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REDUCTION OF BOUNDARY EFFECT DURING STRUCTURAL DAMAGE IDENTIFICATION USING WAVELET TRANSFORM

Abstract: The wavelet transform seems to be effective tool for signal processing of modal data in the problems of structural damage detection and identification. However, the application of wavelet transform is connected with the occurrence of boundary effect, which may cause wrong conclusion about damage localization. In the following paper the problem of boundary effect was discussed and the methods of its reduction were presented. A comparative study of different approaches of the boundary effect reduction was performed and its results were analyzed.

1. Introduction

The wavelet transform (WT) found wide application in engineering problems due to its outstanding properties in comparison with the Fourier transform. The main advantages of WT are a possibility of multiresolution analysis and a possibility of application of different bases (wavelets) depending on the problem and analyzed data type. A wide popularity of WT is observed in signal processing problems, particularly in de-noising and compression, feature extraction and singularity detection, etc., due to its high sensitivity for singularities.

Different types of WT were successfully adapted for problems of structural damage detection, localization and identification [1-3]. WT allows for detection of even small damages, therefore it is a powerful tool for early damage detection in structures of aircraft, civil engineering, mechanical engineering etc. Moreover, the damage detection methods using WT have two main advantages: they ensure non-contact non-destructive inspection and there is not necessary to collect the information about a healthy structure for the comparison with a damaged one. The most of methods used WT are based on vibration testing. First works in this area used the natural frequencies of a structure as an assessment parameter of a damage presence [4], however such a methods may fail in practical applications due to the manufacturing non-repeatability of the structures, especially, when the tested structure was made of polymer composites. The changes in natural frequencies may cause by phenomena other than damages and the information about the natural frequencies of a healthy structure is required. Another approach is based on the analysis of modal shapes of a tested structure. Such an approach allows for detection, precise localization and identification of a damage based on displacement modal shapes. The essence of an approach is an application of WT for the vibration signals in order to identify even subtle local irregularities. Many authors used this approach to develop high-accurate methods for structural damage assessment [1,2,5,6]. It was suitable to generalize WT to two-dimensional (2D), which creates new possibilities of analysis of spatial signals and could be applied for damage identification in 2D structures. Such a generalization is possible when the scaling and wavelet functions are separable. In such problems both continuous wavelet transform (CWT) [6] and discrete wavelet transform (DWT) [3] were used.

One of the important problems of wavelet analysis is the boundary effect occurrence, which is affected by the mathematical specificity of a transform, which will be discussed in detail later. The boundary effect causes increase of details coefficients during the application of DWT and could be a reason of wrong interpretation of results about the damage presence and its location. The aim of the presented study is to analyse possible methods of a reduction or elimination of the boundary effect. A comparative study based on the numerical data was performed in order to evaluate the effectiveness of methods of boundary effect reduction.

2. **Damage identification using discrete wavelet transform**

The damage identification algorithm used in this study is based on DWT, which could be formulated as follows. Consider a function in the 2D space of square-integrable functions $f(x,y) \in L^2(\mathrm{R}^2)$. The multilevel approximation (introduced by Mallat [7]) of the space $L^2(\mathrm{R}^2)$ is a sequence of closed functional spaces $V_j \subset L^2(\mathrm{R}^2)$:

$$\ldots V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \ldots . \tag{1}$$

with properties

$$\bigcup_{j \in \mathrm{Z}} V_j = L^2(\mathrm{R}^2), \quad \bigcap_{j \in \mathrm{Z}} V_j = \{0\} . \tag{2}$$

In the classic one-dimensional (1D) case the functions in the form of $\phi(\bullet - n),\ n \in \mathrm{Z}$, constitute the orthonormal bases in V_0. Considering separability property of applied wavelet bases the orthonormal base in V_0 could be obtained by the convolution in the form:

$$\phi_{0;nm}(x,y) = \phi(x-n)\phi(y-m) . \tag{3}$$

Similarly, the functions which constitutes the orthonormal base in V_j take a form:

$$\phi_{j;nm}(x,y) = \phi_{jn}(x)\phi_{jm}(y) = 2^{-j}\phi\left(2^{-j}x - n, 2^{-j}y - m\right),\ n,m \in \mathrm{Z} . \tag{4}$$

Following this, the scaling function and three wavelets (horizontal, vertical and diagonal) could be presented as follows:

$$\phi(x,y)=\phi(x)\phi(y),\ \psi^H(x,y)=\phi(x)\psi(y),\ \psi^V(x,y)=\psi(x)\phi(y),\ \psi^D(x,y)=\psi(x)\psi(y) \quad . (5)$$

Such an algorithm was used for analysis of spatial displacement modal shape data. Another crucial step of an algorithm is the choice of an appropriate wavelet function suitable for a given problem. Previous studies in this area show, that the best separable compactly supported wavelet function is 2D sixth-order B-spline wavelet [3]. An exemplary results of damage identification in a square layered composite plate clamped on the boundaries were presented in Fig.1, where (a) shows the approximation coefficients set, while (b)-(d) represent details coefficients sets. The 2D signal was generated from the numerical model for the second bending mode of vibration. The rectangular damage was modelled in the bottom-left corner.

