Solving Delivery Problems in Distribution Systems

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The subject matter of this article is the optimization of delivery problems (Vehicle Routing Problems – VRP) with regard to distribution systems. These issues belong to the class of NP-hard problems. Current solutions to various delivery problems (VRP, TSP, MTSP, VRPTW, RDPTW) were analysed. Two examples were presented. In a transport and production task, a marginal cost equalling (MCE) algorithm was used. In the case of a Multi Depot Vehicle Routing Problem (MDVRP), an evolutionary algorithm was used.
Keywords: optimization, transport tasks, delivery problems.

1. PROBLEMS OF LOGISTIC OPERATORS

The dynamic development of logistics and supply chains requires scientific support for many processes. This refers, in particular, to the search for optimum solutions by means of quantitative methods supporting decision-making in those areas decisive for quality of customer service (as broadly understood). Because of their high computational complexity, these issues are classified as NP-hard problems.

According to Kotler, distribution means profit-oriented activity covering the planning, organisation, monitoring and control of the transportation of finished products from the site of their production to the point of sale to final purchasers. Thus, the task of distribution is to supply final purchasers with products they desire (kind, quantity) in places where they want to buy them at times that are convenient to them, according to agreed terms and conditions and at the lowest possible price. Thus, the logistic operator faces the task of setting up a plan of goods transportation that would bring optimum results as a result of the adopted optimization criterion [1], [2], [3]. Apart from this, many other decision-making problems need solutions [4], including:

- problems of minimisation of transport service lead times,
- problems of formation of the transport network,
- problems of distribution of the traffic stream within the network,
- issues of selection of technological equipment and determination of the potential of transport and logistics systems [5].

In the systemic analysis of transport issues, methods from many fields of science are used, including queuing theory, graph theory, linear programming theory, game theory and operational research theory. In addition, methods and algorithms of multi-criteria decision support are used increasingly often to solve transport issues. Extensive information connected with selected methods of optimization (single- and multi-criteria methods) and the evaluation of transport systems is included in this work [4].

When an enterprise has only one distribution centre from which it supplies goods to the market, the determination of the optimum transport plan is concerned with identification of the shortest route between particular purchasers of goods. However, if there are multiple distribution centres, the aim of the task is to determine from which centre the recipient should be supplied and in which sequence recipients should be "served" in order to minimize transport costs, at the same time considering the amount of the resource available in each distribution centre. In such a case, the optimum solution of the problem is a very difficult task because of its computational complexity. Thus, the demand for the delivery of optimum solutions via algorithm to solve such problems have been slackening for some time; instead, approximate solutions that would be acceptably close to optimal are being sought. The weakening of the optimality condition often helps to reduce the time of calculations from exponential time to polynomial time, with a slight loss of optimality. Approximate algorithms based on artificial intelligence are the only realistic method of solving computationally difficult large problems.

There are commonly known methods that help to determine optimum routes between purchasers of goods, called the "Travelling Salesman Problem" in the literature, such as: Little's algorithm, the insertion algorithm, the 2-opt and the 3-opt local search algorithm and others [6], [7]. These methods provide solutions only when all recipients are supplied from one distribution centre. The Travelling Salesman Problem (TSP) is one of the oldest optimization problems occurring in transport activity. In network theory, in order to solve a TSP, it is necessary to find the shortest Hamiltonian cycle in the *n*-vertex complete network. Finding the proper Hamiltonian cycle (often called the Hamiltonian path) is a very difficult task in terms of computation. This task is regarded as an NP-hard problem; a method in which the time to solve the problem would be proportional to the polynomial of the *n* variable has not yet been found. Solution time increases exponentially with an increase in the number of customers, due to which various kinds of heuristic, evolutionary and genetic algorithms allowing for considerable reduction of the calculation time are increasingly sought and used to solve delivery problems [8]. In these solutions the condition of optimality is not specified, but obtained solutions are close to optimal. A more developed version of TSP is the Multiple Travelling Salesman Problem (MTSP). In MTSP the task is performed by many travelling salesmen, but each of them starts and finishes his path in the same depot.

In real logistic systems there are many restrictions not considered in typical algorithms, such as imposed delivery times, the capacity of means of transport, or dimensions of loading units [9]. This results in a continuous search for new algorithms supporting the performance of complex tasks of logistic operators.

