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A PROPOSITION OF A TORSIONAL-BENDING VIBRATIONS MODELLING OF COMBUSTION ENGINES

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Abstract

The article presents the problem of modelling coupled bending-torsional vibrations in crank systems. Because during the design of modern drive systems a growing number of phenomena are taken into account, model description of such vibrations has a practical meaning. Commonly used models of dynamics of systems assume the independence of torsional and bending vibrations, which leads to simultaneous analysis of transverse and angular vibrations. Further analysis is carried out with the use of superposition principle. Such an approach is justified in the case of quite rigid drive shafts, where vibrations are relatively small. Current trends in the design of reducing weight, reduction of toxic emissions and reducing fuel consumption, lead to the situation where shafts in crank systems become less stiff. Therefore, phenomena neglected earlier may have significant meaning.

Analysis of couplings of transverse and torsional vibrations is so important that the occurrence of these phenomena usually leads to new critical states, which may be especially dangerous for engine operation. Considerations on the reasons of the occurrence and kinds of vibration couplings were presented in the introduction of the article. Further part of the article proposes the linear-bending model of the crankshaft, where transverse and angular displacements are dependent. It was tantamount to the assumption of linear relation between the vector of generalized co-ordinates and generalized forces occurring in the system. The next chapter presents the system of equations describing the dynamics of the crankshaft together with a discussion of the co-ordinate system used in the considerations. In addition, there were presented the results of numerical simulations in frequency domain confirming the conclusions taken from the analysis. The whole paper is concluded with synthetic conclusions on the formulated system of equations, simulations and the influence of the coupling on the dynamics of the whole crank system.

Keywords: *rotational system, the system of differential equation, crankshaft systems, numerical solutions*

1. Introduction

The dynamics of the crankshaft is a very important technical problem, because it has a huge impact on the safety of combustion engines operation. Basic parameters of the engine and its work are directly connected with this system. In case of steady rotational speed of the crankshaft, the process of defining forces and displacements occurring in the crankshaft is a simple task. In case of non-steady motion it is a problem much more complicated which requires a lot of research [3, 7, 13, 14, 20].

The crank system has a very complex geometry. This affects both the crankshaft (rotor) and other elements of crank mechanism. As a result, it leads to spatial extortions and vibrational response of the engine. Coupling of particular degrees of freedom makes the analysis difficult. In practice, in the analysis of car engines, the most frequently used simplification is connected with rejecting any dependencies connecting bending and torsional vibrations. Such an approach is typical in initial design calculations. In case of issues related to operation, this simplification is too big. There is a proof of that because significant deviations from theoretical models are observed in vibrations measurements.

Because of coupling of bending and torsional vibrations, there may appear new critical speeds. In addition, torsional vibrations significantly affect transverse displacements (bending). A direct consequence of this motion is vibrations of the whole body. The shift and modulation of eigen frequency in relation to non-coupled system are the result of this. The reason for this is numerous nonlinear or parametric effects occurring in a discussed object.

Knowledge of a structure of not coupled and coupled vibrations allows to determine the differences between them and to study the relationship between the degrees of freedom. Therefore, the authors suggest analysing torsional vibrations of the engine based on spectra of transverse displacements of the body. This problem is important because the measurement of angular vibrations of the crankshaft of the real combustion engine is much more difficult than the measurement of transverse vibrations [4-6, 17, 18, 23].

2. The dynamic model of piston engine with an elastic crankshaft

Due to a complicated geometric and material structure, it is convenient to replace the continuous mass system, which is a crankshaft with a discrete model. In such cases, the masses are usually reduced to selected constructional nodes, whereas the remaining part of the object is treated as a massless deformable structure.

Model of the system of material points is a significant simplification of the continuous system. The difference is that nonfinite (but countable) set of eigenvalues is „replaced” with finite number of eigenfrequencies of a discrete system. It is obvious that it is not possible to replace the continuous system with a model with material points. However, it is necessary to make reduction in a selected frequency band, e.g. in the range of low frequencies. Such a simplification makes it easier to make calculations without introducing errors that are more serious.

Very rigid crankshafts are used in constructions of real combustion engines. Mainly due to the precision required from crank mechanisms. Even small changes in the angular position of the crank may affect the process of combustion in a given system, which directly influences its dynamics. In addition, in vibrating systems there is a risk of resonance with a basic harmonic of extortion, which comes from gas forces. In this case, oversizing of the crankshaft allows moving the natural frequency of vibrations into the area of higher elements of drive moment [1, 2, 8, 9, 11].