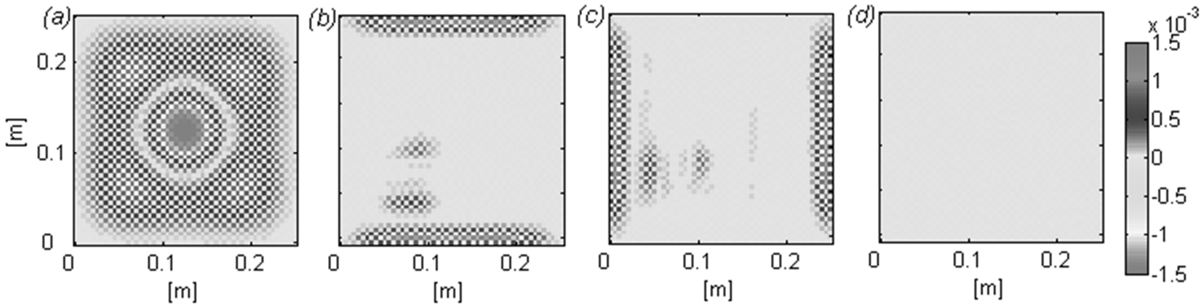

Fig.1. Exemplary results of damage identification using discrete wavelet transform

Analyzing the results presented in Fig.1., it could be clearly detected the damage position on the horizontal coefficients set (Fig.1(b)) and on the vertical coefficients set (Fig.1(c)). The results of DWT were presented in the raw form, i.e. without any reduction of the boundary effect, therefore, the boundary effect is visible on Fig.1(b) and Fig.1(c). The values of details coefficients on the boundaries were even higher than the values of coefficients in the damage location.

3. Boundary effect and methods of its reduction

The problem of a boundary effect reduction is one of the crucial tasks of the damage evaluation. As it could be noticed basing on the presented example (see Fig.1), the boundary effect cause significant increase of the absolute values of details coefficients, which may cause wrong interpretation of a resulted pattern. The occurrence of the boundary effect is resulted by the finite length of the investigated signal and thus, zero-values of a signal outside of the region of interest. When the base function is shifted from zero to non-zero values obtained coefficients values became large. It caused a situation, when the boundaries of these patterns should be excluded from consideration, which results in loss of an information.

There are several parameters, which have an influence on the boundary effect occurrence and its intensification. The length of an effective support of the wavelet used in the analysis plays important role in the accuracy of damage identification. Following to the previous results obtained in [3,8] the most effective wavelets should have possibly short effective support and possibly large number of vanishing moments. The same rule should be applied for the boundary effect analysis since a fact that wavelets with long effective support produce quantitatively more details coefficients with the boundary effect. In other words, when the effective support is short, than the wavelet is translated outside the real signal to the lower distance and produces less and lower-value coefficients with the boundary effect.

For the purpose of reduce or eliminate the boundary effect various methods were proposed. The authors of [2] proposed to neglect the regions, where the boundary effect occurred. However, in the case of a small grid of measurement points neglecting of the regions where the boundary effect occurs may reduce the useful information much.

The most simple and commonly used methods of reducing the boundary effect are zero-padding, constant-padding, reflection, antisymmetric reflection and periodization [9,10]. All of these methods are based on simple operations with the real signal. Zero-padding and constant-padding methods are based on the extension of real signal outside the original support with zero-values and values on the boundary respectively, while reflection and antisymmetric reflection simply reflects the signal outside the original support with values on the boundaries of a signal. The reflection (or symmetrization) method was recommended by the authors of [11] as a suitable method for reducing the boundary effect in damage identification problems. The periodization procedure produces a periodic extension of a signal outside the support. These methods are available in many wavelet-based professional software for signal processing, e.g. in Matlab® or PyWavelets. However, applying of zero-padding method causes discontinuities on the boundary of a signal, while other above-mentioned methods cause discontinuities in derivatives. Nevertheless, in many practical problems they could be used as well.

The second group of methods uses extrapolation algorithms at the boundaries of neighbour measurement points. The authors of [6] proposed a technique based on cubic spline extrapolation of four neighbour points. Such an approach gives continuous extension over the original support of a signal, also for its derivatives.

The last group of methods is based on specific techniques which modify the algorithm of WT in order to substitute wavelets on the boundaries. Some works were presented such wavelets and adapted lifting techniques (e.g. [12,13]), which are called edge wavelets or interval wavelets. One of the families of such wavelets is called Sturm-Liouville wavelets following to the name of Sturm-Liouville boundary problems [9]. The authors of [10] stated that such a method gives superior results in comparison with previous ones; however the method needs replacement of the wavelets translated to the border by the edge wavelets, which results in more complicated and slower algorithm. Due to the practical usability of such an algorithm it was not considered in the following study.

