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DIMENSION, EGALITARIANISM AND DECISIVENESS OF EUROPEAN VOTING SYSTEMS

An analysis of three major aspects has been carried out that may apply to any of the successive voting systems used for the European Union Council of Ministers, from the first one established in the Treaty of Rome in 1958 to the current one established in Lisbon. We mainly consider the voting systems designed for the enlarged European Union adopted in the Athens summit, held in April 2003 but this analysis can be applied to any other system. First, it is shown that the dimension of these voting systems does not, in general, reduce. Next, the egalitarian effects of superposing two or three weighted majority games (often by introducing additional consensus) are considered. Finally, the decisiveness of these voting systems is evaluated and compared.

Keywords: *voting systems, simple games, weighted majority games, Shapley–Shubik power index, dimension, egalitarianism, decisiveness*

1. Introduction

The successive enlargements undergone by the European Union raise many interesting questions concerning not only politics, but also the mechanisms used to make decisions. Cooperative game theory, and more particularly simple games, provide suitable tools to analyze some of them. Among the decision–making organisms of the Union, the Council of Ministers appears, each time, as one of the main battlefields in the design of the enlarged structure.

In this paper, we are interested in the study of three major aspects of the sophisticated voting rules that concern the Council of Ministers. These rules are defined

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by combining in each case two or three elementary mechanisms (weighted majority games), giving rise to much more complicated and restrictive ones. We will adopt here a normative viewpoint, so that no strategic behavior of the involved countries will be assumed.

First, we shall deal with the dimension of the simple games defining these voting rules and the possibility of simplifying them. Second, we will focus on a delicate point, particularly since the agents are countries and not individual people: the egalitarianism level of these rules, rather than the specific fraction of power they allocate in the Council to each of the countries that will form the future Union. Third, we will evaluate the (structural) decisiveness that the rules show as decision-making procedures, given that their structure suggests a strong component of inertia. In all cases, the effect of the imposed restrictions will be our main interest.

These three aspects, which are the object of analysis in this work, complement many others treated in several references, among them: Bertini et al. [1, 2], Chakravarty et al. [7], Freixas and Pons [14], Freixas et al. [18], Freixas and Gambarelli [16], Gambarelli [20, 21] or Owen [28].

The organization of this article is as follows. In Section 2, technical preliminaries concerning the three points of our research are given. Section 3 provides a summary of the voting rules adopted in Athens and the corresponding simple games. Section 4 is devoted to the study of the dimension. Section 5 refers to egalitarianism. Section 6 deals with decisiveness. Finally, the conclusions are given in Section 7.

2. Preliminaries

In this section, in order to make a self-contained article, we recall some basic definitions and properties of simple games, dimension, egalitarianism and decisiveness.

2.1. Simple games

Definition 2.1. A (monotonic) simple game is a pair (N, v) where $N = \{1, 2, ..., n\}$ is a finite set of players, every $S \subseteq N$ is a coalition, 2^N is the set of all coalitions, and $v : 2^N \to \{0, 1\}$ is a characteristic function which satisfies $v(\emptyset) = 0$, $v(N) = 1$, and $v(S) \le v(T)$ if $S \subset T \subseteq N$. A coalition S is winning if $v(S) = 1$ and losing otherwise. If W denotes the set of winning coalitions in v, then $\emptyset \notin W$, $N \in W$, and $T \in W$ whenever $S \subset T \subseteq N$ and $S \in W$. A coalition S is blocking if $N - S \notin W$. Wherever N is clearly fixed, we will simplify the notation and speak of the game v. For additional material on simple games, the reader is referred to Shapley [31], Carreras and Freixas [5], Taylor and Zwicker [36] and Carreras [3].

Definition 2.2. A simple game (N, v) is a weighted majority game (WMG, for short) if there are nonnegative weights $w_1, ..., w_n$ attached to the players and a positive quota $q \le w_N$ such that

$$v(S) = \begin{cases} 1 & \text{if} \quad w_S \ge q \\ 0 & \text{if} \quad w_S < q \end{cases}$$

where $w_S = \sum_{i \in S} w_i$ for every $S \subseteq N$. We then write $(N, v) = [q; w_1, ..., w_n]$. It is well known that only for $n \le 3$ every simple game is a WMG. In the sequel, we will always assume that $w_1 \ge \ldots \ge w_n$ and, in the case of having different weights $w_1 > ... > w_r$ repeated $k_1, ..., k_r$ times, respectively (so that $k_1 + ... + k_r = n$), we will often write $(N, v) = [q; w_1(k_1), ..., w_r(k_r)]$ for short. In particular, a k-out-of-n game is a special case of WMG: in fact, the expression k -out-of- n refers to the description of the game in which each one of the n players is given a weight of 1 and the quota is set at k, i.e. $(N, v) = [k; 1(n)]$.

Taylor and Zwicker [34] established that, among simple games, the WMGs are precisely those where winningness is “robust” with respect to general trades (appropriate changes of coalition members, see below).

Definition 2.3. A simple game (N, v) is k-trade robust for some positive integer k if there is no exchange of members among any collection of $j \le k$ winning coalitions $R_1, ..., R_j$ that leads to losing coalitions $T_1, ..., T_j$ in such a way that

$$|\{p : i \in R_p\}| = |\{p : i \in T_p\}| \quad \text{for each} \quad i \in N$$

A simple game is trade robust if it is k-trade robust for all k.