2. DELIVERY PROBLEMS – AN OVERVIEW OF SOLUTION METHODS

The most typical delivery problem – the Vehicle Routing Problem (VRP) – is concerned with the minimization of costs of transport from one depot to any number of recipients (customers). Since 1987, when the problem was defined by Solomon [10], intensive research has been carried on for the purpose of elaborating algorithms that would represent the changing requirements of suppliers and recipients – participants in the supply chain. Initially, the solution of the problem was sought with the use of tabu search heuristics proposed by Cordeau and Solomon [11] and developed by, among others, Nowicki [12]. The applied tabu search consists of searching for the optimal solution within the space of all possible solutions. The algorithm assumes the existence of tabu (i.e. prohibited) movements, so it avoids repeating solutions found earlier. In contrast to MTSP, in the VRP task each means of transport (vehicle) has a defined load capacity and the customer has a defined demand. This means that the total demand of customers cannot exceed the load capacity of the vehicles. Speaking in most general terms, the solution is based on the minimisation of the number of vehicles, the number of routes and the total length of routes.

Currently there are many different kinds of delivery problems. The most commonly known are:

- Capacity Vehicle Routing Problem (CVRP) – all vehicles have identical load capacities,
- Open Vehicle Routing Problem (OVRP) – the task is finished after the last customer is served; the vehicle does not return to the original depot,
- Vehicle Routing Problem with Time Windows (VRPTW) – an extension of CVRP with time windows for each customer (the time interval in which the customer can be served),
- Site-Dependent Vehicle Routing Problem (SDVRP) – an extension of CVRP;

vehicles have different load capacities, which causes restrictions in service to some customers,

- Multi Depot Vehicle Routing Problem (MDVRP) – there are many central depots,
- Multi Depot Vehicle Routing Problem with Time Windows (MDVRPTW),
- Stochastic Vehicle Routing Problem (SVRP),
- Vehicle Routing Problem with Stochastic Demands (VRPSD),
- Rich Delivery Problem with Time Windows (RDPTW) – customers' windows are defined, the weight of one loading unit is defined, and vehicles have different load capacities.

In other versions of the RDPTW problem [Woch] the existence of many depots is taken into account, loading units and vehicles are described additionally by overall dimensions and each vehicle collects cargo only from one depot, at which it starts and finishes the performance of the logistic task. To solve the problem, it is necessary only to minimize the following three criteria:

- Number of vehicles,
- Cumulative length of routes (paths) of all vehicles,
- Cost of task performance (or defined cost index).

Currently various hybrid algorithms are used for solving such complex problems. These are most often genetic algorithms, evolutionary algorithms, Adaptive Large Neighbourhood Searches (ALNS), Simulated Annealing and Ant Colony Systems. A task can be considered in many ways, depending on the problem to be solved.

3. EXAMPLES OF SOLUTIONS

3.1. TRANSPORT AND PRODUCTION TASK (DELIVERY AND INCINERATION OF MEDICAL WASTE)

An enterprise processing a homogeneous material has m points where this material is collected and n facilities where this material is processed.

It is necessary to determine a plan of deliveries of material to each facility and processing of material in these facilities so that the total costs of transport and processing are minimal.

The following symbols have been adopted:

i - collection point number (supplier number),
j - processing facility number (recipient number),
x_{ij} - quantity of material sent from the supplier i to the recipient j,
x_j - quantity of material processed by the recipient j,
a_i - quantity of material that must be sent by the supplier i,
c_{ij} - unit cost of transport from the supplier i to the recipient j,
$f_j(x_j)$ - cost of processing of x_j units of material in the facility j (at the site of the recipient j).

