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The Influence of Internal and Constructional Supports Damping on the Γ-type Frame Vibrations

Wojciech SOCHACKI
Institute of Mechanics and Fundamentals of Machinery Design
University of Technology, Czestochowa, Poland
sochacki@imipkm.pcz.pl

Piotr ROSIKOŃ
Institute of Mechanics and Fundamentals of Machinery Design
University of Technology, Czestochowa, Poland
p.rosikon@imipkm.pcz.pl

Sandra TOPCZEWSKA
Institute of Mechanics and Fundamentals of Machinery Design
University of Technology, Czestochowa, Poland
s.topczewska@imipkm.pcz.pl

Abstract

The paper presents the formulation and solution of Γ-type frame damping vibration. The physical system model takes into account the energy dissipation of the vibrating frame due to the internal vibration damping of the viscoelastic frame material and the constructional damping in the place of frame bolt support. As the results of the problem solution, the damping and system geometry effects on the first frame eigenvalue (damped frequencies and coefficients of amplitude decay factor) were presented.

Keywords: vibration damping, Γ-type frame, eigenvalue, amplitude decay factor.

1. Introduction

The constructional damping vibration problems of frames are extremely significant from the point of view of mechanical structural designs. Also the structures of frames in a square Γ-type [1], T-type [2, 3, 4] or other two or three bar frames [5] form have been described in many scientific publications. Experimental, theoretical and numerical study associated with Γ type frame with reference to stability and free vibrations, have been carried out in the monograph [1].

The type of instability of a T-type frame with joint mass M subjected to a compressive follower force P applied at the joint was researched in the work [2]. In paper [3] a formulation and solution for the problem of damped vibration in T-type frame was presented. The energy dissipation in a vibrating frame as a result of constructional damping in the points of the frame mounting and the supports in physical model was also took into account. T-type frame theoretical, numerical and experimental research on the stability and free vibrations was also described in [4]. The author investigated frame loaded by longitudinal force in relation to its bolt.

Importance of two-bar frames research was emphasized by describing the variational method for investigation of the stability of a rectangular two-bar frame in the work [5].

Also interesting studies in the field of numerical procedure for the complex frequencies and vibration modes evaluation were carried out in the article [6]. Interesting research results related to the effects of small both internal and external damping on the stability of disturbed non-conservative systems could be found in the paper [7].

In this paper the formulation and solution of Γ-type frame damped vibration was presented. In the vibration model, internal damping of viscoelastic material in frame (rheological model by Kelvin-Voigt) and constructional damping in the place of frame bolt support was taken into account. As the results of the problem solution, the damping and system geometry effects on the first frame eigenvalue were presented. The results obtained in the study were presented in 2D figures and spatial presentations.

2. Physical and mathematical model

Physical model of the considered system is shown in Fig. 1. Considered frame consists of a column with an l_1 length and l_2 long bolt. Constructional damping of the bolt support vibrations was modelled by viscous rotary damper with a damping factor denoted as C_R. Viscoelastic material has been characterized by the Young's modulus E_i and the viscosity coefficient E_i* of frame material.

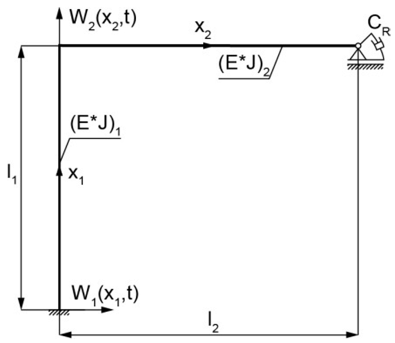

Figure 1. Physical model of the Γ-type frame

The equations of motion of the individual frame beams can be written as:

$$E_i J_i \frac{\partial^4 W_i(x,t)}{\partial x^4} + E_i^* J_i \frac{\partial^5 W_i(x,t)}{\partial x^4 \partial t} + \rho_i A_i \frac{\partial^2 W_i(x,t)}{\partial t^2} = 0 \tag{1}$$

where:

$W_i(x,t)$ – the lateral displacement for individual beams of frame, $i = 1,2$,
A_i – the cross-section area of the beams,
J_i – the moment of inertia for beam section,
E_i – Young's modulus,
E_i* – material viscosity coefficient,
ρ_i – the density of the beam material,
x – space coordinate,
t – time.