For instance, if $N = \{1, 2, 3, 4\}$ and v is the simple game in which the winning coalitions are: $R_1 = \{1, 2\}$ and $R_2 = \{3, 4\}$ plus those extended by monotonicity, it follows that both $T_1 = \{1, 3\}$ and $T_2 = \{2, 4\}$ are losing and can be obtained from R_1 and R_2 by swapping players 2 and 3 between them. Thus this game is not 2-trade robust and therefore it is not weighted.

Theorem 2.4 [34]. A simple game is a WMG if and only if it is trade robust.

2.2. Dimension

The following notion was introduced for graphs in the late 1970s. Its extension to hypergraphs (equivalent to simple but not necessarily monotonic games) is due to Jereslow [23]. Nevertheless, the definition of dimension for simple games is reminiscent of the definition of the dimension of a partially ordered set as the minimum number of linear orderings whose intersection is the given partial ordering [11].

Definition 2.5. The dimension of a simple game (N, v) is the least k for which there exist k WMGs $(N, v_1), \ldots, (N, v_k)$ such that $v = v_1 \cap \ldots \cap v_k$.

Theorem 2.6. Every simple game has a dimension which is bounded by the number of maximal losing coalitions of the game (see, for example, [33] or [35]) . □

The dimension of a simple game can be seen as a measure of its complexity. In the books by Taylor [33] and Taylor and Zwicker [35], the authors deal with dimension theory for simple games. In Freixas and Puente [15], the dimensions of several types of composite games are computed. Most real voting systems are described by simple games of dimension one or two: the United Nations Security Council is of dimension 1, and interesting examples of dimension 2 are the United States federal system (see, for instance, Taylor [33]) and the Victoria Proposal, the procedure to amend the Canadian Constitution (see [24, 33]).

2.3. Linear games and egalitarianism

Definition 2.7. The individual desirability relation D, introduced by Isbell [22] and generalized later on by Maschler and Peleg [26], is the partial preorder on the player set N defined, for each game v on N, by iDj in v if and only if $v(S \cup \{i\}) \geq v(S \cup \{j\})$ for every $S \subseteq N - \{i, j\}$.

Definition 2.8. Games for which D is complete (or total, i.e. satisfying that for every $i, j \in N$ either iDj or jDi or both) have been given various names in the literature (ordered, complete). We will refer to them here as linear games. It is clear that every WMG is linear, but for any $n \geq 6$ there are linear games that are not WMGs. In the sequel, when considering linear games, we will always assume that $1D2$, $2D3$, ..., $(n-1)Dn$. For a characterization of any linear simple game in terms of numerical invariants, the reader is referred to [5].

Definition 2.9. A simple game v on N is a linear game with consensus if

$$v = u \cap [q; 1(n)]$$

where u is a simple game on N such that v becomes linear. This notion, introduced in [6], is slightly more general than the one considered by Peleg [29].

Notice that u is not required to be linear. If iDj in u, then iDj in v, but the converse is not true. From the fact that every WMG is linear, it follows that if u is a WMG then v is a linear game with consensus. Furthermore, if u is the intersection of two WMGs u^1 and u^2 and the weights, respectively, defining them satisfy $w_i^k \geq w_{i+1}^k$ for $i = 1, 2, ..., n-1$ and $k = 1, 2$ (as mentioned in Definition 2.2), then u is linear and v becomes a linear game with consensus. This is especially important for the voting systems we will study below.

Definition 2.10. The well-known Shapley–Shubik index of power, introduced in [32] (see also [30]), is the allocation rule that assigns to every simple game (N, v) the n-vector $\Phi[v] = (\Phi_1[v], ..., \Phi_n[v])$ defined by

$$\Phi_i[v] = \sum_{S \subseteq N: S \ni i} \gamma_n(s)[v(S) - v(S - \{i\})] \text{ for each } i \in N$$

where $s = |S|$ and $\gamma_n(s) = \dfrac{(s-1)!(n-s)!}{n!}$. It is worth mentioning the axiomatic characterization of this allocation rule by means of the efficiency, symmetry, null player and transfer properties stated by Dubey [10]. In addition, Shapley and Shubik interpreted $\Phi_i[v]$ as the probability of player i being pivotal when all permutations of the players are equally likely (player $i = \pi_k$ is pivotal in permutation $\pi = (\pi_1, ..., \pi_n)$ for game v if $\{\pi_1, ..., \pi_k\}$ is winning in v but $\{\pi_1, ..., \pi_{k-1}\}$ is not).

Definition 2.11. An n-vector $x = (x_1, ..., x_n)$ Lorenz-dominates $y = (y_1, ..., y_n)$ if $\sum_{i=j}^{n} x_i \geq \sum_{i=j}^{n} y_i$ for $j = 1, ..., n$. In symbols, xLy.

Let u be a linear game on N, $v^1 = u \cap [q_1; 1(n)]$ and $v^2 = u \cap [q_2; 1(n)]$, with $1 \leq q_1 < q_2 \leq n$. Peleg [29] proved that $\Phi[v^2]\ L\ \Phi[v^1]$ (see also [39]). From the efficiency and Lorenz-domination, it follows that

$\Phi_1[v^1] \geq \Phi_1[v^2]$ and

$\Phi_n[v^2] \geq \Phi_n[v^1]$

(For any other player, i.e. $i \neq 1, n$, there are no generally valid inequalities like these; for details, see Proposition 3.1 in [6].) This can be interpreted as reflecting that, from the viewpoint of the Shapley–Shubik index, the game v^2 is more "egalitarian" than v^1, in the sense that

$$\Phi_1[v^1] - \Phi_n[v^1] \geq \Phi_1[v^2] - \Phi_n[v^2]$$

To cope with this idea, we introduce some notions.