It was also assumed that the convex cost

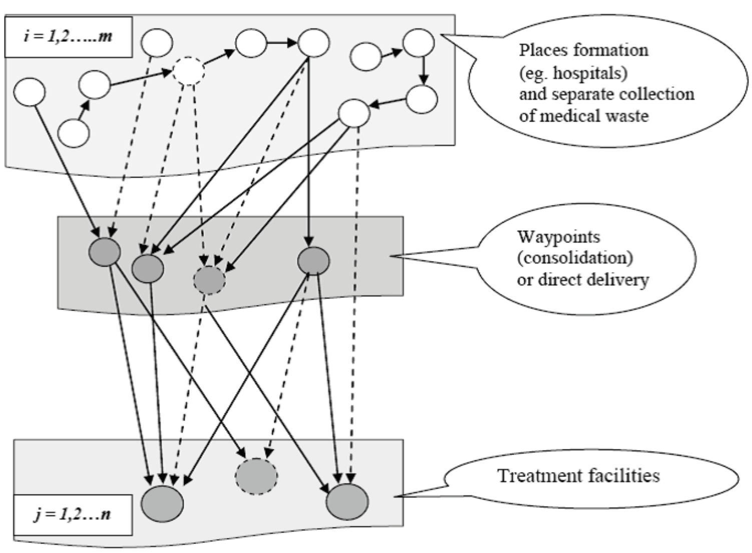

Fig. 1. Scheme of connections in the TPT task

function f_i is a second-degree polynomial in the following form:

$$f_j(x_j) = c_j x_j + e_j x_j^{\,2}, \quad c_j, e_j > 0 \tag{1}$$

where:

c_j - describes the minimum unit cost of processing,

e_j - determines the rate of growth of the unit cost.

The problem of determination of the optimal plan of deliveries and processing of material can be presented in the form of a non-linear decision-making task. The aim is to find such values for variables x_{ij} and x_j that:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{j=1}^{n} f_j(x_j) \longrightarrow min, \tag{2}$$

with conditions:

$$\sum_{j=1}^{n} x_{ij} = a_i; \quad (i=1,\ldots,m), \tag{3}$$

$$\sum_{i=1}^{m} x_{ij} = x_j; \quad (j=1,\ldots,n), \tag{4}$$

$$x_{ij} \geq 0, \; x_j \geq 0; \; (i=1,\ldots,m; \, j=1,\ldots,n). \tag{5}$$

The function of objective (2) minimizes total transport and processing costs. The task is a quadratic programming task with a special transport structure. It can be solved by applying the marginal cost equalling (MCE) algorithm.

Marginal cost is the cost incurred by a producer in connection with an increase in the value of production of a given commodity by one unit. It constitutes the growth of total costs connected with the production of one additional unit of goods. If a facility increases its production by one unit, total production costs will increase. The difference in the amount of costs incurred by the producer previously and the amount of costs incurred after the increase in production constitutes marginal cost. It is, therefore, the cost of production of an additional unit of goods. The concept of marginal cost can also be formulated with regard to the consumer; in such a case, it means the cost of acquisition of an additional unit of goods.

Marginal cost is an important microeconomic concept. It has been observed that marginal costs initially decrease along with an increase in production for typical economic processes, until the technological minimum is achieved. However, further expansion of production beyond the technological minimum entails increasingly higher unit costs at successive stages of production growth, which in turn increases marginal costs. This observation is important in the microeconomic analysis of the producer's behaviours and the determination of the optimal production level. According to economic theory, marginal cost cannot be negative. This means that increases in production cannot entail any decrease in total costs.

The MCE method is based on:

a) the determination of the best possible and acceptable initial solution,
b) improvement of successive solutions X^1, X^2, **…,** through shifts equalling marginal costs.

The sequence of successive solutions X^1, X^2, …, X^r, **…,** that we obtain by using the MCE method is not necessarily finite. Thus, it is imperative to cease calculations at a certain point. However, the final solution should not differ too much (in terms of the value of the function of the objective) from the optimal solution.

The MCE algorithm is based on the performance of the following steps:

1. Determination of the initial solution:
   a) a route with a minimal marginal cost is determined for the supplier i (i = 1,…, m),
   b) the entire supply of the supplier i is placed on the selected route,
   c) the marginal cost is updated in the column with the selected route.
2. Checking whether the current solution X^r fulfils the criterion of optimality. If yes, the final solution is optimal. If no, move on to step 3.
3. Checking whether the solution X^r is ε-accurate. If yes, computations are finished. If no, move on to step 4.
4. Improvement of the solution through cost-equalling shifts and return to step 2.

After the determination of the solution X^r and the marginal cost matrix K^r, the difference between the maximal incurred cost and the minimal cost is determined for each supplier.

The task is solved with the use of the MCE computer program developed in MatLab and provided with a graphic user interface [13]. An example of transport and production task data for the solution of the transport problem and the incineration of medical waste in Poland's Podkarpackie province is presented in Table 1.