Beam model of a single crank of the crankshaft of the piston engine was shown schematically in Fig. 1.

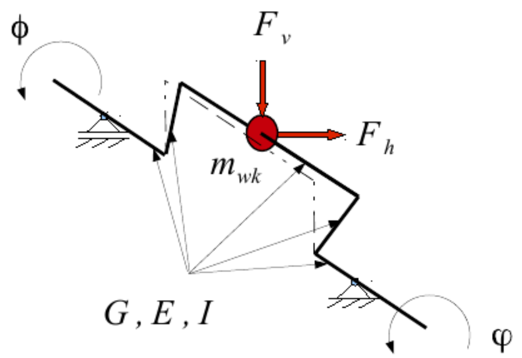

Fig. 1. The model of the crankshaft of the one piston

In real crank systems, displacements occurring in the shafts are very small. Therefore, it is reasonable to assume small deformations. For most constructional materials used in automotive industry and methods of forming, stresses are proportional to displacements. This allows using linear-elastic model to describe deformations of the crankshaft [10, 12, 15, 19]. Thus, the relationship between the generalized forces and the generalized coordinates is as follows:

$$F = K \cdot u\,, \tag{1}$$

where:
K – stiffness matrix,
F – generalized force vector,
u – generalized displacement vector.

Vector of generalized forces and displacements are as follows:

$$F = \begin{bmatrix} F_h \\ F_v \\ M_\phi \\ M_\varphi \end{bmatrix},\ u = \begin{bmatrix} u_n \\ u_\tau \\ \phi \\ \varphi \end{bmatrix}, \tag{2}$$

where:
ϕ – rotation angle of the left end of crank,
φ – rotation angle of the right end of crank,
u_n – radial deformation of the crank,
u_τ – tangential deformation of the crank.

For generalized coordinates, stiffness matrix of the proposed model of the crank has the following form:

$$K = \begin{bmatrix} k_{nn} & 0 & 0 & 0 \\ 0 & k_{\tau\tau} & -k_{\tau\theta} & 0 \\ 0 & -k_{\tau\theta} & k_{\theta\theta} & -k_{\theta\theta} \\ 0 & 0 & -k_{\theta\theta} & k_{\theta\theta} \end{bmatrix}. \tag{3}$$

Analysed crankshaft has two cranks. The scheme of analysed object will look like in Fig. 2.

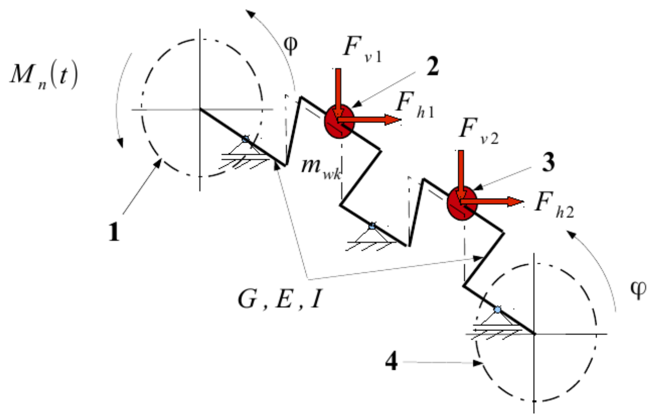

Fig. 2. The model of the crankshaft with two pistons

This is a serial connection of two cranks. With the use of conditions of forces and internal moments, it is possible to determine the coefficients of a global stiffness matrix. The matrix is given with the following formula:

$$K = \begin{bmatrix} k_{n1n1} & -k_{n1n2} & 0 & 0 & 0 \\ -k_{n1n2} & k_{n2n2} & 0 & 0 & 0 \\ 0 & 0 & k_{\tau1\tau1} & -k_{\tau1\tau2} & -k_{\tau1\theta} \\ 0 & 0 & -k_{\tau1\tau2} & k_{\tau2\tau2} & -k_{\tau2\theta} \\ 0 & 0 & -k_{\tau1\theta} & -k_{\tau2\theta} & k_{\theta\theta} \end{bmatrix}. \tag{4}$$

Figure 3 shows the displacement of the crank described in the moving coordinate system.