Definition 2.12. Let (N, v) be a linear simple game.

- The range of (N, v) is the range of the set of numbers $\Phi_1[v], ..., \Phi_n[v]$, i.e.

$$\text{rang}[v] = \Phi_1[v] - \Phi_n[v]$$

- The egalitarianism of (N, v) is

$$\text{egal}[v] = 1 / \text{rang}[v]$$

Notice that $1 \leq \text{egal}[v] \leq \infty$ for all v. In fact, $\text{egal}[v] = 1$ iff v is a dictatorship and $\text{egal}[v] = \infty$ iff v is a k-out-of-n game. We will be interested in studying the increase in egalitarianism when passing from a linear game with consensus v^1 to another linear game with a higher level of consensus v^2. The over-egalitarianism percentage, defined by

$$\text{oep}\ [v^1, v^2] = \frac{\text{egal}\ [v^2] - \text{egal}\ [v^1]}{\text{egal}\ [v^1]} \times 100$$

reflects this increase. The definition makes sense unless v^1 is a k-out-of-n game. In this case, v^2 would also be a k'-out-of-n game, and we could take $\text{oep}\,[v^1, v^2] = 0$ as a convention.

At this point, we recall from [6] an important result to be used below.

Theorem 2.13 [6]. Let $v^1 = u \cap [q_1; 1(n)]$ and $v^2 = u \cap [q_2; 1(n)]$ be linear games with consensus with $1 \leq q_1 < q_2 \leq n.$ Then:

- $0 \leq \Phi_1[v^1] - \Phi_1[v^2] \leq \dfrac{q_2 - q_1}{n}$
- $0 \leq \Phi_n[v^2] - \Phi_n[v^1] \leq \dfrac{1}{n}$ □

2.4. Structural decisiveness

Let us finally refer to the notion of decisiveness, introduced in [4] (see also [9]). As real life experience shows, two main tendencies arise in the design of voting systems.

The first one tries to strengthen the agility of the mechanism in order to take decisions, and usually applies to national and regional parliaments, town councils, and many other committee systems. The second tendency is rather interested in protecting the rights of certain minorities, even at the cost of introducing a remarkable inertia into the mechanism, and is found particularly often in supranational organizations. It seems therefore interesting to measure, and of course to compare, the agility/inertia of such decision--making procedures, and the decisiveness index is intended to this end.

Definition 2.14 [4]. The (structural) decisiveness index is the map that assigns to every simple game (N, v) the number

$$\delta(N, v) = 2^{-n}\,|W|$$

The number $\delta(N, v)$, or simply $\delta[v]$, will be called the decisiveness degree of game (N, v).

If f is the multilinear extension of game v [27], then $\delta[v] = f(\ 1/2, ..., \ 1/2)$. Thus, $\delta[v]$ merely gives the probability of a proposal being socially accepted by N under the acceptance rules stated by v when each agent votes independently of each other for the motion with probability 1/2. Nevertheless, it is precisely this formal approach, that does not take into account any strategic behavior by the players, which is the tool best suited to analyze voting systems from just a structural viewpoint.

The decisiveness index is a normalized measure, as $0 < \delta[v] < 1$ for any simple game v. More precisely, for a given N the minimum decisiveness degree is attained for the unanimity game u_N (where N is the only winning coalition) and is $\delta[u_N] = 2^{-n}$, whereas the maximum degree is attained for the individualistic game u_N^* (the dual game of u_N, where any $S \neq \varnothing$ is winning) and is $\delta[u_N^*] = 1 - 2^{-n}$. Notice that all so-called decisive games (that is, those where $S \in W$ iff $N - S \notin W$) show a decisiveness degree of 1/2. In general, all proper (i.e. superadditive simple) games have a degree lower than, or equal to, 1/2. The lower $\delta[v]$, the more difficult it is to take decisions in v. For the main properties of the decisiveness index, in particular referring to standard ways of combining simple games, several axiomatic characterizations, and an alternative computation procedure, we refer the interested reader to [4].

3. Provisions of the Accession Treaty on voting in the Council

As was pointed out in the first part of this article, the normative methodology proposed may be used for any binary voting system or simple game resulting from the intersection of at least two WMGs. Dimension and consensus then become interesting issues to be analyzed, while decisiveness applies to all simple games, no matter whether they decompose or not as the intersection of two or more WMGs.

As Taylor [33] noted in his book entitled *Mathematics and Politics*: *the interest of dimension lies in the fact that all known voting simple games in practice have small dimension: either one or two*. This observation makes dimension a very interesting notion, since a dimensionally efficient representation is a compact, intuitive and simple way to represent almost any real voting simple game.

Two voting systems of the European Union Council, that entered into effect on February 1st 2003 under the Nice rules, became the first known real-world examples of dimension 3 [13]. Other real-world examples with dimension 3 appeared later on. Indeed, Cheung and Ng [8] proved that the voting system in Hong Kong, which is not a complete simple game, also has dimension 3. Kurz and Napel [25] have proven that the Lisbon voting system of the Council of the European Union, which became effective in November 2014, cannot be represented as the intersection of six or fewer weighted games, i.e., its dimension is at least 7 and determination of the exact dimension has been posed as a challenge to the community. This sets a new record for real-world voting bodies.