Table 1. Unit transport costs, supply and demand

Suppliers	**Supply** A_i [Mg]	**Incinerating plants**					
		Variant v1		**Variant v2**		**Variant v3**	
		S1 (RZ)	**S2 (JE)**	**S1 (RZ)**	**S2 (JE)**	**S1 (RZ)**	**S2 (JE)**
D1 (Rzeszów)	500	5	60	5	60	5	60
D2 (Dębica)	80	40	60	40	60	40	60
D3 (Jasło)	200	70	15	70	15	70	15
D4 (Krosno)	400	70	5	70	5	70	5
D5 (Sanok)	120	100	50	100	50	100	50
D6 (Przemyśl)	300	100	80	80	100	80	100
Supply B_j [Mg]	**1600**	**700**	**1500**	**1000**	**1500**	**700**	**1500**

A number of variants were considered in these calculations: the supply of waste remained constant, whereas both the demand (processing capacity) and processing costs (description of the function) changed. For instance, the following demand was assumed for the variant v1 according to Table 1: 700 Mg/year for incinerating plant S1 and 1,500 Mg/year for incinerating plant S2. Moreover, on the basis of performed studies, the following forms of processing functions were assumed:

$$f_1(x_1) = 15x_1 + 0.2\,x_1^2\,,$$
$$f_2(x_2) = 15x_2 + 0.1\,x_2^2\,.$$

If the solution is optimal or ε-accurate, the result of the calculations is a table presenting the following information:

- volume of processing in individual facilities,
- total waste transport and processing costs,
- waste transport cost,
- waste processing cost,
- average costs,
- marginal costs,
- waste distribution method.

Results of calculations obtained for the variant v1 are presented in Figure 2 (the final results screen).

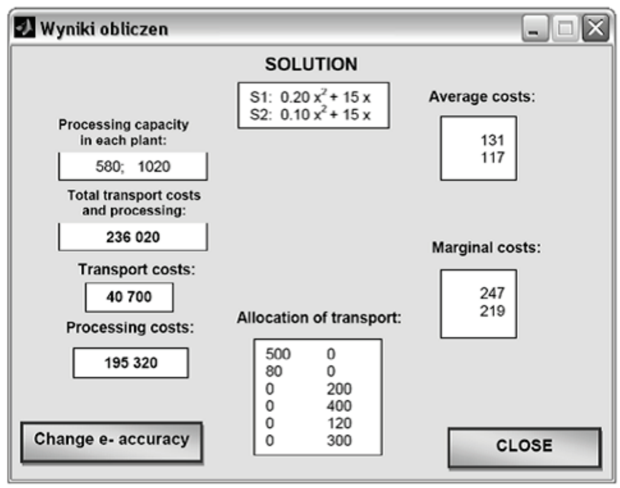

Fig. 2. Results of computations in the MCE program for the variant v1

An approach using the solution of a production and transport task with the convex cost function proposed in this work should make it easier to make decisions in waste management processes.

3.2. TRAVELLING SALESMAN PROBLEM FOR A NUMBER OF DISTRIBUTION CENTRES

If an enterprise has several distribution centres (depots), the problem of the logistic operator lies in the allocation of relevant recipients to suppliers such that the total demand of recipients does not exceed the supply capacities of particular suppliers and that the entire transport cost is as low as possible.

For the purposes of this study, this problem was called the *"Travelling Salesman Problem for a* *number of distribution centres"* (in abbreviated form: *TSP-ZT*), and its graphic interpretation is presented in Figure 3. The TSP-ZT task corresponds to the Multi Depot Vehicle Routing Prblem (MDVRP).

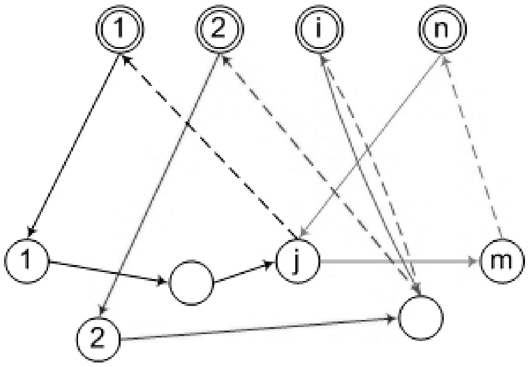

Fig. 3. A graph of the structure of the TSP for a number of distribution centres

There are n suppliers, each of which has a_i commodity units, respectively *(i=1, 2,...,n)*, and m recipients, each of which reports demand for goods in the quantity of b_j units *(j=1, 2,...,m)*. Each supplier can supply any recipient and, conversely, each recipient can receive goods from any supplier. Each travelling salesman can depart from his base only once and must return to this base after delivering goods to recipients. Each town can be visited by any supplier only once, but towns can be visited in any order. It is assumed that distances and, consequently, costs of transport between each pair of places are known, that the sum of deliveries to recipients is equal to their demand and that the sum of deliveries sent by suppliers does not exceed the amount suppliers have at their disposal. The total transport cost is equal to the sum of transport costs of each supplier.