In order to analyze presented techniques for the boundary effect reduction and identify the most suitable method for the investigated problem an exemplary signal presented in Fig.1 was taken into consideration. The techniques described before were implemented in Matlab® and the results for a signal after reduction of the boundary effect were presented in Fig.2. For the simplicity and clarity of the analysis of resulted details coefficients patterns (vertical, horizontal and diagonal), they were added following the previously used technique presented in [3].

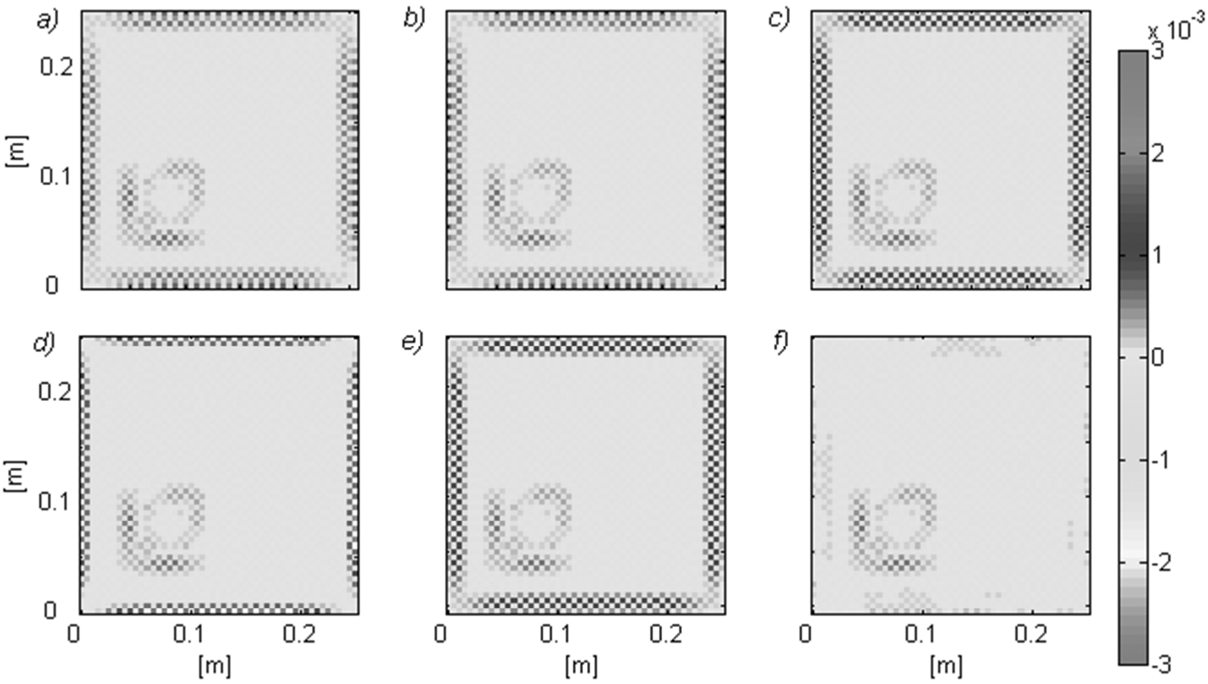

Fig.2. Results of reduction of the boundary effect using: a) zero-padding, b) constant-padding, c) reflection, d) antisymmetric reflection, e) periodization, f) cubic spline extrapolation methods.

Results of the analysis show that the signal extension algorithms, presented before as a first group of the boundary effect reduction methods, presented merely effectiveness, which was resulted by the occurrence of discontinuities of the extended signal and its derivatives. The last method based on cubic spline extrapolation gives excellent results: the boundary effect was avoided almost completely. For the additional analysis the maximal values of sums magnitudes of details coefficients in the regions of the boundary effect occurrence were tabulated in Tab.1.

Tab. 1. Comparison of sums of details coefficients in the regions of the boundary effect occurrence

Zero-padding	Constant-padding	Reflection	Antisymmetric reflection	Periodization	Cubic spline extrapolation
$7.8918 \cdot 10^{-4}$	$7.8918 \cdot 10^{-4}$	$1.5118 \cdot 10^{-3}$	$2.0656 \cdot 10^{-3}$	$1.5573 \cdot 10^{-3}$	$2.8563 \cdot 10^{-4}$

As it could be noticed from Tab.1, the best result was achieved using cubic spline extrapolation. The maximal value for this method in the regions of the boundary effect occurrence is almost lower three times lower than the summarized details coefficients in the region of damage occurrence. Therefore, if the damage is located in the region of the boundary effect occurrence it could be identified as well.

4. **Conclusion**

In the presented study the methods of reduction of boundary effect after an application of 2D DWT were discussed. Some of the most common methods used in signal and image processing were compared in order to evaluate the most effective method for analysis of damage identification problems. Basing on presented results the most effective method for the reduction of boundary effect is a cubic spline extrapolation. The study was executed on the simulation data and need to be experimentally verified, which is planned in the further research.

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