The Athens treaty was signed on April 16th 2003 in Athens, Greece and came into force on May 1st 2004, the day of the enlargement of the European Union. It modified a significant number of points that originally dealt with the Treaty of Nice. This treaty, chronologically situated between the treaties of Nice and Lisbon, will be taken as the basis for the theoretical discussions that follow in this article.

The Athens Treaty amended the system of qualified majority voting to apply from 2004. We consider rules regarding two different scenarios for enlargement: the transitional period and the period from November 1st 2004. For each assumed scenario, a WMG is at the core of the system, but some additional conditions must also be met in terms of the number of countries supporting a proposal and, in some cases, their population.

Using the appropriate terminology from game theory, the players are: Germany, United Kingdom, France, Italy, Spain, Poland, The Netherlands, Greece, Czech Republic, Belgium, Hungary, Portugal, Sweden, Austria, Slovak Republic, Denmark, Finland, Ireland, Lithuania, Latvia, Slovenia, Estonia, Cyprus, Luxembourg, Malta, which we will represent by the set $\{1, 2, ..., 25\}$, where 1 stands for Germany, 2 stands for the United Kingdom, and so on.

3.1. Transitional period: May 1st 2004–October 31st 2004

We quote the relevant text from [37], article 26:

For their adoption, acts of the Council shall require at least:

• *88 votes in favour (of a total of 124 votes) where this Treaty requires to be adopted on a proposal from the Commission,*

• *88 votes in favour (of a total of 124 votes), cast by at least two-thirds of the members, in other cases.*

In the event that fewer than ten new Member States accede to the Union, the threshold for the qualified majority for the period until October 31st 2004 shall be fixed by Council decision so as to correspond as closely as possible to 71.26% of the total number of votes.

The first game

$$u_1 = [88; 10(4), 8(2), 5(6), 4(2), 3(8), 2(3)] \tag{1}$$

corresponds to a given vote distribution among countries and a majority of 70.97%, i.e. the threshold for the qualified majority is as close as possible to 71.26%. Let $v_1 = [13; 1(25)]$ and $v_2 = [17; 1(25)]$ be the games that correspond to a simple majority and a two-thirds majority of the members, respectively. Notice that $u_1 \cap v_1 = u_1$. The second game is

$$u_2 = u_1 \cap v_2 \tag{2}$$

3.2. From November 1st 2004

We quote the relevant text from [37], article 12:

Acts of the Council shall require for their adoption at least 232 votes in favour cast by a majority of the members where this Treaty requires them to be adopted on a proposal from the Commission.

In other cases, for their adoption acts of the Council shall require at least 232 votes in favour, cast by at least two-thirds of the members.

When a decision is to be adopted by the Council by a qualified majority, a member of the Council may request verification that the Member States constituting the qualified majority represent at least 62% of the total population of the Union. If that condition is shown not to have been met, the decision in question shall not be adopted.

In the event of fewer than ten new Member States acceding to the European Union, the threshold for the qualified majority shall be fixed by Council decision by applying a strictly linear, arithmetical interpolation, rounded up or down to the nearest vote, between 71% for a Council with 300 votes and the level of 72.27% for an European Union of 25 Member States.

The first game is

$$v_3 = [232; 29(4), 27(2), 13(1), 12(5), 10(2), 7(5), 4(5), 3(1)]$$

Let

$$v_4 = [620; 182, 132, 131, 128, 87, 86, 35, 23(3), 22(2),$$
$$20, 18, 12(2), 11, 8(2), 5, 4, 3, 2, 1(2)],$$

where weights are proportional to populations and a majority of 62% is demanded.

The voting systems to be used will correspond to

$$u_3 = v_3 \cap v_4 \cap v_1 \tag{3}$$

and

$$u_4 = v_3 \cap v_4 \cap v_2 \tag{4}$$

Systems (3) and (4) should therefore be thought of as requiring triple majorities: (a) weights must meet or exceed the threshold (a super-majority about 72.27% of the sum of voting weights); (b) a super-majority of 62% of the total EU population with the weights and quota being based on the appropriate rate per thousand; and (c) either a simple majority of the number of countries or a super–majority of 2/3 of this number.

In this model, we assume that the planned referenda will result in all the 10 candidate countries joiningthe European Union.

The two Athens rules from November 1st 2004 that we deal with here, u_3 and u_4, require the agreement of three sorts of majorities. Among other results, we shall prove that the noted complexity of both systems is irreducible, thus proving the existence of real voting systems of dimension three. As for the rule u_2, used in the transitional period, we will also prove that this system is irreducible.

4. On the dimension of the Council

In this section, we prove that the dimension of u_2 is two and the dimension of u_3 and u_4 is three and, therefore, none of these games can be described using fewer WMGs. Similar calculations have already been done in Freixas [13] for the initially foreseen enlargement to 27 members, agreed in December 2000 at the summit of Nice.

For variants, e.g., the notion of codimension and theoretical background on the notion of dimension, we refer the reader to Freixas and Marciniak [17].

4.1. The dimension for the transitional period

Theorem 4.1. The dimension of game u_2 is 2.