Geometric boundary conditions and continuities are as follows:

$$W_1(0,t) = W_2(l_2,t) = 0,$$

$$\left.\frac{\partial W_1\,(x_1,t)}{\partial x_1}\right|^{x_1=l_1} = \left.\frac{\partial W_2\,(x_2,t)}{\partial x_2}\right|_{x_2=0},$$

$$\left.\frac{\partial W_1\,(x_1,t)}{\partial x_1}\right|_{x_1=0} = 0 \tag{2}$$

The boundary issues are complemented by the natural boundary conditions of the form:

$$(E_2 + jE_2^*\omega^*)J_2 \left.\frac{\partial^2 W_2\,(x_2,t)}{\partial x_2^2}\right|^{x_2=l_2} = -\,C_R \left.\frac{\partial^2 W_2\,(x_2,t)}{\partial x_2 \partial t}\right|^{x_2=l_2}$$

$$(E_1 + jE_1^*\omega^*)J_1 \left.\frac{\partial^3 W_1\,(x_1,t)}{\partial x_1^3}\right|^{x_1=l_1} = 0,$$

$$(E_1 + jE_1^*\omega^*)J_1 \left.\frac{\partial^3 W_1\,(x_1,t)}{\partial x_1^3}\right|^{x_1=l_1} - (E_2 + jE_2^*\omega^*)J_2 \left.\frac{\partial^2 W_2\,(x_2,t)}{\partial x_2^2}\right|_{x_2=0} = 0, \tag{3}$$

$$(E_2 + jE_2^*\omega^*)J_2 \left.\frac{\partial^3 W_2\,(x_2,t)}{\partial x_2^3}\right|_{x_2=0} = 0$$

The last boundary condition for x_2=0 in many publications is assumed to be $W_2(0,t) = 0$ (cf. [8]). Adoption of such condition requires the assumption that during the vibration the vertical rod (pole) of the frame at the end of x_1=l_1 performs so small vibrations (displacement) that they could be identified as negligible. However, assuming that displacement is $W_1(l_1,t) \neq 0$ and taking into account the restoring force of the bending frame (slender system) in x_2=0, then one of the variances of the potential energy element is $\left.(E_2 + jE_2^*\omega^*)J_2 \frac{\partial^3 W_2\,(x_2,t)}{\partial x_2^3}\delta W_2\,(x_2,t)\right|_{x_2=0}$. Assuming that the variation $\left.\delta W_2\,(x_2,t)\right|_{x_2=0} \neq 0$ the natural boundary condition becomes $\left.(E_2 + jE_2^*\omega^*)J_2 \frac{\partial^3 W_2\,(x_2,t)}{\partial x_2^3}\right|_{x_2=0} = 0$ [1, 5].

3. The solution to the problem

The solutions of the equation (1) are as follows:

$$W_i\ (x,t) = w_i(x)e^{j\omega^* t} \tag{4}$$

where: ω^* – the complex eigenvalue of the system, $j = \sqrt{-1}$.

By substituting (4) to (1-3) we obtain:

$$w_i^{IV}(x) - \gamma_i w_i(x) = 0,\ i = 1,2 \tag{5}$$

where:

$$\gamma_i = \frac{\rho_i A_i \omega^{*2}}{(E_i + jE_i^* \omega^*)J_i}\ . \tag{6}$$