Thus, for this transport task it is mainly necessary to find the minimum of the function:

$$min\ z = \sum_i \sum_j \sum_k (c_{0ki}\ x_{0ki} + c_{j0i}\ x_{j0i} + c_{jk}\ x_{jki}) \quad (6)$$

where:

- i – the supplier belonging to the set of suppliers *N,*
- j, k – recipients between which goods are transported, belonging to the set of recipients *M,*
- $00c_{jk}$ – cost of transport between recipients j and k,
- c_{0ki} – cost of transport between the supplier i and the recipient k,
- c_{j0i} – cost of transport between the recipient j and the supplier i,
- x_{jki} – the binary variable specifying if the supplier i transports goods between recipients j and k,
- x_{0ki} – the binary variable specifying if goods are transported between the supplier i and the recipient k,
- x_{j0i} – the binary variable specifying if goods are transported between the recipient j and the supplier i.

For the purpose of solving the task of a travelling salesman serving a number of distribution centres, a Vitrans computer program was developed with the use of Borland C++ Builder, a programming tool designed for creating applications in C++ language [14]. Figure 4 presents the main window of the application with sample data (4 distribution centres: Krakow, Wroclaw, Lodz, Warsaw and 25 customers in Poland).

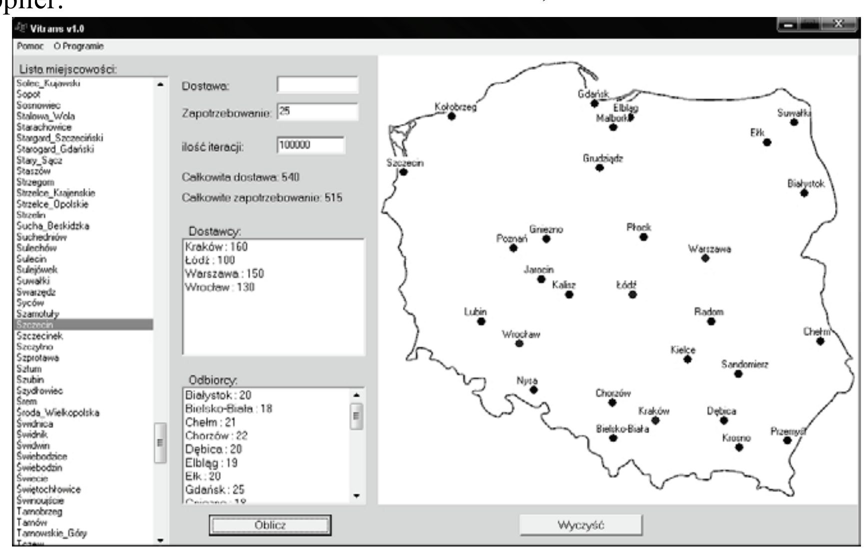

Fig. 4. The main window of the program with entered data

Based on properties of evolutionary algorithms, Vitrans makes it possible to determine transport tasks for a standard TSP and for its extended version with a number of distribution centres (TSP-ZT).

The adaptation of an individual is calculated on the basis of information with which every individual is provided (genotype, phenotype). For the problem under consideration, the genotype stands for places representing suppliers and quantity of goods, which totalled 515 units, was assigned to each recipient. After the distribution of goods among recipients, a stock of 25 units of goods remained in depots, of which 11 units remained in Krakow, 5 in Lodz, 6 in Warsaw and 3 in Wroclaw. The total length of the routes travelled by salesmen was 3,449.17 km.

The solution of the task for 10^5 iterations is presented in Figure 5.

