Kotkowska, L., Olszak, I., 2009. Nowe wyniki datowań TL a wiek wysoczyzny glacjalnej SE części regionu łódzkiego. [New results of TL dating and the age of the glacial plateau in the SE part of the Łódź Region]. [In:] A. Kostrzewski & R. Paluszkiewicz (Eds): Geneza, litologia i stratygrafia utworów czwartorzędowych. [Origin, lithology and stratigraphy of Quaternary sediments]. *Geografia* 88, 577–588.

Wachecka-Kotkowska, L., Górska-Zabielska, M., 2011. Extent of the Middle Polish Glaciations (Saalian, MIS 6) in central Poland on the basis of petrographic analysis. Abstracts IAG/AIG Regional Conference 'Geomorphology for Human Adaptation to Changing Tropical Environments' (Addis Ababa, 2011), 161.

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