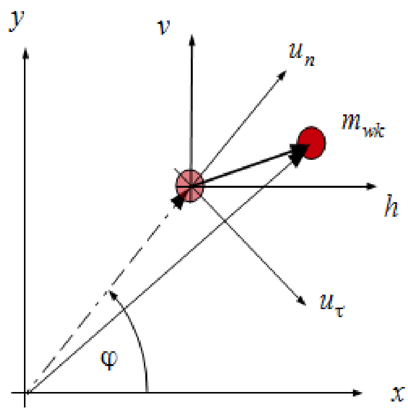

Fig. 3. The model of the single crank of the considered crankshaft

It is possible to find the equation of motion for the system presented in scheme 2 with the use of any formalism of analytical mechanics. The Lagrange equations of the second kind are used because the model is linear and only holonomic bonds exist in the system. On this basis, the following dynamic model is determined:

$$m_w \ddot{u}_{n1} - m_w R \dot{\varphi}_L^2 + k_{n1n1} u_{n1} - k_{n1n2} u_{n2} = P_{r1}, \tag{5}$$

$$m_w \ddot{u}_{n2} - m_w R \dot{\varphi}_L^2 - k_{n1n2} u_{n1} + k_{n2n2} u_{n2} = P_{r2}, \tag{6}$$

$$I_{kl} \ddot{\varphi}_L + m_{wk} R^2 \ddot{\varphi}_L + m_w R \ddot{u}_\tau - k_{\tau 1\theta} u_{\tau 1} - k_{\tau 2\theta} u_{\tau 2} + k_{\theta\theta}(\varphi_L - \varphi_P) = M_0, \tag{7}$$

$$m_w \ddot{u}_{\tau 1} + m_w R \ddot{\varphi}_L + k_{\tau 1\tau 1} u_{\tau 1} - k_{\tau 1\tau 2} u_{\tau 2} - k_{\tau 1\theta}(\varphi_P - \varphi_L) = P_{\tau 1}, \tag{8}$$

$$m_w \ddot{u}_{\tau 2} + m_w R \ddot{\varphi}_L - k_{\tau 1\tau 2} u_{\tau 1} + k_{\tau 2\tau 2} u_{\tau 2} - k_{\tau 1\theta}(\varphi_P - \varphi_L) = P_{\tau 2}, \tag{9}$$

$$I_{kP} \ddot{\varphi}_P - k_{\tau 1\theta} u_{\tau 1} - k_{\tau 2\theta} u_{\tau 2} + k_{\theta\theta}(\varphi_P - \varphi_L) = 0. \tag{10}$$

3. Simulation analysis of transverse vibrations of the crankshaft

The series of numerical simulations was carried out for a proposed system of equations. At the beginning there were made the analyses of transverse vibrations of the crankshaft without the coupling of bending and torsional vibrations. Angular and transverse vibrations of the crank of the crankshaft are presented in plots 4-8.

If the coupling is taken into account, the spectrum structure of transverse vibrations is much more complicated. The spectrum of displacements of transverse vibrations in a moving coordinate system is shown in plots 7 and 8. As it was assumed, additional frequencies connected with torsional vibrations may be observed.

4. Conclusion

The phenomenon of coupling of bending and torsional vibrations in vibrating systems is usually omitted in model calculations. Such calculations are justified at the design stage, when it is necessary to pre-define the basic dimensions of the system for further designing process. However, in reality, the dynamics of the actual motion of the crank system is much more complex. Therefore, it is necessary to use the model, which includes more phenomena and allows for more detailed analysis of vibrations; occurring in combustion engines. For this purpose, the authors used linear-elastic description of the crankshaft for a description [16, 21, 22, 24, 25].