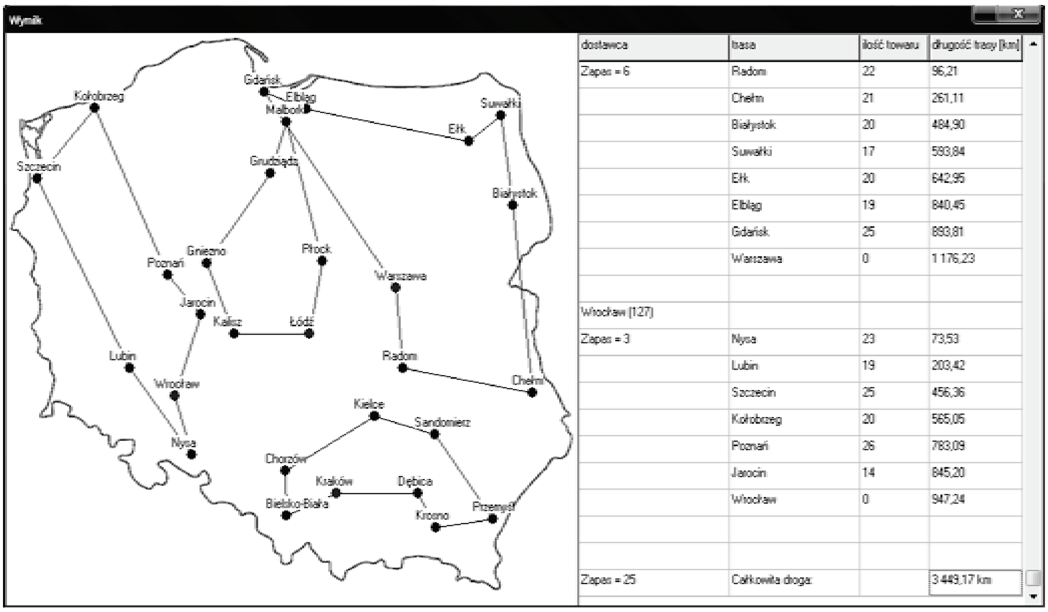

Fig. 5. The window of the program with the solution for 10^5 iterations.

recipients of goods, whereas the phenotype stands for connections (routes) between these places. In an evolutionary algorithm, a population consisting of one individual is processed. In the main loop, the basic individual is reproduced; consequently, its copy is created and stored in memory, while the basic individual is subjected to mutation processes. Mutation occurs in three stages. Each stage is repeated until the stop condition is fulfilled. The stopping of the loop at each stage of mutation contains an element of randomness; therefore, it is not possible to determine the number of repetitions of each stage, but only the probability of its occurrence.

For the purpose of checking the functioning of the Vitrans program and presenting the quality of results generated by the program, goods transport routes were determined for a certain data group (4 distribution centres, 25 recipients). The number of iterations of the program was set at 100,000 steps. The quantity of the entire resource that suppliers had at their disposal amounted to 540 loading units, of which Krakow had 160, Lodz 100, Warsaw 150 and Wroclaw 130. The required

For a small number of steps (100) the program generated a solution almost 30% worse than for 10^5 steps. However, a further increase of the number of iterations above 10^5 steps failed to generate a better solution, and in the case of 10^6 steps the solution was even slightly worse, which may suggest that any further increase of the number of steps, instead of producing a considerably better solution, will only extend the time of computations performed by the program. This may mean that the solution for which the total transport route is 3,449.17 km long is an optimal solution or is very close to such a solution.

4. CONCLUSION

The problem of determination of an optimum plan of goods transport between suppliers and recipients is an NP-hard problem, which means that there is no algorithm to solve this problem in polynomial time. When such large problems are solved, the fulfilment of the condition of provision of an optimal solution by the algorithm is abandoned and an approximate solution close to the optimal one is assumed. The use of heuristic optimization algorithms often helps to reduce the time of computation from exponential time to polynomial time, with a slight loss of optimality. The functions that are important for the elaboration of this class of algorithms are the initial generation function and the transition function. The transition function allows us to search the space of solutions. In order to bring solutions obtained as a result of the use of various genetic, evolutionary, heuristic and hybrid algorithms closer to the expectations of logistic operators, it is necessary to extend VRP delivery problems with parameters reflecting actual problems, e.g.:

- dimensions (logistic characteristics) of vehicles,
- dimensions of loading units (cargoes) in transport,
- costs (cost indexes) of vehicles in operation.

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