The boundary conditions (after the separation of variables) of considered system, are as the following:

$$\begin{gathered}
w_1(0) = w_2(l_2) = 0,\\
w_1^I(l_1) = w_2^I(0),\\
w_1^I(0) = 0,\\
(E_2 + jE_2^*\omega^*)J_2\, w_2^{II}(l_2) = -C_R\, j\, \omega^* w_2^I(l_2),\\
(E_1 + jE_1^*\omega^*)J_1\, w_1^{III}(l_1) = 0,\\
(E_1 + jE_1^*\omega^*)J_1\, w_1^{II}(0) - (E_2 + jE_2^*\omega^*)J_2\, w_2^{II}(0) = 0,\\
(E_2 + jE_2^*\omega^*)J_2\, w_2^{III}(0) = 0
\end{gathered} \tag{7}$$

The solution of equations (5) is expressed in the form of functions:

$$w_i(x) = D_{1i}e^{\lambda_i x} + D_{2i}e^{-\lambda_i x} + D_{3i}e^{j\bar{\lambda}_i x} + D_{4i}e^{-j\bar{\lambda}_i x} \tag{8}$$

The substitution of the solution (8) into equation (7) leads to a system of equations because of the constant D_{ki} (k=1,2-4). The solution of such a system is the solution of boundary problem and it leads determine the eigenvalues of studied system, in the form of damped frequencies $Re(\omega^*)$ and the amplitude decay factor $Im(\omega^*)$.

4. The results of numerical computations

The study of the analyzed frame damping vibrations were performed for the following geometrical and material data: $(EJ)_i$ = 6.443 [Nm^2], $(\rho A)_i$ = 15.433 [kg/m] and for the beam lengths: l_1 =2 and l_2 = 0.5. Calculations were made after the adoption of the dimensionless damping coefficients and the relationship of moments of inertia of the column sections and bolt frame J in the form of:

$$\eta = \frac{E_i^*}{aE_i}, \quad a^2 = (l_1 + l_2)^4 \frac{\sum_i (\rho A)_i}{\sum_i (EJ)_i}, \quad \mu = \frac{C_R}{l_2 \sqrt{(\rho A)_i (EJ)_i}}, \quad J = \frac{J_2}{J_1}. \tag{9}$$

In Figures 2-5 results of the calculation are shown. In Fig. 2 the results of research of dependency between the frame eigenvalues and constructional damping in the place of bolt support.

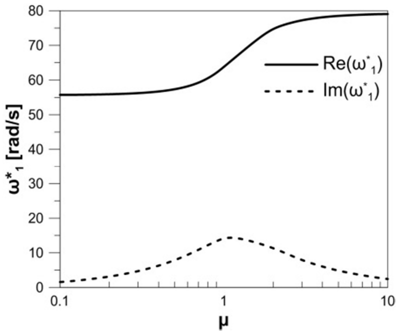

Figure 2. The dependence between the real parts ($Re(\omega_1^*)$) and imaginary parts ($Im(\omega_1^*)$) of the first beam eigenvalue and the constructional damping μ

In the next figure (Fig. 3) the results of frame eigenvalues (with the selected constructional damping value μ = 0.2) changes along with the bolt length (l_2) changes were presented.

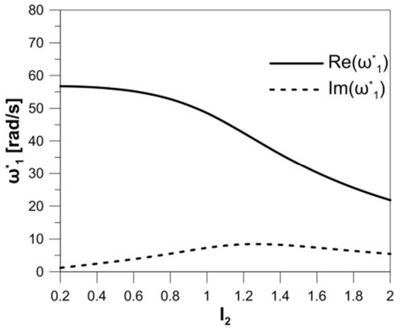

Figure 3. The dependence between the real parts ($Re(\omega_1^*)$) and imaginary parts ($Im(\omega_1^*)$) of the first beam eigenvalue and the horizontal beam l_2

The results of studies on the impact of changes in stiffness of the bolt and the column of the frame on the eigenvalues of the system are shown in Fig. 4. The constructional damping factor in this case also was μ = 0.2. By changing the relation between the moments of cross-section inertia J, in each case constant inertia moment J_2 was taken.

In Figure 5 the results of research on the frame's viscoelastic material internal damping influence on its eigenvalues were presented.