Proof. Obviously, the dimension of $u_2 = u_1 \cap v_2$ is at most 2. If $i \leq j$ in N, let us denote $[i, j] = \{k \in N : i \leq k \leq j\}$. Consider the following coalitions: $A = [5, 25]$, $B = [1, 16]$, $A' = A - \{24, 25\} \cup \{4\}$, and $B' = B - \{4\} \cup \{24, 25\}$. The weights of these coalitions in games u_1 and v_2 are as follows:

	A	B	A'	B'
u_1	84	100	90	94
v_2	21	16	20	17

Assume now that u_2 has dimension 1, i.e. that u_2 is a WMG. Coalition A is losing in u_1 and B is losing in v_2 and, hence, both coalitions are losing in u_2. But, after the described trades, A and B convert into the winning coalitions A' and B'. Consequently, the game u_2 cannot be a WMG according to Theorem 2.4.□

4.2. The dimension from November 1st 2004

Theorem 4.2. The dimension of game u_3 is 3.

Proof. Since $u_3 = v_3 \cap v_4 \cap v_1$, its dimension is at most 3. Consider the following coalitions:

- $A = [1, 3] \cup [7, 23]$, $B = [1, 12]$ and $C = \{1\} \cup [5, 25]$.
- $A' = A - \{13, 20, 21\} \cup \{4\}$, $B' = B - \{4\} \cup \{13, 20, 21\}$,
and $C' = C - \{5, 25\} \cup \{3\}$.
- $A'' = A - \{3\} \cup \{5, 25\}$, $B'' = B - \{2\} \cup \{13, 14\}$, and $C'' = C - \{13, 14\} \cup \{2\}$.

The weights of these coalitions in games v_1, v_3 and v_4 are stated as follows:

	A	B	C	A'	B'	C'	A''	B''	C''
v_1	20	12	22	18	14	21	21	13	21
v_3	231	243	234	242	232	233	232	234	243
v_4	696	894	608	795	795	651	653	800	702

Assume first that u_3 has dimension 1. Coalition A is losing in v_3 and so is B in v_1. Consequently, A and B are losing in u_3. However, after trades, A and B convert into A' and B' that are both winning in u_3.

Assume now that u_3 has dimension 2, i.e. it can be represented as the intersection of two WMGs. It follows that at least one of the following statements should be true:

1. A and B are losing in the same WMG.
2. A and C are losing in the same WMG.
3. B and C are losing in the same WMG.

Statement 1 cannot be true because, as we have seen above, A' and B' are both winning in u_3, which is not possible in a WMG.

Statement 2 is impossible, because A'' and C' are both winning in v_3, v_1 and v_4 and, thus, coalitions A, C, A'' and C' show that trade robustness does not hold.

Finally, statement 3 is impossible for the same reason by considering coalitions B, C, B'' and C''. □

Theorem 4.3. The dimension of game u_4 is 3.

Proof. This proof follows the same approach as that of the proof of Theorem 4.2. For the sake of completeness, we indicate the coalitions we use to make trades and their corresponding weights in games v_2, v_3 and v_4:

- $A = [1, 2] \cup [4, 5] \cup [7, 12] \cup [16, 23]$, $B = [1, 16]$ and $C = [2, 3] \cup [6, 25]$.
- $A' = A - \{22, 23\} \cup \{3\}$, $B' = B - \{3\} \cup \{22, 23\}$ and $C' = C - \{6, 25\} \cup \{4\}$.
- $A'' = A - \{4\} \cup \{6, 25\}$, $B'' = B - \{1\} \cup \{24, 25\}$ and $C'' = C - \{24, 25\} \cup \{1\}$.

	A	B	C	A'	B'	C'	A''	B''	C''
v_2	18	16	22	17	17	21	19	17	21
v_3	231	277	236	252	256	235	232	255	258
v_4	730	956	602	856	830	643	689	776	782 □

4.3. Two surprising facts about the dimension of the Council

Let us analyze the initial enlargement of the European Union planned at the summit of Nice. There, 27 countries were supposed to form the future EU: the countries considered in Athens with the addition of Romania and Bulgaria. These two countries were assigned 14 and 10 votes in the Council, respectively. The 25 other countries were assigned the same number of votes as in game v_3.

We quote the relevant text from [38], p. 164.

Acts of the Council shall require for their adoption at least 258 votes in favour, cast by a majority of members, where this Treaty requires them to be adopted on a proposal from the Commission. [...] *When a decision is to be adopted by the Council by a qualified majority, a member of the Council may request verification that the Members States constituting the qualified majority represent at least 62% of the total population of the Union. If that condition is shown not to have been met, the decision in question shall not be adopted.*

If we consider $N = \{1, 2, ..., 27\}$ and games

$$\begin{aligned} v^1 &= [14; 1(27)], \\ v^3 &= [258; 29(4), 27(2), 14(1), 13(1), 12(5), 10(3), 7(5), 4(5), 3(1)], \\ v^4 &= [620; 170, 123(2), 120, 82, 80, 47, 33, 22, 21(4), 18, 17(2), \\ &\quad 11(3), 8(2), 5, 4, 3, 2, 1(2)], \end{aligned}$$

then the game that represents the full rules is $u = v^3 \cap v^4 \cap v^1$ (notice that, e.g. the notation for the voting rule based on a weight per thousand inhabitants changes from v_4 to v^4 when including both new countries). For simplicity, we assume that no relevant population changes have occurred since 2001. Notice that the threshold of 258 implies a super–majority of 74.78%, which is quite high and, therefore, it is rather difficult to reach agreements. Game u reduces to a single WMG [12], so that its dimension is 1.

Assume for a while that some states (Romania and Bulgaria) delay their incorporation into the EU, but the ratio between the threshold and the sum of weights used in each of the three games is not modified: 50.01% for game v^1, 74.78% for v^3 and 62% for v^4. This means that the *spirit* of the rule is maintained. How does the reduction of the number of countries affect the dimension? Will the dimension of this game with two fewer players be necessarily 1 or, instead, can it be greater than 1?