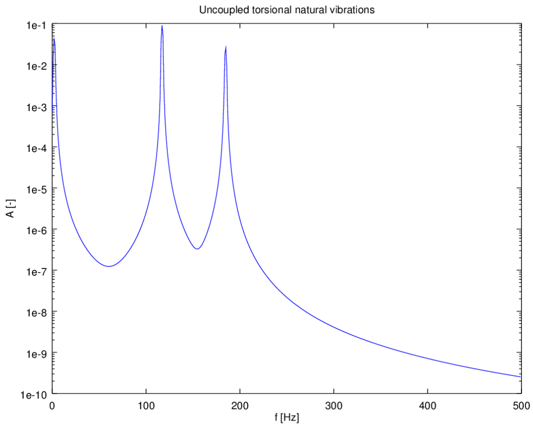

Fig. 4. The spectrum of natural displacements of torsional vibrations of the model of the crankshaft without the coupling

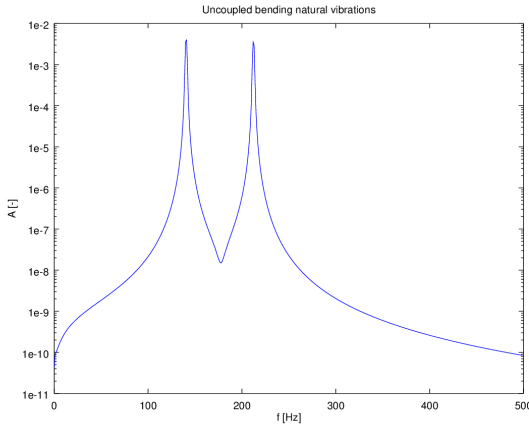

Fig. 5. The spectrum of natural displacements of bending vibrations of the model of the crankshaft without the coupling

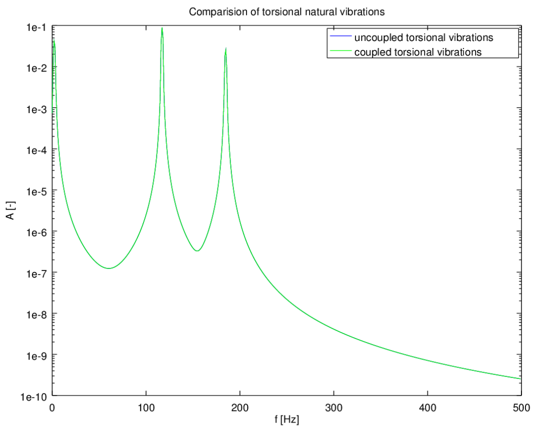

Fig. 6. The spectrum of natural displacements of torsional vibrations of the model of the crankshaft with the coupling

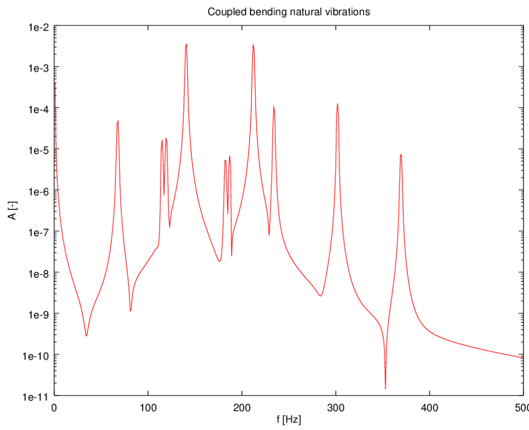

Fig. 1. Bending vibrations of the crankshaft of the system with a coupling

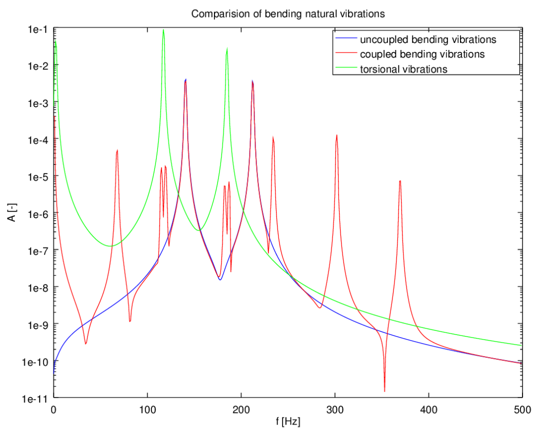

Fig. 8. Comparison of natural displacements of torsional and bending vibrations of the model of the crankshaft with and without coupling

The proposed system of equations of dynamics in moving coordinate system is possible to be solved analytically. Part of the equations is uncoupled and linear.

The simulations clearly show the impact of taking into account the coupling on transverse displacements of the crankshaft. The frequencies of torsional vibrations are transferred to bending oscillations. This allows drawing conclusions about the frequencies occurring in the spectral structure of angular vibrations only based on the measurements of body vibration. The proposed model can be used successfully in a diagnostics of combustion engines.

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