In Figure 6 the summary graphs of the dependence of the real ($Re(\omega_1^*)$) and imaginary parts ($Im(\omega_1^*)$) of the first beam eigenvalue in analyzed system, relative to the constructional damping μ parameter and internal damping coefficient η, were presented.

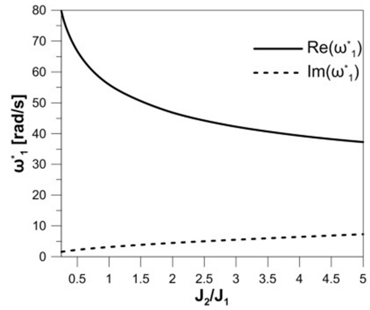

Figure 4. The dependence between the real parts ($Re(\omega_1^*)$) and imaginary parts ($Im(\omega_1^*)$) of the first beam eigenvalue and the beam cross-section J moment of inertia

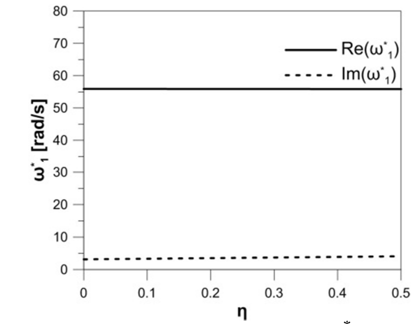

Figure 5. The dependence between the real parts ($Re(\omega_1^*)$) and imaginary parts ($Im(\omega_1^*)$) of the first beam eigenvalue and internal damping coefficient η

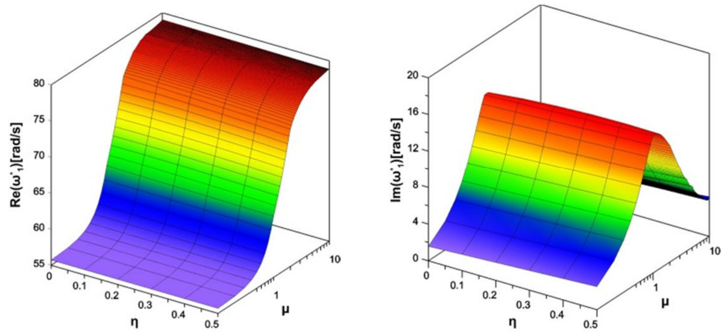

Figure 6. The dependence between the real parts ($Re(\omega_1^*)$) and imaginary parts ($Im(\omega_1^*)$) of the first beam eigenvalue and the constructional damping parameter μ and internal damping coefficient η

4. Conclusions

The paper presents a model of damped vibrations of Γ-type frame. Based on the obtained results it could be concluded that including constructional damping mounting, causes significant changes in the frame eigenvalues. The change of the damping coefficient μ, significantly affects on the first eigenvalue (both on the damped frequency $Re(\omega_1^*)$ and the amplitude decay factor $Im(\omega_1^*)$).The damped frequency $Re(\omega_1^*)$ is increasing to a value corresponding to the two-sided rigid frame mounting. It can be seen that with the increase of the rotary damper damping coefficient, the amplitude decay coefficient rises to a maximum value and then tends to 0 when $\mu \to \infty$.

Analyzing the impact of the frame bolt length on its eigenvalues, it could be concluded that that suitable damped vibrations $Re(\omega_1^*)$ decrease with the elongation of the bolt (l_2), which was to be expected. However, the amplitude decay factor increases to a maximum value, and then decreases.

Significant changes in the eigenvalues of research system could be seen when changing relations of cross-section inertia moments of the two parts of frame. The increase in the ratio of J moments causes stronger vibration damping in the system (growth of coefficient $Im(\omega_1^*)$). The inclusion of internal damping in the frame vibration model, causes a slight change in the first eigenvalue (damped vibrations $Re(\omega_1^*)$ as well as the amplitude decay factor $Im(\omega_1^*)$).

Based on the research it could be determined such geometric parameters of the frame, for which the amplitude decay factors are greatest, and hence it is possible to design systems providing minimum vibration amplitudes.

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