Notice that, in general, if a simple game has dimension k, i.e. it can be expressed as an intersection of k WMGs, and some players are removed, but the threshold is left invariant in each WMG, then the dimension of the reduced game is at most k. However, we will see that the dimension of the reduced game may be greater than k if the thresholds are modified in order to preserve the corresponding proportion of the sum of weights required in each of the WMGs. Game u will help us to show this fact.

Let us consider the following game with the 25 Member States

$$u' = (v^3)' \cap v_4 \cap v_1$$

where $(v^3)' = [240; 29(4), 27(2), 13(1), 12(5), 10(2), 7(5), 4(5), 3(1)]$ and v_1 and v_4 are the games that we described in Section 3.

Game u' represents the reduction of u in the way we mentioned above. In fact, the threshold for the 25-player game $(v^3)'$, 240, is a proportion 100×240/321 = 74.77% of the number of votes, and this ratio almost coincides with that of the 27-player game v^3. Games v_1 and v_4 also have ratios that almost coincide with those in games v^1 and v^4. The first, rather surprising, fact is shown in the next result.

Theorem 4.4. The dimension of game u' is 2.

Proof. This comes from the following properties:

1. Each winning coalition in $(v^3)'$ is also winning in v_4, i.e. $(v^3)' \cap v_4 = (v^3)'$ and hence $u' = (v^3)' \cap v_1$.

2. $(v^3)' \cap v_1$ does not reduce to a single WMG.

To see property 1, it suffices to check that if S is a coalition with a weight lower than 82 in $(v^3)'$, then its weight in v_4 is lower than 380, and this allows us to considerably reduce the number of coalitions to examine to 106 relevant ones (of course we omit this tedious but easy part). Thus, $(v^3)' \cap v_4 = (v^3)'$.

Property 2 follows from the fact that coalitions $A = [1, 3] \cup [7, 25]$ and $B = [1, 12]$ are both losing in u' but, after trades, convert into the winning coalitions $A' = A - \{13, 14, 24\} \cup \{6\}$ and $B' = B - \{6\} \cup \{13, 14, 24\}$. This proves that $u' = (v^3)' \cap v_1$ does not reduce to a WMG. □

Let us explain now a second surprising phenomenon that might happen. If we consider that significant changes in the populations are possible, imagine, for instance, that the population of each state tends to be proportional to the weight assigned to this state in the original weighted game. Thus it will be possible that the weighted–votes game and the population game are the same WMG. In consequence, if the populations change, it is theoretically possible to reach a game with the same member states, but with a smaller dimension than the original one.

In conclusion, the behavior of the dimension is sensitive to both the addition/removal of members and small changes in the population percentages. Therefore, eventual reductions of the dimension hardly justify simplification of the voting mechanisms intended for the Council.

5. On the egalitarianism of the Council

In this section, we are interested in the effect of requiring consensus in the voting systems planned for the enlargement of the EU to 25 members. We will quantify the egalitarianism of each voting system and how much it changes when the level of consensus required by the voting rule increases. As in Section 3, we study both of the foreseen scenarios separately. Three articles dealing with the issue of egalitarianism are by Peleg [29], Carreras and Freixas [6] and Freixas and Marciniak [19].

5.1. On the effect of the consensus required in the transitional period

Recall that game u_1 represents the basic WMG to be applied for a proposal coming from the European Commission (if a straight majority is also required, u_1 does not change: i.e. $u_1 = u_1 \cap v_1$), and u_2 is the same game, but with a threshold of 2/3 $(u_2 = u_1 \cap v_2)$: this applies for motions not coming from the European Commission.

Table 1. Distribution of power
and egalitarianism for the transitional period

Weight	$\Phi_i[u_1]$	$\Phi_i[u_2]$
10	0.0830	0.0759
8	0.0651	0.0602
5	0.0397	0.0391
4	0.0325	0.0337
3	0.0234	0.0265
2	0.0157	0.0207
Egalitarianism	14.8588	18.1159

Table 1 shows the distribution of power, according to the Shapley–Shubik index of power, and the egalitarianism index for games u_1 and u_2. The players are represented by their weights and the power indices are rounded to four decimal places.

As intuition would predict, the higher level of consensus required in game u_2 makes it more egalitarian than u_1. The over-egalitarianism percentage quantifies how much more:

$$\mathrm{oep}[u_1, u_2] = 21.92\%$$

Notice that the fall in the power index of Germany, United Kingdom, France and Italy (the main players) is 0.0071, while that of Malta's (the weakest player) increases by 0.0050. According to Theorem 2.13, these two quantities could reach 0.1600 and 0.0400, respectively.

5.2. On the effect of the consensus required from November 1st 2004

Table 2 shows the distribution of power among the players for the most interesting games involved in the rules of the voting systems applied since November 1st 2004. Again, the players are represented by their weights, but now the Shapley–Shubik index of power is rounded to six decimal places because, if only four decimals were taken into account, certain differences in power that really exist would appear to be zero for some non-equivalent players.

Table 2. Distribution of power and egalitarianism from November 1st 2004

Population	Weight	$\Phi_i[v_4]$	$\Phi_i[v_3]$	$\Phi_i[v_4 \cap v_3]$	$\Phi_i[u_3]$	$\Phi_i[u_4]$
182	29	0.197955	0.092926	0.094941	0.094930	0.085770
132	29	0.134660	0.092926	0.093708	0.093698	0.084537
131	29	0.133462	0.092926	0.093704	0.093693	0.084532
128	29	0.129934	0.092926	0.093694	0.093683	0.084523
87	27	0.085358	0.086136	0.086715	0.086705	0.078377
86	27	0.084452	0.086136	0.086715	0.086705	0.078377
35	13	0.033029	0.039829	0.039515	0.039511	0.038448
23	12	0.021359	0.036479	0.036142	0.036138	0.035901
23	12	0.021359	0.036479	0.036142	0.036138	0.035901
23	12	0.021359	0.036479	0.036142	0.036138	0.035901
22	12	0.020384	0.036479	0.036142	0.036138	0.035901
22	12	0.020384	0.036479	0.036142	0.036138	0.035901
20	10	0.018452	0.030241	0.029926	0.029927	0.031159
18	10	0.016505	0.030241	0.029903	0.029904	0.031135
12	7	0.011085	0.020984	0.020655	0.020662	0.024126
12	7	0.011085	0.020984	0.020655	0.020662	0.024126
11	7	0.010137	0.020984	0.020651	0.020657	0.024121
8	7	0.007297	0.020984	0.020646	0.020652	0.024116
8	7	0.007297	0.020984	0.020646	0.020652	0.024116
5	4	0.004468	0.011892	0.011695	0.011704	0.017524
4	4	0.003615	0.011892	0.011695	0.011704	0.017524
3	4	0.002723	0.011892	0.011695	0.011704	0.017524
2	4	0.001819	0.011892	0.011695	0.011704	0.017524
1	4	0.000911	0.011892	0.011695	0.011704	0.017524
1	3	0.000911	0.008938	0.008741	0.008750	0.015410
	Egalitarianism	5.0750	11.9065	11.6009	11.6036	14.2126

Let us first refer to games v_4 (population game) and v_3 (weight game). The egalitarianism of game v_4, given by $\text{egal}[v_4] = 5.0750$, reflects the great difference in power between the main and the weakest players. Game v_3 is more egalitarian, since

$\text{egal}[v_3] = 11.9065$. Note that there is an increase of 134.61% in egalitarianism when passing from v_4 to v_3.

If we cross v_3 and v_4, we can see that v_4 is highly affected by v_3, as $\text{egal}[v_4 \cap v_3] = 11.6009$. Nevertheless, the intersection of v_4 and v_3 is somewhat superficial, since the egalitarianism of v_3 remains almost unvaried when intersected with v_4.

We can also see that the requirement of a majority by means of v_1 is almost negligible, because $\text{egal}[u_3] = 11.6036$. In fact, the weight game v_3 is more egalitarian than the one obtained when there are also population and majority requirements.

The demand of a 2/3 consensus changes the situation a little bit. In this case, we get a higher level of egalitarianism, $\text{egal}[u_4] = 14.2126$, which represents an increase of 22.48% with respect to u_3. As shown in Table 2, there are visible modifications in the players' power with respect to the other games.

6. On the decisiveness of the Council

We finally analyze the (structural) decisiveness of the different voting systems involved in the Council's decision-making procedures. All of these games are proper (i.e., do not contain disjoint winning coalitions) and most of them are weak (i.e., admit blocking coalitions). As a matter of comparison, we note that the previous 15-member voting systems of the Council show decisiveness degrees of 0.0778 and 0.0704 depending on whether the proposal at stake comes from the European Commission or not (for details, see [4]).

Table 3. Decisiveness degree of several games

Game	Description (when necessary)	Structural decisiveness
u_N	unanimity game	3×10^{-8} (minimum)
v_1	transitional 1/2-majority	0.5000 (maximum)
v_2	transitional 2/3-majority	0.0539
$u_1 = u_1 \cap v_1$	transitional qualified majority	0.0349
$u_2 = u_1 \cap v_2$	transitional qualified majority + 2/3-majority	0.0259
v_3	qualified majority of weights	0.0359
v_4	qualified majority of population	0.2397
$v_3 \cap v_1$		0.0359
$v_3 \cap v_2$		0.0222
$v_4 \cap v_1$		0.1988
$v_4 \cap v_2$		0.0404
$v_3 \cap v_4$		0.0359
$u_3 = v_3 \cap v_4 \cap v_1$		0.0359

Game	Description (when necessary)	Structural decisiveness
$u_4 = v_3 \cap v_4 \cap v_2$		0.0222
u_1'	straight majority of transit. weights	0.4863
v_3'	straight majority of weights	0.5000
v_4'	straight majority of population	0.5000

Table 4. Loss in decisiveness [%]

	v_1	u_1'	u_1	v_3'	v_4'	v_3	v_4	$v_3 \cap v_4$	u_3
v_2	89								
$u_1 = u_1 \cap v_1$		93							
$u_2 = u_1 \cap v_2$			26						
v_3				93					
v_4					52				
$v_3 \cap v_1$						0			
$v_3 \cap v_2$						38			
$v_4 \cap v_1$							17		
$v_4 \cap v_2$							83		
$v_3 \cap v_4$						0	85		
$u_3 = v_3 \cap v_4 \cap v_1$						0	85	0	
$u_4 = v_3 \cap v_4 \cap v_2$						38	91	38	38

Table 3 displays the decisiveness degrees of several games. Among them, we have included the games v_1, v_2, u_1, u_2, v_3, v_4, u_3, u_4 and some combinations of these, as well as the unanimity game u_N for $n = |N| = 25$, which gives the minimum degree of decisiveness, and the games u_1', v_3' and v_4' that correspond to u_1, v_3 and v_4 by replacing the appropriate qualified majority with a straight one.

Table 4 presents the percentage loss in decisiveness obtained when passing from a given game (to be found in the upper row) to a less decisive one (to be found in the left column). Together with the results from Table 3, these results will be the basis for our subsequent comments.

6.1. Decisiveness for the transitional period

Although the decisiveness degrees of games u_1 and u_2 are far from the minimum (attained by u_N), they are less than 1/2 of those corresponding to the previous Council. This decrease seems hard to justify, due to the provisional nature of the transitional period.

Incidentally, notice that using the straight majority game u_1' instead of u_1 would take the decisiveness degree to 0.4863, while the maximum for proper games is 0.5 (as, for example, in game v_1): this difference is due to the fact that u_1' admits blocking coalitions, since the total number of votes is even. The percentage loss in decisiveness when passing from u_1' to u_1 is 93%.

When comparing the two real procedures given by u_1 and u_2, we should first notice that passing from v_1 to v_2 implies a loss of 89% in decisiveness. However, passing from $u_1 = u_1 \cap v_1$ to $u_2 = u_1 \cap v_2$ only gives a loss of 26%, so that the negative effect on decisiveness derived from the additional requirement of a 2/3-consensus could be considered, after all, quite reasonable.

6.2. Decisiveness after November 1st 2004

We first note that the very low degree of decisiveness derived from imposing a qualified majority on the weights (game v_3) represents a loss of 93% with regard to the straight majority game v_3'. Instead, the qualified majority based on population (game v_4) gives rise to a decisiveness degree of 0.2397 and hence to a clearly smaller loss of 52% with regard to the straight majority game v_4'.

Among the intermediate intersections, $v_3 \cap v_1$, $v_3 \cap v_2$, $v_4 \cap v_1$, $v_4 \cap v_2$ and $v_3 \cap v_4$, only $v_4 \cap v_1$ presents a decisiveness degree which is clearly greater than the others (0.1988), with a loss of 17% with respect to v_4. Especially striking are the losses of decisiveness for the games $v_4 \cap v_2$ and $v_3 \cap v_4$ compared to v_4 (83% and 85%, respectively).

The actual procedures u_3 and u_4 are also interesting to analyze. First, their decisiveness degrees are again very small and hardly 1/2 of the corresponding previous procedures. Thus, the enlarged Union does not seem designed to be especially effective in decision-making processes. The equalities

$$\delta[u_3] = \delta[v_3] = \delta[v_3 \cap v_4] = \delta[v_3 \cap v_1] = 0.0359$$

are also worth mentioning, and mean that intersections often cause no loss of decisiveness. Finally, u_3 (respectively, u_4) implies a loss of 85% (respectively, 91%) with respect to v_4, whereas the loss of u_4 with respect to v_3, $v_3 \wedge v_4$ and u_3 is 38%.

7. Conclusions

Several features of the voting rules for the Council of Ministers of the European Union adopted at the Athens summit have been analyzed here. We have studied, from a strictly normative viewpoint, dimension, egalitarianism and decisiveness. Two periods have been considered in each case: the transitional one (until October 31st 2004) and the definitive one (from November 1st 2004).

As to dimension, one of the transitional voting rules has been found to be of dimension 2, as its definition suggests, but the other reduces to a one-dimensional game (i.e., a WMG). On the other hand, both of the rules adopted in the definitive period were shown to be of dimension 3 and, therefore, provide examples of real voting systems of this dimension (not easy to find). A rather surprising fact is also stated, namely that changes of dimension are possible in two cases: the first by either adding or removing countries but maintaining the proportion between the quota and the total weight, and the second resulting from changes in populations. We conclude that dimension is a very sensitive notion, and hence eventual changes in its value do not justify simplification of the voting mechanisms.

With regard to egalitarianism, for the transitional period we find that increasing the required level of consensus implies, of course, increasing the egalitarianism of the rule, but this difference is not especially relevant. Only the power of the four main countries and the single least powerful one changes appreciably with the required change in the level of consensus. Things seem therefore balanced enough for this period. Instead, in the definitive period, v_3 is much more egalitarian than v_4. Intersecting these two games, there is a great increase in egalitarianism with respect to v_4 and a small decrease with respect to v_3. A further intersection with v_1 does not affect egalitarianism, whereas intersecting with v_2 clearly increases this characteristic. In general, there are only noticeable differences in individual power indices between game v_4 and the other games: v_3, $v_4 \cap v_3$, u_3 and u_4.

Finally, concerning decisiveness, in the transitional period we find very low degrees, even less than 1/2 of the decisiveness degrees of the previous rules used for the 15-member Union (already very low). The increase in consensus implies a loss in decisiveness of 26%. The decisiveness of the corresponding rules adopted since November 1st 2004 are even lower. Only v_4 shows a relatively high degree of 0.2397, but there are drastic losses for u_3 and u_4 with regard to that population game. When intersected with other voting rules, game v_1 does not affect the degree of $v_3 \cap v_4$ but there is a decrease if the majority rule used in game v_2 is adopted. The loss from u_3 to u_4 is 38%. In our opinion, if the voting procedures intended for the European Union Council of Ministers have to be really useful for taking decisions, then the best games for the definitive period would be $v_4 \cap v_1$ and $v_4 \cap v_2$. The Athens rules should thus be subjected to new, sound analysis and maybe modified by the European Union. Notice that our conclusion is similar to the proposal contained in the first draft of the European Constitution.

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